Tensor Network based approaches to Quantum Many-Body Systems
Max Planck Institute for the Physics of Complex Systems (MPIPKS)
Dresden, Germany, November, 2018

Pre-homework—to be done before the first lecture

1. Review the singular value decomposition, which is sometimes left out of undergraduate linear algebra classes. Exercise: given a rectangular matrix ψ_{ij}, define its left density matrix as $\rho^L_{ii'} = \sum_j \psi_{ij}\psi^*_{i'j}$. Find the relation between the eigenvalues of ρ^L and the singular values of ψ. Do the same for a similarly defined ρ^R.

2. Download Julia on your laptop (from julialang.org) and read through the beginning documentation and write some simple programs. Most of the school will be done using Julia.

3. Learn about the Lanczos method for iteratively finding the largest or smallest eigenvector and eigenvalue of a Hermitian matrix. In Julia, the package KrylovKit.jl, written by Jutho Haegeman, has Lanczos implemented, so you don't need to write your own function. Another similar package is Arpack.jl. For a 4 site Heisenberg model (spin 1/2), with Hamiltonian

$$H = \sum_{i=1}^{3} \vec{S}_i \cdot \vec{S}_{i+1} \tag{1}$$

write a Lanczos exact diagonalization in Julia to find the ground state of this system. (Note the use of open boundary conditions.)