Tensor Networks, Dresden 2018, Tutorial Problems

1. Write a Julia program to use a Lanczos method to find the ground state of small, open Heisenberg spin chains (using $J = 1$) of even length $N = 2, \ldots, 12$. Splitting the system down the middle, perform an SVD that does the Schmidt decomposition on the ground state ψ; and calculate the entanglement entropy S. From this data make a plot of $S(N)$ versus N. Include in your plot the maximum possible S for that N, $\frac{N}{2} \ln 2$.

2. Use your Julia program to get the ground state for a pretty long chain, say $N = 16$. From the Schmidt decomposion for a split down the middle, form the approximate ground state that comes from truncating the Schmidt decomposition, keeping only m states, for m ranging from 1 up to $2^{N/2}$. After you make the approximation to the matrices, you can multiply them back together to get an approximation to the exact ground state. For each m, calculate (1) the truncation error, namely the sum of discarded squared singular values

$$\delta_m = \sum_{j=m+1}^{2^{N/2}} \lambda_j^2;$$

(2) the error in the ground state squared, i.e. vecnorm(psi-psiapprox)^2; (3) the error in the ground state energy from using the approximate ground state, using

$$E = \langle\psi|\, H \,|\psi\rangle \,/\, \langle\psi|\, \psi\rangle.$$

Make a plot of ΔE_m versus δ_m; you should find a roughly linear relationship. This plot is widely used in DMRG to extrapolate the energy to $m \to \infty$.

3. Take your ground state of problem 2, and find its MPS representation by repeatly SVD'ing it to break it up to site-tensors, with a bond dimension $m = 10$. Within the MPS representation, contracting tensors left to right, calculate the norm of the state. Reconstruct the full tensor representation of the wavefunction of the MPS, by sequentially multiplying up the site-tensors. Compare the energy of the MPS to the true ground state energy of the chain.