APPLYING MPS TO LATTICE GAUGE THEORIES

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In this talk…

Why using TNS/MPS for LGT?

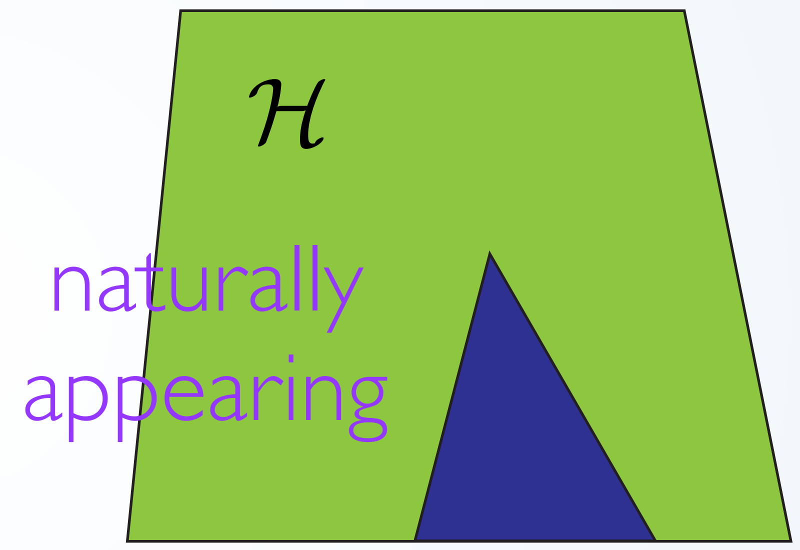
testbench:
Schwinger model

- spectral calculations
- finite temperature
- real-time
- chemical potential

WHY SHOULD TNS BE USEFUL?

States appearing in Nature are peculiar

State at random from Hilbert space is not close to product



We look for the particular "*corner*" of the Hilbert space

- TNS = Tensor Network States

FINDING A GOOD ANSATZ

Which properties characterize ground states of relevant Hamiltonians?

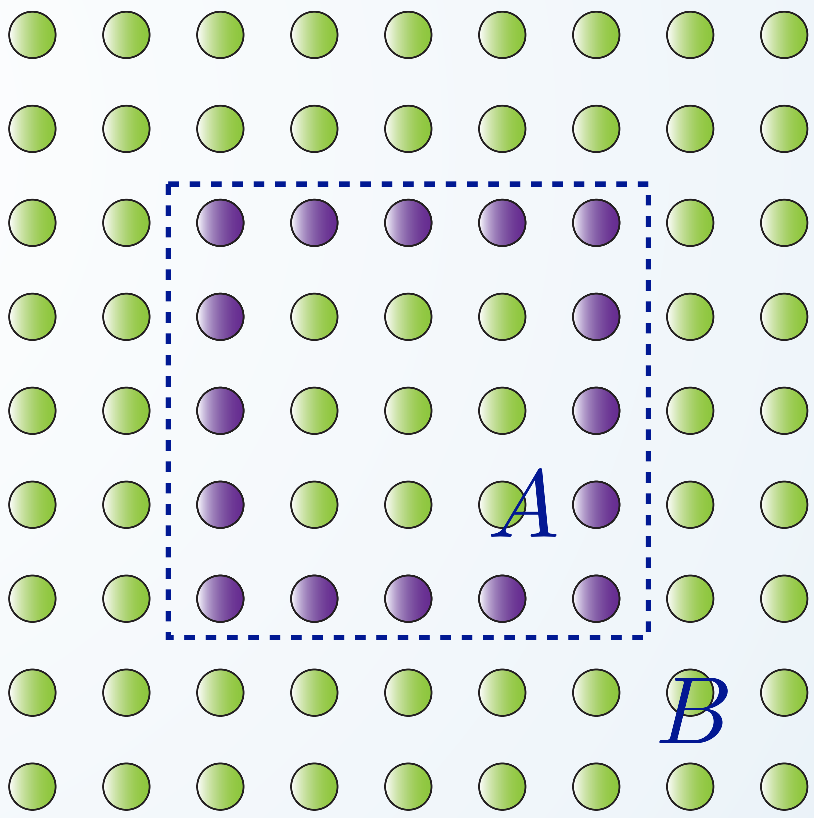
local gapped Hamiltonians

have ground states

with little entanglement

$S_{A_{\max}} \propto |\partial A|$ Hastings 2007

Area law



FINDING A GOOD ANSATZ

Which properties characterize ground states of relevant Hamiltonians?

local gapped Hamiltonians

have ground states

with little entanglement

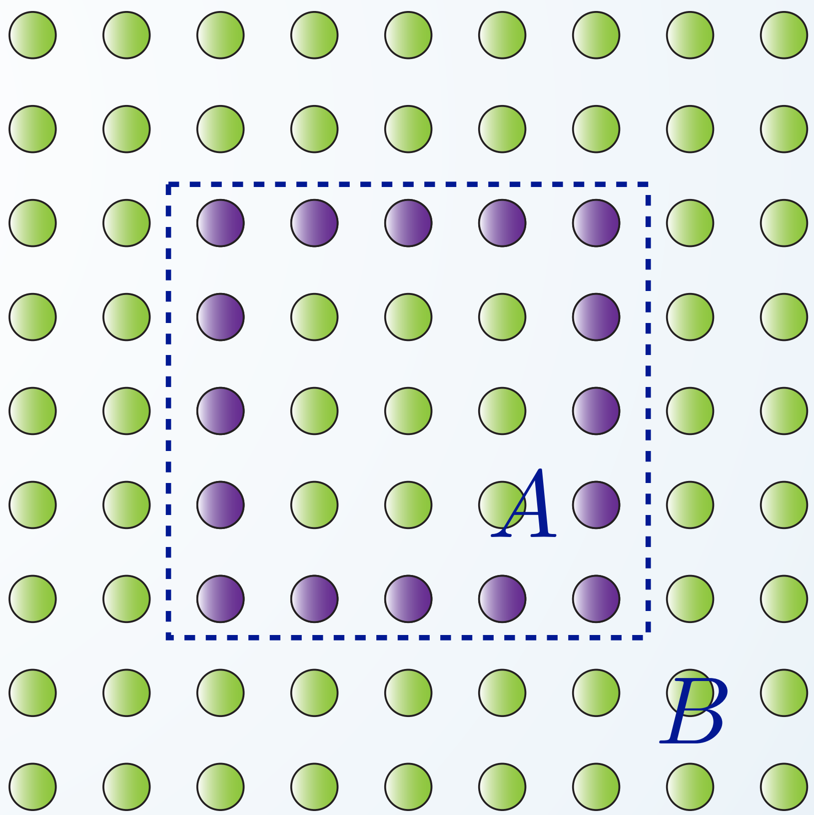
$$S_{A_{\max}} \propto |\partial A|$$

Hastings 2007

in 1D critical systems, logarithmic corrections

$$S_{A_{\max}} \propto |\partial A| \log |A|$$

Area law

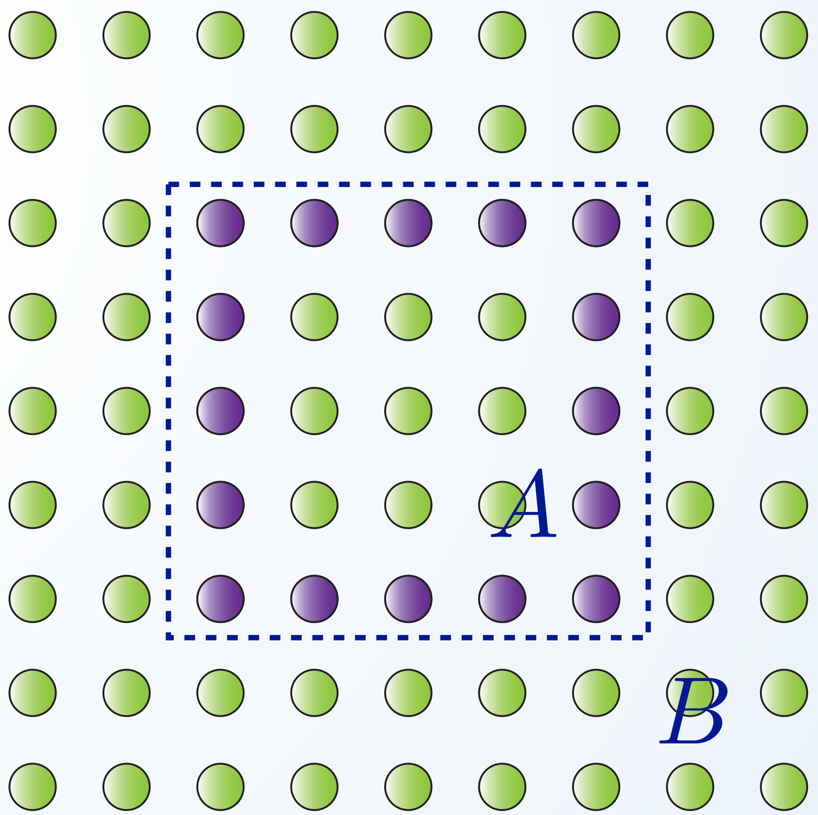


Calabrese, Cardy 2004

Wolf 2006

FINDING A GOOD ANSATZ

Area law



MPS and PEPS satisfy the area law by construction

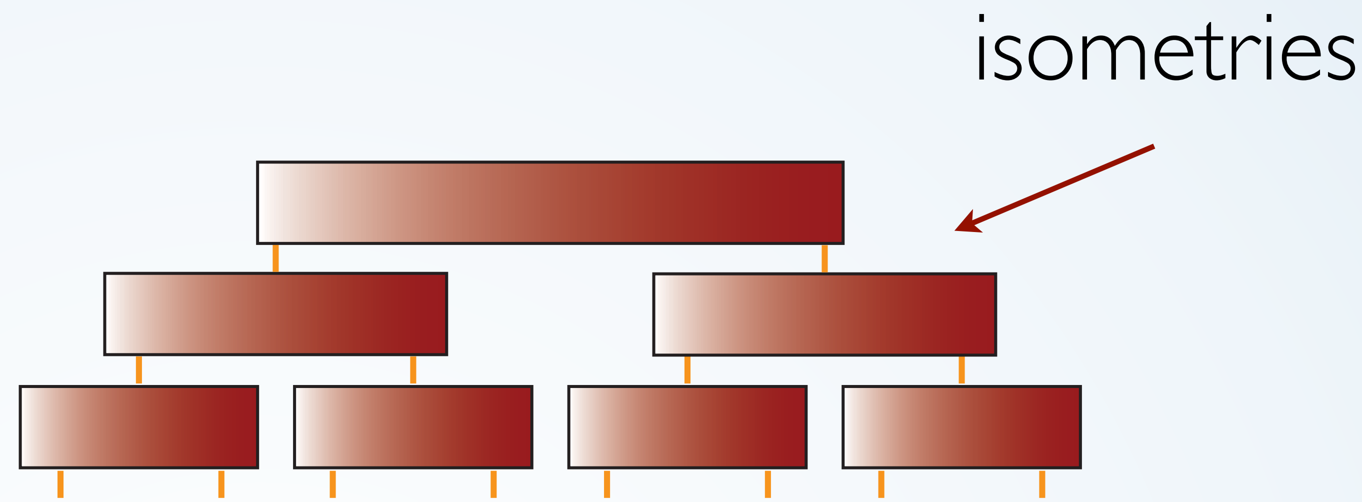
TNS = entanglement based ansatz

TNS = entanglement based ansatz

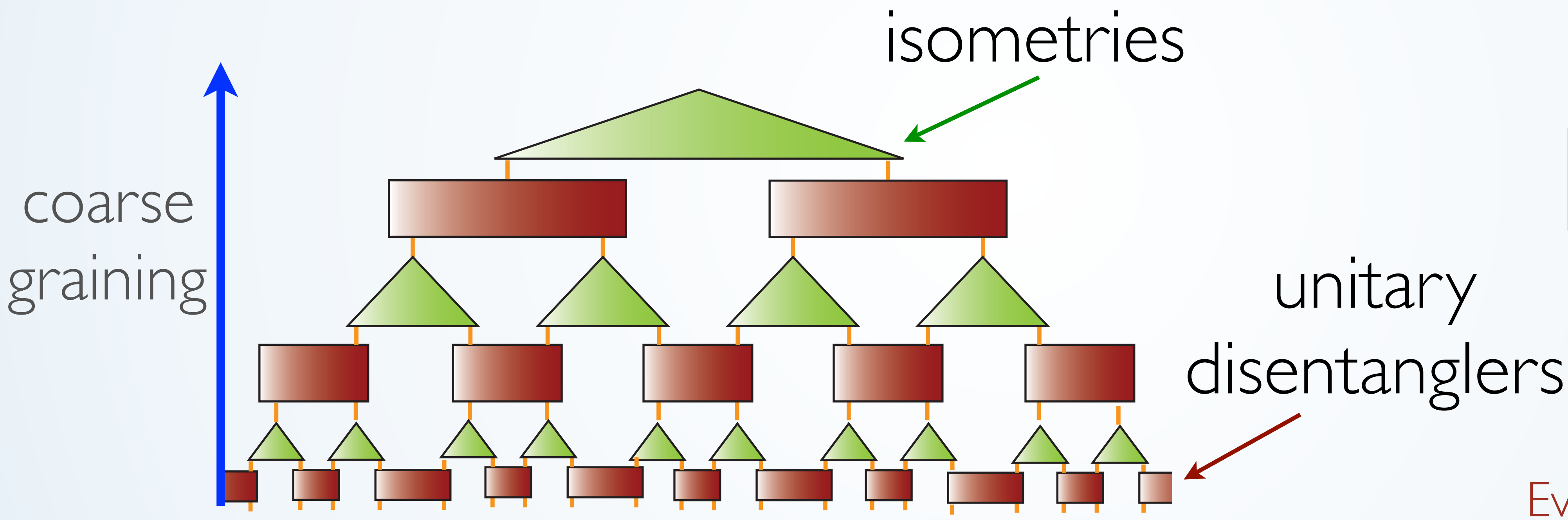
OTHER TNS

TTS

Shi, Duan, Vidal, PRA 2006



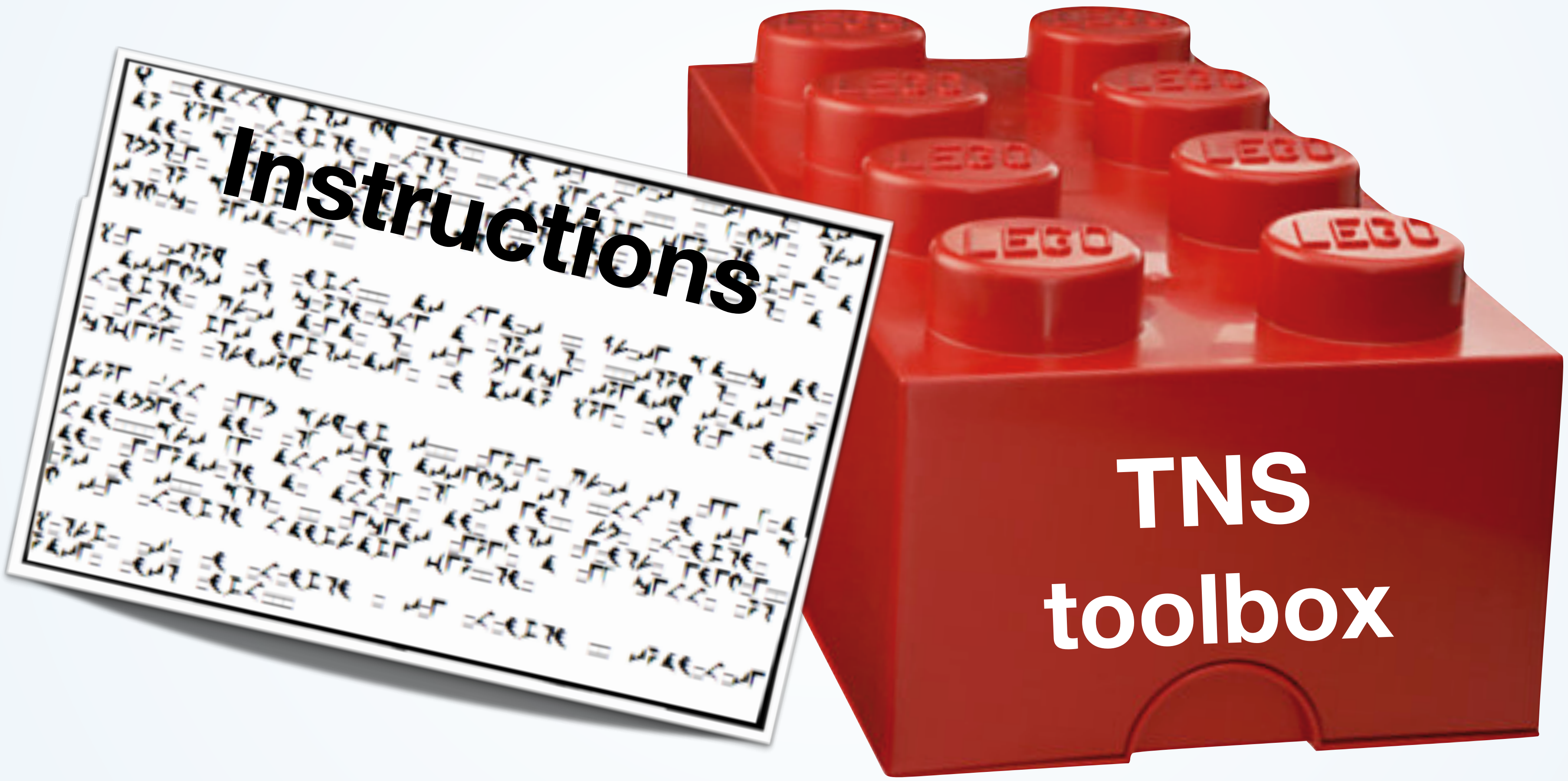
MERA



Vidal PRL 2007
Evenbly, Vidal, PRB2009

efficient contraction

violate area law logarithmically (in 1D) in principle, they can all be used for LGT simulations



start with the simplest case: 1D (1+1) LGT

MPS

MPS PROPERTIES

- MPS = Matrix Product States

MPS

good approximation of ground states

Verstraete, Cirac, PRB 2006

Hastings, J. Stat. Phys 2007

gapped finite range Hamiltonian ⇒
area law (ground state)

Cramer, Eisert, Plenio, RMP 2009

efficient calculation of expectation values

exponentially decaying correlations

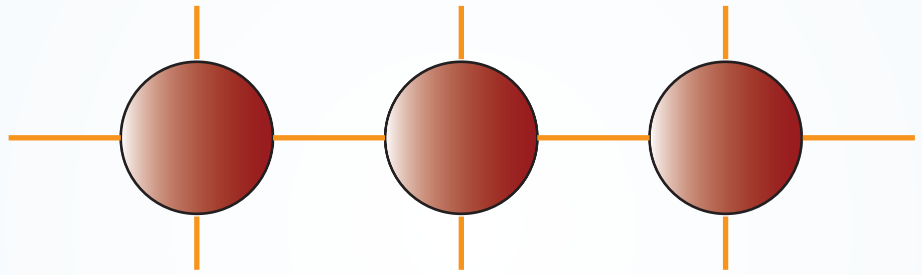
can be prepared efficiently

ALSO FOR OPERATORS

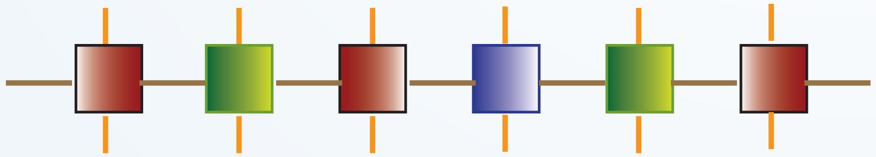
MPO

- MPO = Matrix Product Operator

Same kind of ansazt for operators



$$\hat{M} = \sum_{i_1, j_1 \dots i_N, j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \dots M_N^{i_N j_N}) |i_1 \dots i_N\rangle\langle j_1 \dots j_N|$$



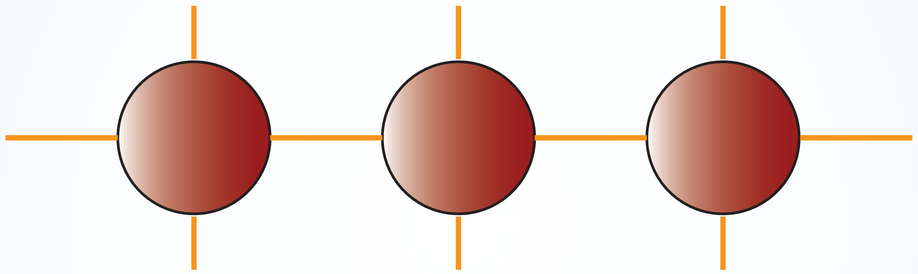
mixed states, H and $U(t)$

efficient exact MPO representation for local, NN, ...

MIXED STATES

- MPDO = Matrix Product Density Operator

Use for density operators



need some properties

$$\rho = \sum_{i_1,j_1\ldots i_N,j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \ldots M_N^{i_N j_N})|i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

can we impose them *locally*?

$\rho = \rho^\dagger$ ✔

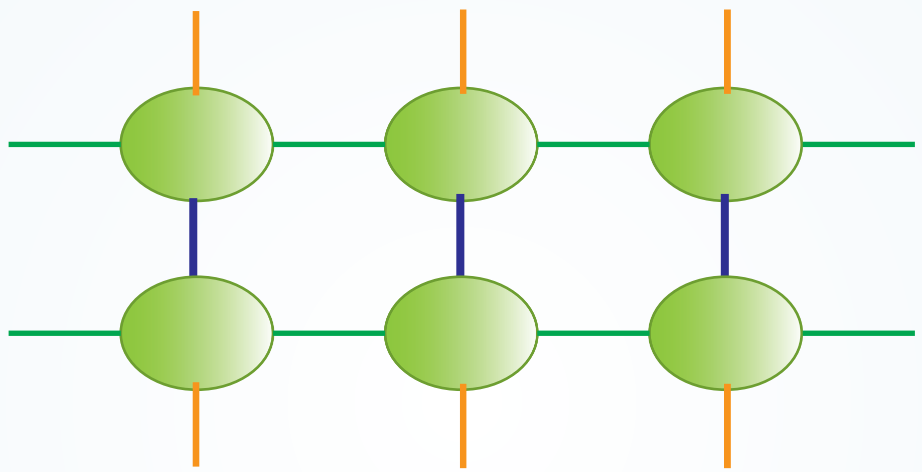
$\mathrm{tr}\rho = 1$ ✔

$\rho \geq 0$ ✘

MIXED STATES

- MPDO = Matrix Product Density Operator

purification



need some properties

$$\rho = \sum_{i_1,j_1\ldots i_N,j_N} \mathrm{tr}(M_1^{i_1 j_1} M_2^{i_2 j_2} \ldots M_N^{i_N j_N})|i_1 \ldots i_N\rangle\langle j_1 \ldots j_N|$$

can we impose them *locally*?

$\rho = \rho^\dagger$ ✓

$\mathrm{tr}\rho = 1$ ✓

$\rho \geq 0$

in a way

$$\rho_S = \mathrm{tr}_A|\Psi_{SA}\rangle\langle\Psi_{SA}|$$

Verstraete et al., PRL 2004

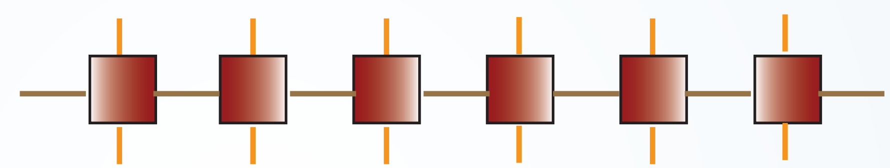
Zwolak and Vidal, PRL 2004

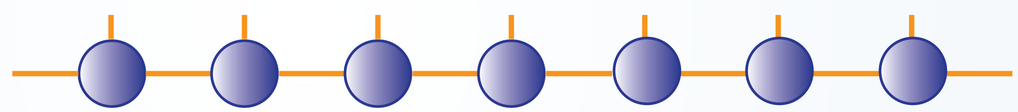
WHAT CAN WE DO WITH THEM?

BASIC ALGORITHMS

variational minimization of energy

local Hamiltonian

$H =$ 

$|E_0\rangle \simeq$ 

ground state
excitations

apply local operators → simulate time evolution

imaginary time → ground state
thermal state

alternatively: TDVP

REGARDING DYNAMICS

BASIC ALGORITHMS

Simulate time evolution

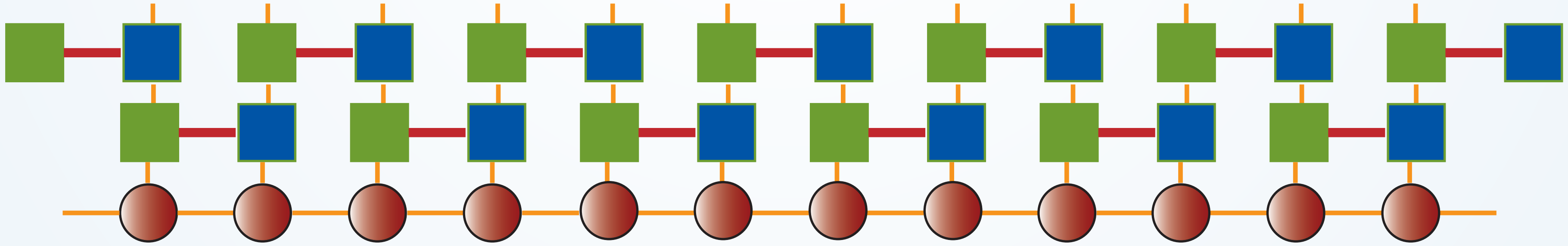
$$U(t) \to [U(\delta)]^M$$

apply evolution step

$$H = H_e + H_o$$

$$U(\delta) = e^{-iH_e\delta} e^{-iH_o\delta}$$

Suzuki-Trotter



TEBD
t-DMRG

alternatively: TDVP

Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

Haegeman et al, PRL 2011

Entanglement growth in non-equilibrium scenarios limits the applicability of MPS

ALSO FOR MIXED STATES

MIXED STATES

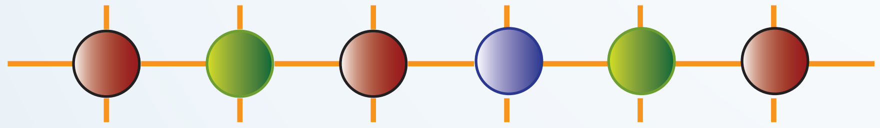
- MPO = Matrix Product Operator

Similar problems can be attacked

equilibrium → thermal states

imaginary time evolution

time-dependent → real time evolution



unitary $\rho(t) = U(t)\rho(0)U(t)^\dagger$

non-unitary $$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho)$$

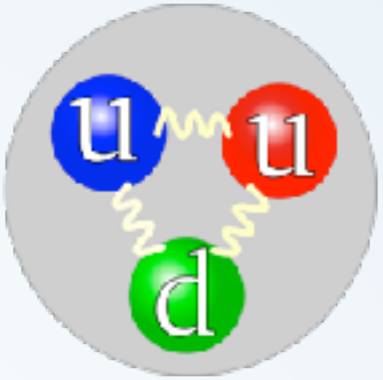
Verstraete et al., PRL 2004
Prosen, Znidaric PRL 2008
Cai, Barthel, PRL 2013,...

TNS FOR LGT???

Motivation for LGT: QCD

Wilson, 1974

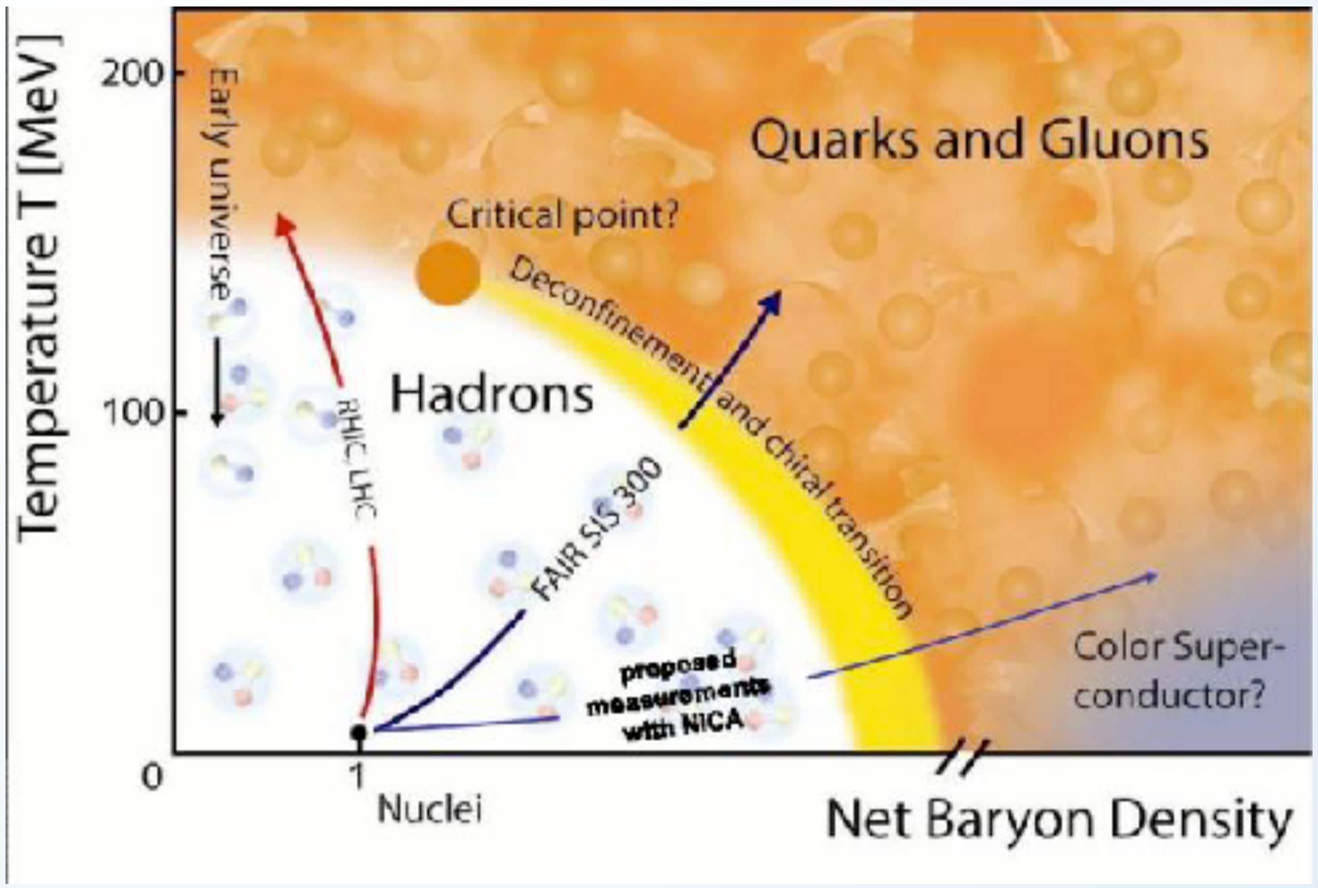
non perturbative at low energy



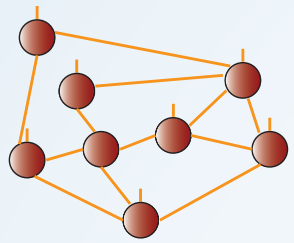
LQCD

successful spectral calculations

limitations: time, finite density



WHY?



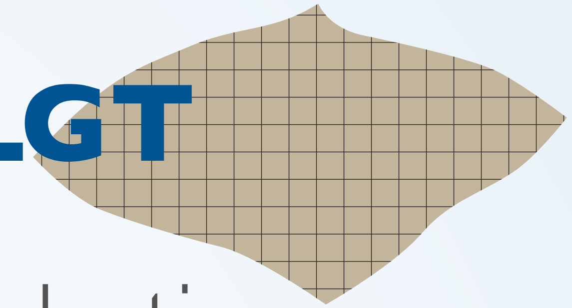
TNS

Non-perturbative for Hamiltonian systems

Extremely successful for 1D systems (MPS)

Promising improvements for higher dimensions

ground states
low-lying excitations
thermal states
time evolution



LGT

Non-perturbative way of solving QFT (QCD)

Mostly path-integral formalism & MC

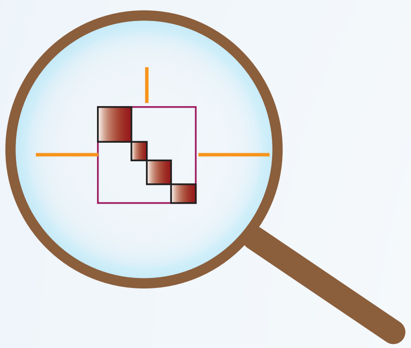
4D lattice

spectrum
finite T
big 3+1 dimensional
chemical potential
time evolution

How can we use TNS for LGT?

USING TNS FOR QMB

a formal approach



classifying tensors

constructing states

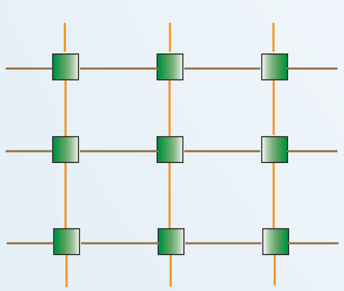
great descriptive power: phases, topological chiral states, anyons...

Chen et al PRB 2011
Schuch et al PRB 2011
Wahl et al PRL 2013; Yang et al PRL 2015
Haegeman et al, Nat. Comm. 2015

no sign problem

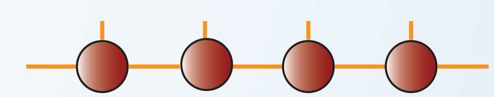
numerical algorithms

tensor networks describe partition functions (observables)



need to contract a TN
TRG approaches

Nishino, JPSJ 1995
Levin & Wen PRL 2008
Xie et al PRL2009; Zhao et al PRB 2010



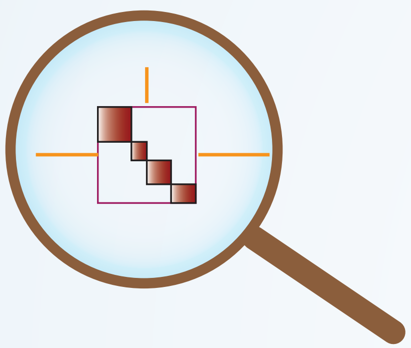
TNS as ansatz for the state

efficient algorithms for GS, low excited states, thermal, dynamics

White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

USING TNS FOR LGT

a formal approach



gauging the symmetry

explicitly invariant states

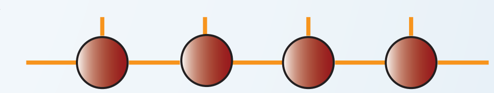
Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

general prescriptions, U(1), SU(2)

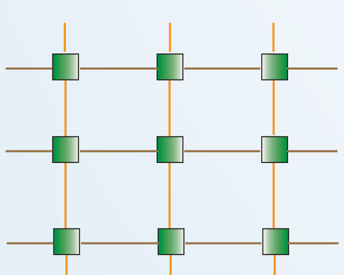
no sign problem

numerical algorithms

tensor networks describe
partition functions (observables)



TNS as ansatz for the state



TRG approaches to classical
and quantum models

Liu et al PRD 2013
Shimizu, Kuramashi, PRD 2014
Kawauchi, Takeda 2015

next...

Related: proposals for quantum simulation of LGT with ultracold atoms

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

TNS as ansatz for physical states

Earlier related work

DMRG on Schwinger model

Byrnes et al. PRD 2002

best precision for GS, vector

DMRG on $\lambda\Phi^4$

Sugihara NPB 2004

TN→extensions

time evolution, finite T

MPS for LGT Z_2

Sugihara JHEP 2005

Tagliacozzo PRB 2011

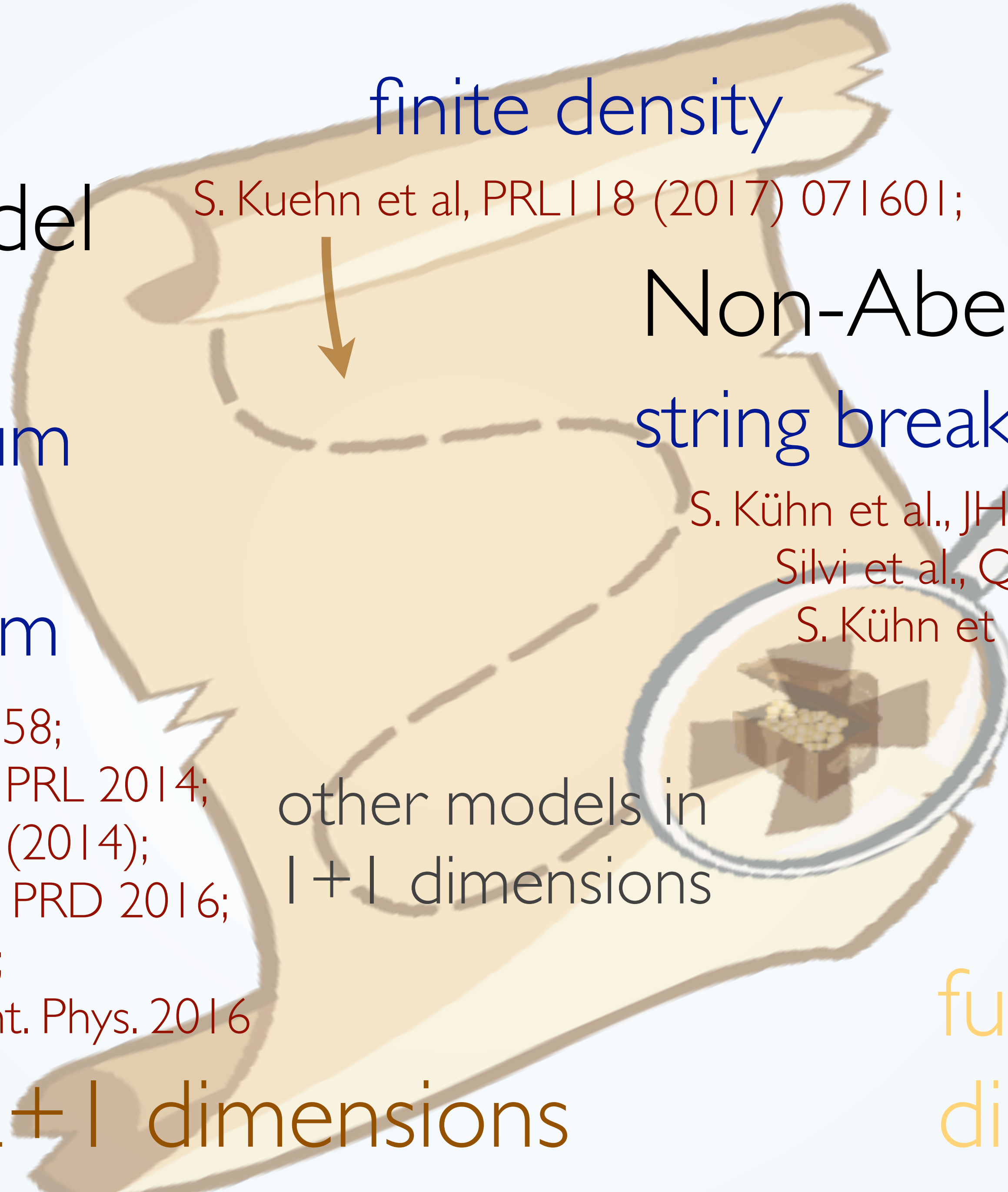
MPS for critical QFT

Milsted et al. 2013

TNS for classical gauge models

Meurice et al. 2013

current: an ongoing LGT-TNS roadmap...



Schwinger model U(1) in 1D

precise equilibrium simulations, feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
S. Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016

finite density

S. Kuehn et al, PRL118 (2017) 071601;

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

other models in 1+1 dimensions

2+1 dimensions

full LQCD in 3+1 dimensions

SCHWINGER MODEL AS LABORATORY

SCHWINGER MODEL

Schwinger '62

Simplest gauge theory with matter

QED in 1+1 dimensions

electrons & photons

Shows some of the features of *full* QCD

confinement → bound states (massive bosons)

fermion condensate

A testbench for lattice techniques

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

in 1+1 D single adimensional parameter ***m/g***

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$

U(1) gauge invariance

$$\Psi(x) \to e^{-ig\phi(x)}\Psi(x)$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \phi(x)$$

equations of motion

$$\partial_\alpha \frac{\partial \mathcal{L}}{\partial \Phi_{,\alpha}} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 \quad \text{for} \quad \Phi = A_\mu,\ \Psi$$

$$(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\,\Psi = 0$$

$$\partial_\mu F^{\mu\nu} = g\bar{\Psi}\gamma^\nu \Psi$$

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu\partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

in 1+1 D single adimensional parameter m/g

$$\partial_\mu F^{\mu\nu} = g\bar{\Psi}\gamma^\nu\Psi$$

Hamiltonian quantization $\mathcal{H} = \sum_\Phi \Pi_\Phi \dot{\Phi} - \mathcal{L}$

$$\Pi_\Phi = \frac{\partial\mathcal{L}}{\partial\dot{\Phi}}$$

no $\partial_0 A_0$ $\Rightarrow$ fix temporal gauge: $A_0 = 0$

A_0 not in H, but EoM imposes additional constraint

Gauss law

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

in 1+1 D single adimensional parameter ***m/g***

only component: electric field

$$E = -F^{01} = -\dot{A}^1 = \frac{\partial \mathcal{L}}{\partial \dot{A}_1}$$

canonical conjugates

$$\Rightarrow \quad \mathcal{H} = E\dot{A}_1 + i\bar{\Psi}\gamma_0\partial_0\Psi - \mathcal{L}$$

$$F^{\mu\nu}F_{\mu\nu} = -2E^2$$

constraint (Gauss law)

$$\partial_1 E = g\bar{\Psi}\gamma^0\Psi \quad \Rightarrow \quad E = g\int dx\, j_0(x) + \text{const}$$

fixed up to background field

SCHWINGER MODEL

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Hamiltonian formulation

$$A^0 = 0$$

$$E = -\dot{A}^1$$

$$H = \int dx \left[-i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1 A_1\Psi + m\bar{\Psi}\Psi + \tfrac{1}{2}E^2 \right]$$

fermion kinetic term

fermion-photon coupling

fermion mass

electrostatic energy

plus a constraint: $\partial_1 E = g\bar{\Psi}\gamma^0\Psi$ Gauss' law

quantization

$$\{\Psi_i(x), \Psi_j^\dagger(y)\} = \delta_{ij}\delta(x-y)$$
$$\{\Psi_i(x), \Psi_j(y)\} = 0$$
$$[A_1(x), E(y)] = i\delta(x-y)$$

discretize

SCHWINGER MODEL

on the lattice

discrete Hamiltonian (staggered) formulation

$$x$$

$$\begin{pmatrix} \Psi^{(1)}(x) \\ \Psi^{(2)}(x) \end{pmatrix}$$

SCHWINGER MODEL

on the lattice

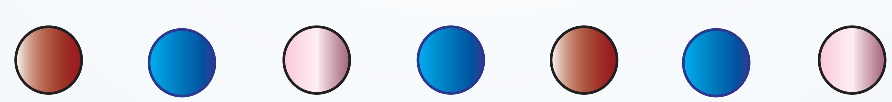
discrete Hamiltonian (staggered) formulation

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$

Φ_{2n} Φ_{2n+1}



$$\frac{1}{ga}\theta_n \rightarrow -A^1(x)$$

$$gL_n \rightarrow E(x)$$

$$[\theta_n, L_m] = ig\delta_{nm}$$

SCHWINGER MODEL

on the lattice

discrete Hamiltonian (staggered) formulation

x

$$U(x, x+\epsilon) = e^{ig\epsilon A_1(x)}$$

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$

SCHWINGER MODEL

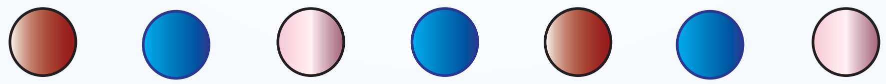
on the lattice

discrete Hamiltonian (staggered) formulation

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$



$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \to -A^1(x)$$

$$gL_n \to E(x)$$

$$[\theta_n, L_m] = ig\delta_{nm}$$

SCHWINGER MODEL

on the lattice

MPS formulation needs a (finite dimensional) basis for each dof

fermions

Fock space

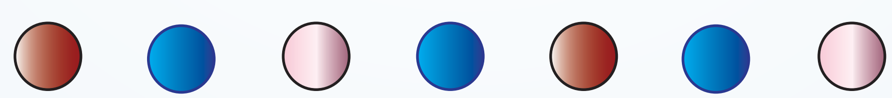
$\{|0\rangle, |1\rangle\}$

$|1\rangle = \Phi^\dagger |0\rangle$

$L|\ell\rangle = \ell|\ell\rangle \quad \ell \in \mathbb{Z}$

$e^{i\theta}|\ell\rangle = |\ell + 1\rangle$

$\Phi_{2n} \quad \Phi_{2n+1}$



$U_{n,n+1} = e^{i\theta_n}$

cQED

$[\theta_n, L_m] = ig\delta_{nm}$

We have the discrete Hamiltonian formulation…

SCHWINGER MODEL

on the lattice

discretized

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \text{h.c.}\right) + m\sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

plus constraint: Gauss' Law

spinless fermions

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2}\left[1 - (-1)^n\right]$$

choice

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

equivalently in terms of spins…

notice: not strictly necessary for TNS

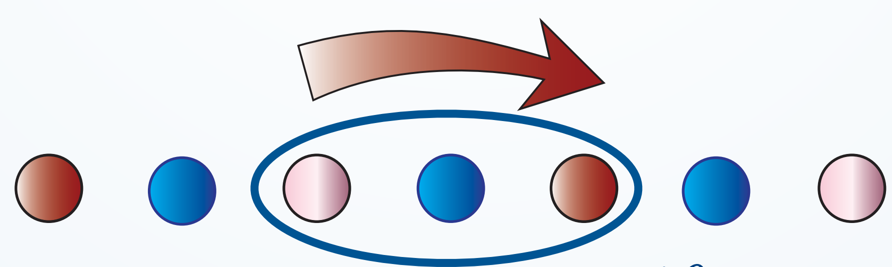
SCHWINGER MODEL

on the lattice

Jordan-Wigner → spin model

$$H = \frac{1}{2a}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$

$$+\frac{m}{2}\sum_n \left(1+(-1)^n\sigma_n^3\right) + \frac{ag^2}{2}\sum_n L_n^2$$



hopping

$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \to -A^1(x)$$

$$gL_n \to E(x)$$

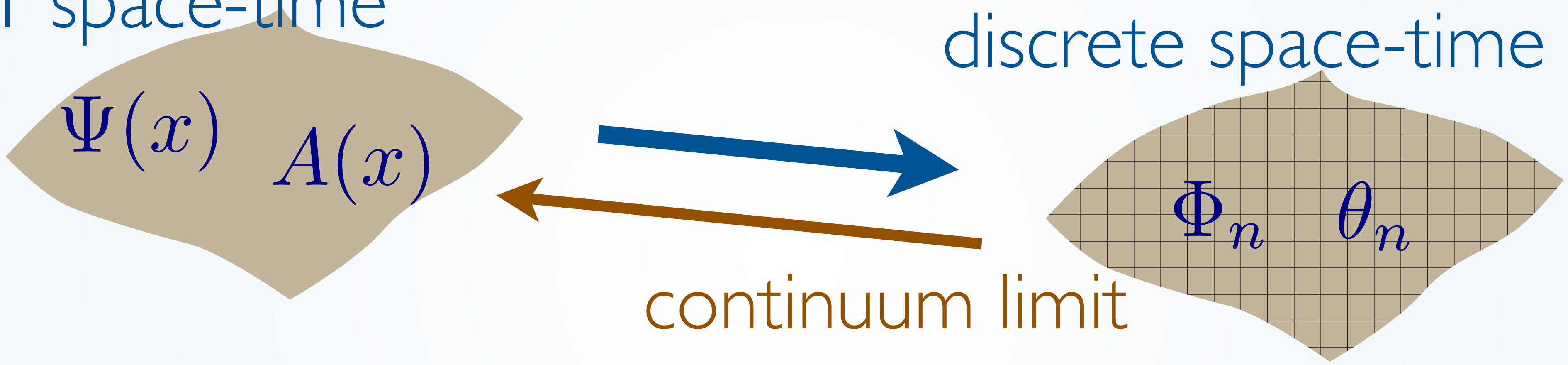
SCHWINGER MODEL

on the lattice

continuum QED

1+1 space-time

discrete space-time



fermions

photons

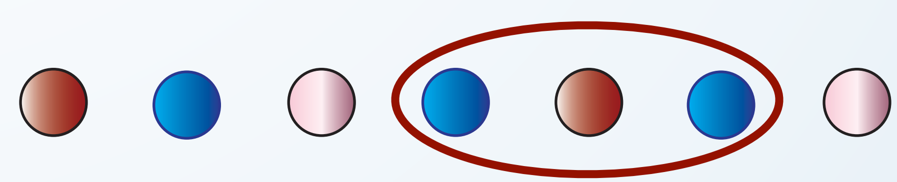
lattice spacing $ag \to 0$

spin Hamiltonian

+ Gauss law

fermion mass m/g

adimensional

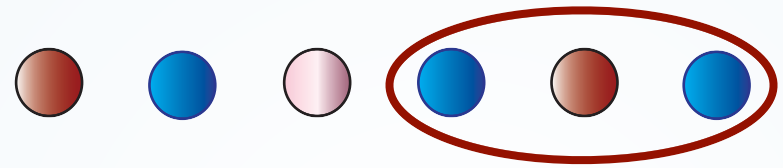


Hamiltonian

+ Gauss law

for a TNS we need a basis

SCHWINGER MODEL



basis $|\ldots s_e \;\ell\; s_o \;\ell\; s_e \;\ell\; s_o \ldots\rangle$

but Gauss' law fixes *photon* content

SCHWINGER MODEL

basis $|\ldots s_e \quad s_o \quad s_e \quad s_o \ldots\rangle$

but Gauss' law fixes *photon* content

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n} \sigma_n^3 + \ldots$$

$\Rightarrow$ eliminate gauge dof

introducing long range interactions

SCHWINGER MODEL

MPS representation for OPEN BOUNDARIES

$$|\ell_0 \ldots \boldsymbol{s}_e \ \boldsymbol{s}_o \ \boldsymbol{s}_e \ \boldsymbol{s}_o \ldots\rangle$$

non-local terms

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n}\sigma_n^3$$

Long range interactions

$$\sum_n \sum_{k<n} (N-n)\sigma_k^3\sigma_n^3$$

can be written as a MPO of D=5

both possibilities

SCHWINGER MODEL
on the lattice

basis $|\ldots s_e\,\ell\,s_o\,\ell\,s_e\,\ell\,s_o\ldots\rangle$ all terms are local

infinite dimensional: truncation

Gauss' law needs to be imposed

works by Buyens et al., PRL 2014; arXiv:1509.00246
Rico et al., PRL 2014; NJP 2014

or integrating out the gauge dof

basis $|\ell_0\ldots s_e\;s_o\;s_e\;s_o\ldots\rangle$ non-local terms

exact physical subspace

✗ does not generalize to bigger dimensions

SPECTRAL PROPERTIES

SCHWINGER MODEL

Bound states

vector First excited state over GS

Different C, P charges from GS

scalar In same C, P sector as GS

lattice breaks symmetries

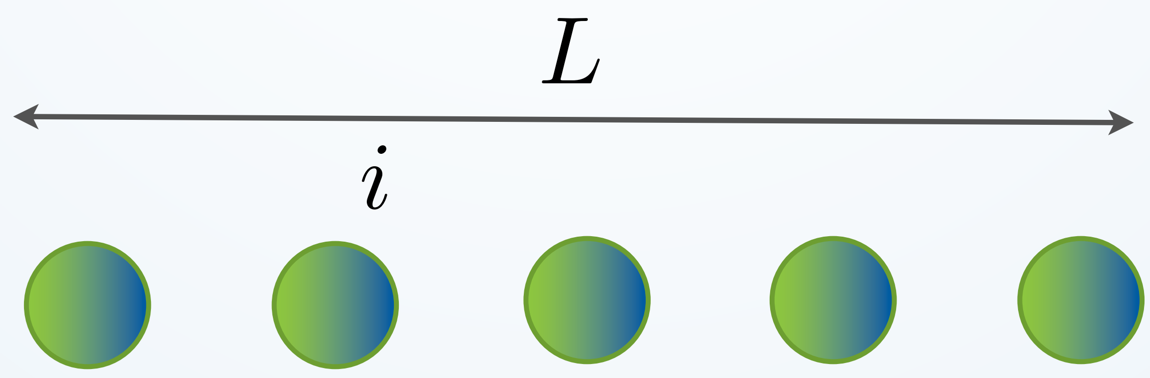
PBC → some remain

OBC → unique sector

Stability and efficiency

How to do the continuum calculation?

discrete system → finite lattice spacing

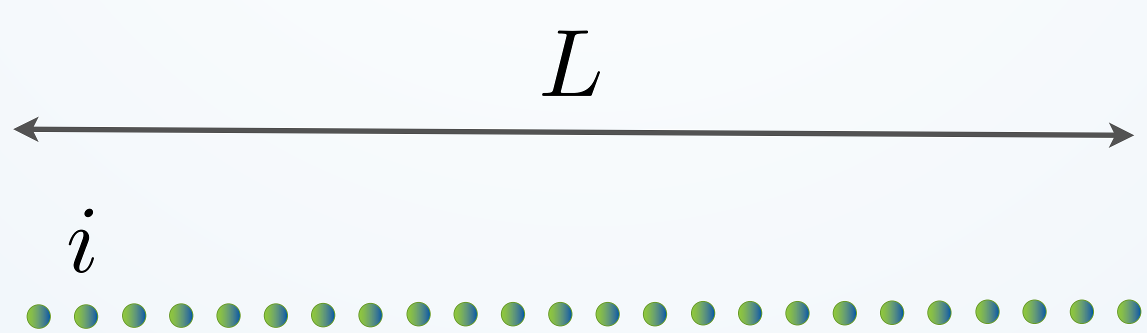


How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size

alternative: infinite size



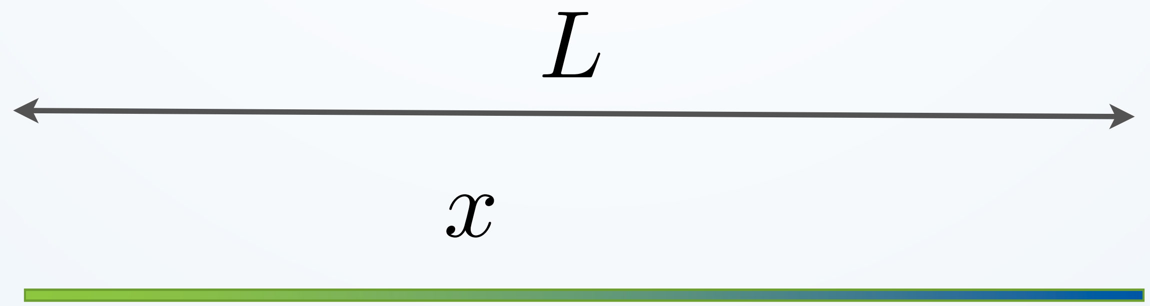
How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size

alternative: infinite size

extrapolate to vanishing spacing → possible divergences



SCHWINGER MODEL

dimensionless Hamiltonian

$$\frac{2}{ag^2}\left(H = \frac{1}{2a}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right) + \frac{m}{2}\sum_n \left(1 + (-1)^n \sigma_n^3\right) + \frac{ag^2}{2}\sum_n L_n^2 \right)$$

SCHWINGER MODEL

dimensionless Hamiltonian

$$W = \frac{1}{g^2a^2}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$

$$+\frac{m}{ag^2}\sum_n \left(1+(-1)^n\sigma_n^3\right) + \sum_n L_n^2$$

x

V

W_0

$$x = \frac{1}{g^2a^2} \to 0$$

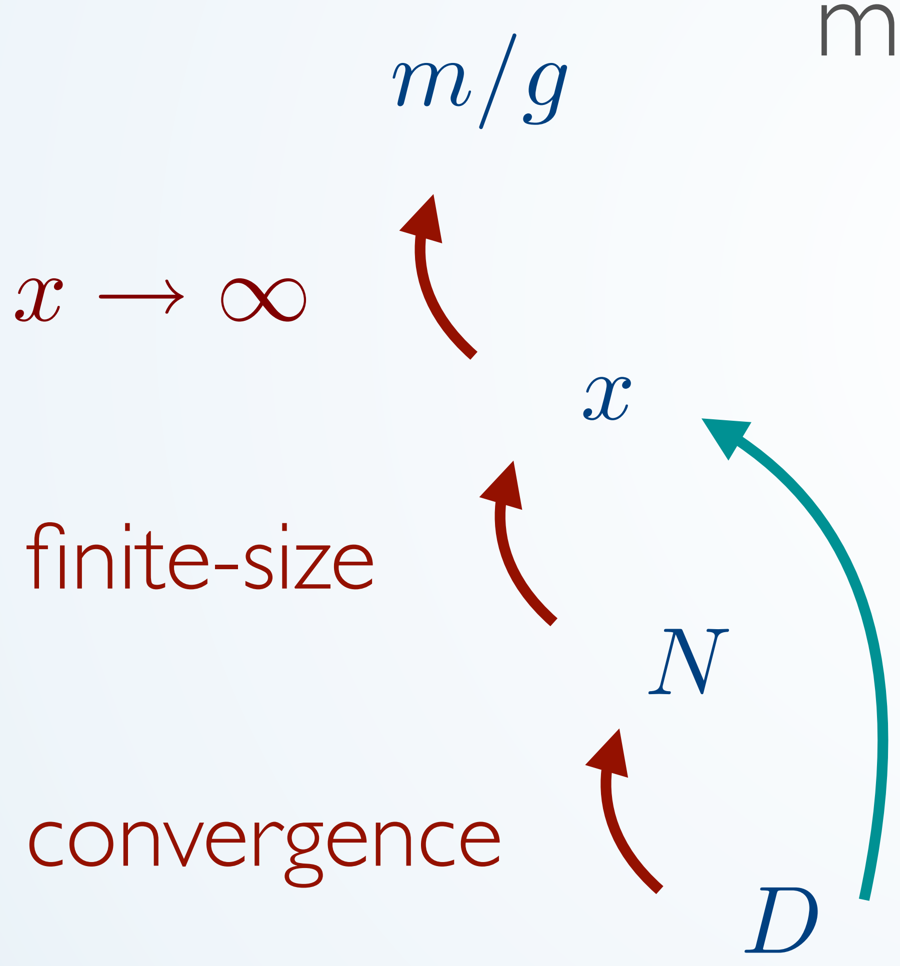
$g \to \infty$

strong coupling expansion

ground state $|\ldots 1\; 0\; 0\; 0\; 1\; 0\; 0\ldots\rangle$

COMPUTING THE SPECTRUM WITH MPS

Scan parameters



mass gaps and GS properties in the continuum $x \to \infty$

x as large as possible

$N \propto x$

alternative: infinite size

JHEP11(2013)158

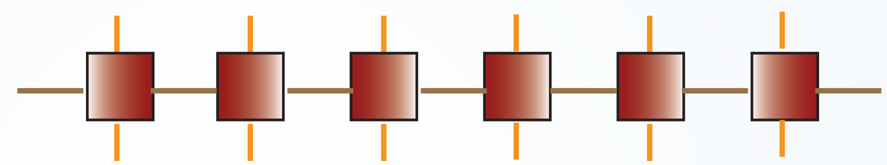
PRL113 (2014) 091601

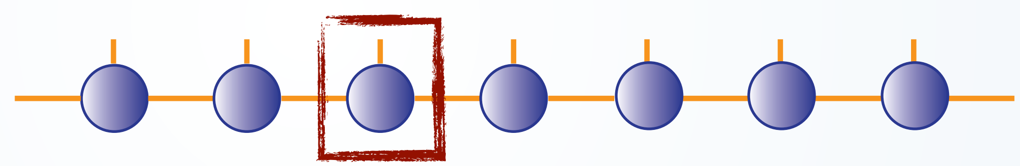
for each set $(m/g, x, N, D)$ run the basic algorithm

ALGORITHM

Variational minimization of energy

MPO
Hamiltonian

$H =$ 

$|E_0\rangle \simeq$ 

Variational principle

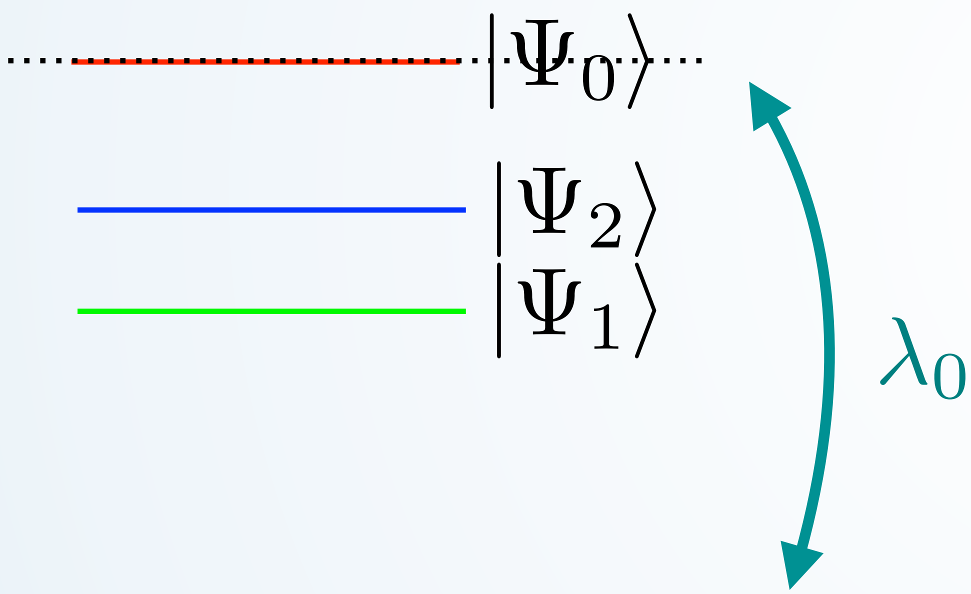
$$\min_{\{A\}} \frac{\langle\Psi|H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \longrightarrow \min_{A} \frac{\bar{A} H_{\text{eff}} A}{\bar{A} N_{\text{eff}} A}$$

sweep back and forth
over tensors

White, PRL 1992
Verstraete, Porras, Cirac, PRL 2004
Schollwöck, RMP 2005, Ann. Phys. 2011

SIMILAR FOR EXCITATIONS

$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$



$$H_1 = H + \lambda_0|\Psi_0\rangle\langle\Psi_0|$$

has $|\Psi_1\rangle$ as GS

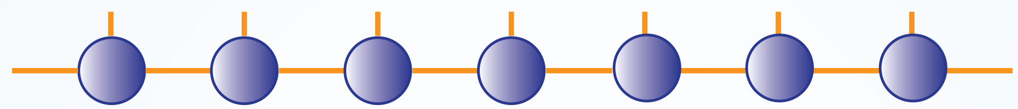
assuming (offset) $\lambda_0 + E_0 > E_1$

equivalent: next orthogonal eigenstate

alternative: working with symmetric tensors in the TD limit

uniform MPS (uMPS)

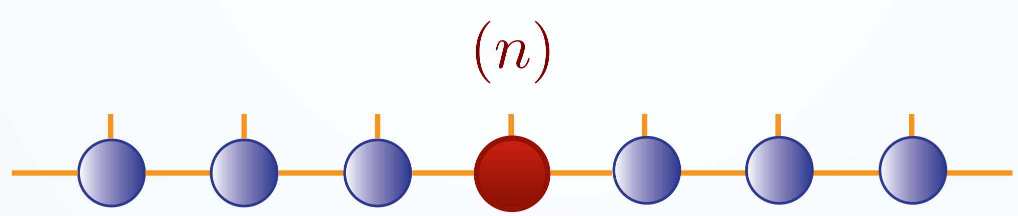
Ansatz determined by a single tensor



GS can be found by iTEBD, iDMRG, TDVP…

for excitations:

$$\sum_n e^{ikn}$$

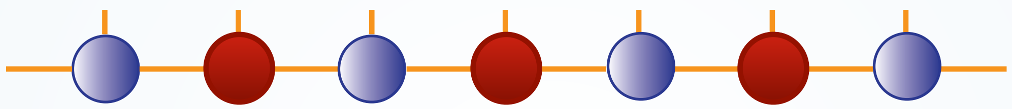


span tangent space

well defined momentum

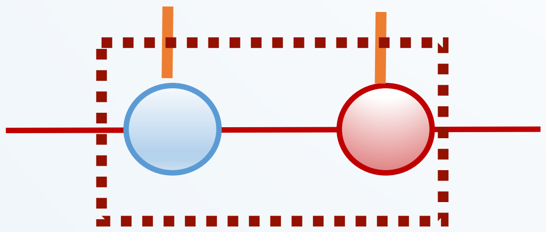
uniform MPS (uMPS)

cannot integrate out gauge $|\ldots s_e\, \ell\, s_o\, \ell\, s_e\, \ell\, s_o \ldots\rangle$



but physical states have to satisfy Gauss' law

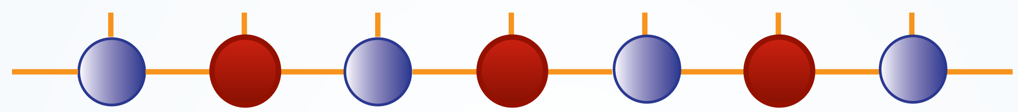
⇒e.g. symmetries



$A^{i\,\ell}_{\alpha\beta}$

uniform MPS (uMPS)

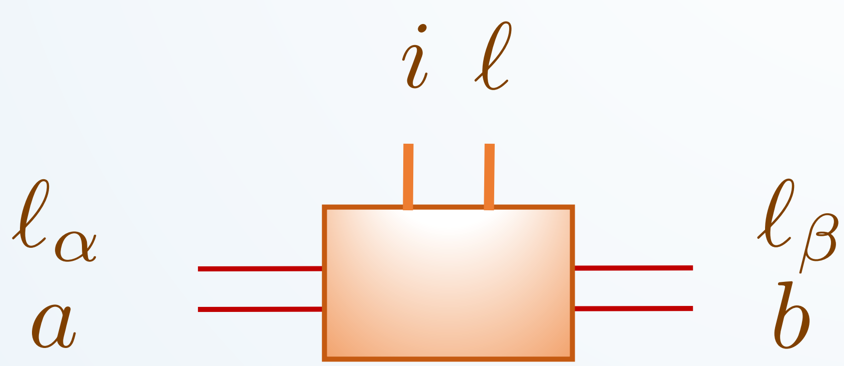
cannot integrate out gauge $|\ldots s_e\, \ell\, s_o\, \ell\, s_e\, \ell\, s_o \ldots\rangle$



truncate

but physical states have to satisfy Gauss' law

⇒e.g. symmetries



$A^{i\,\ell}_{\alpha\beta}$

$\alpha = (\ell_\alpha\, a)$

$\beta = (\ell_\beta\, b)$

$$\ell = \ell_\beta \qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}$$

IDENTIFYING THE LEVELS

lattice with PBC or TI

orthogonal subspaces

GS & scalar $\longleftrightarrow$ vector

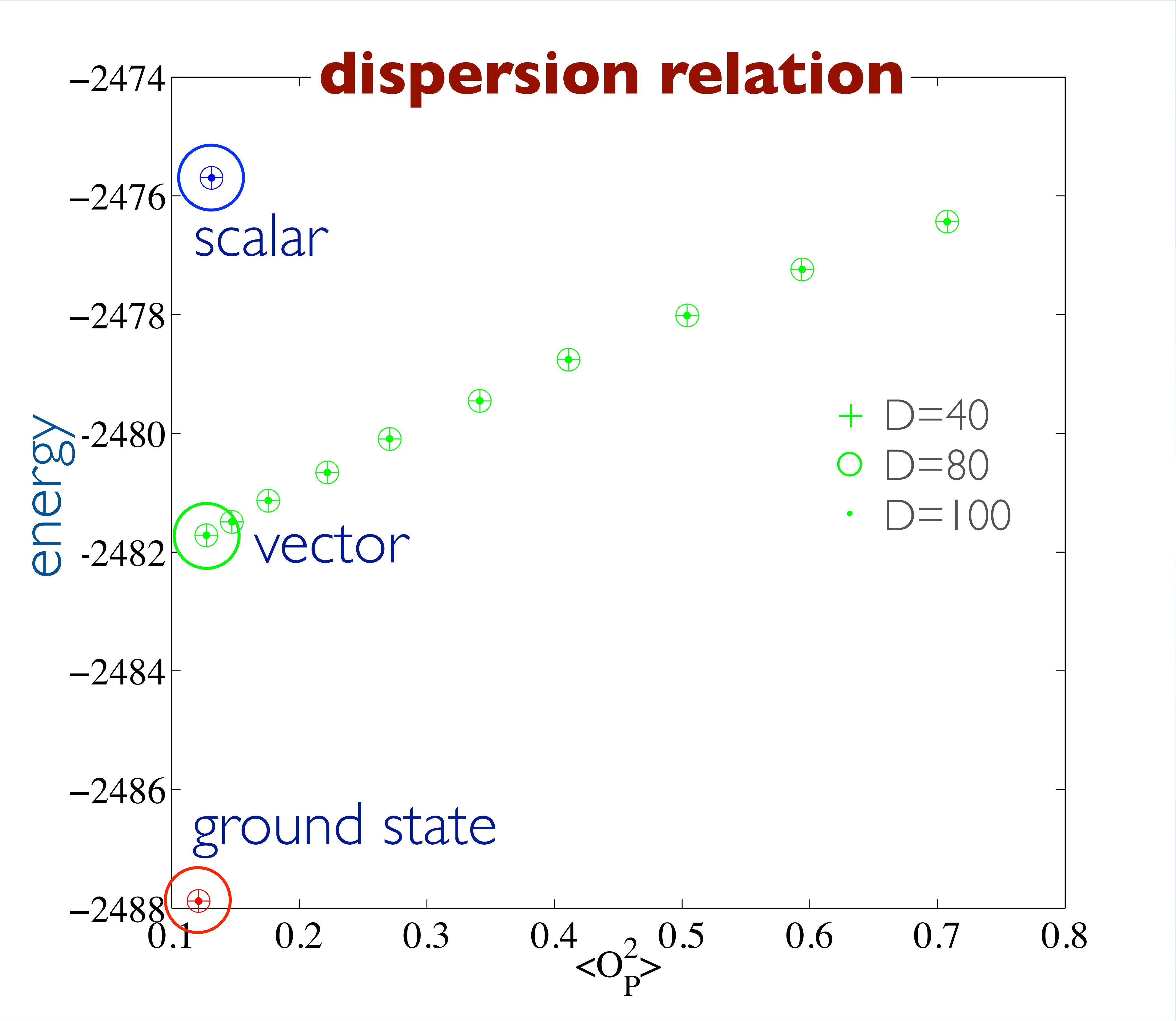
lattice with OBC

intermediate states need to be reconstructed before reaching the scalar

momentum excitations of the vector

need to *recognize* the scalar

approximate symmetry transformations



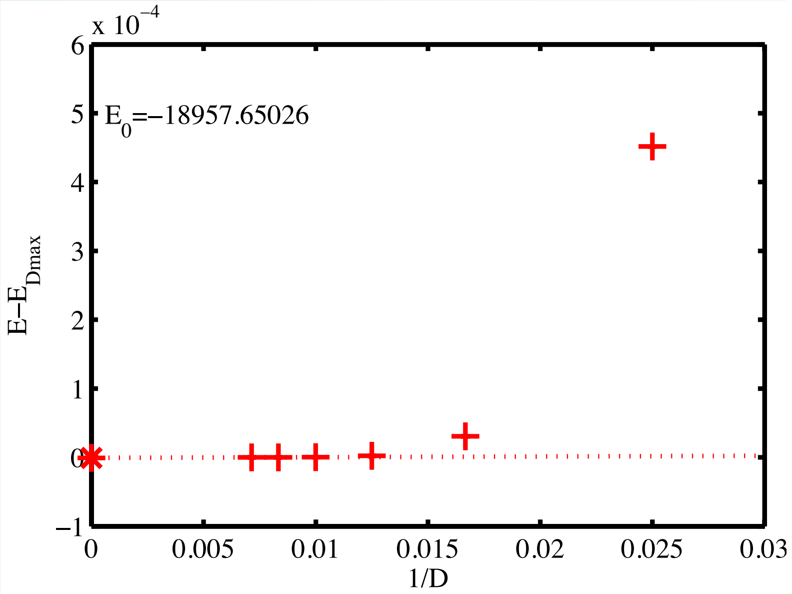
dispersion relation
−2474
−2476
−2478
-2480
-2482
−2484
−2486
−2488
energy
scalar
vector
ground state
D=40
D=80
D=100
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$<O_P^2>$

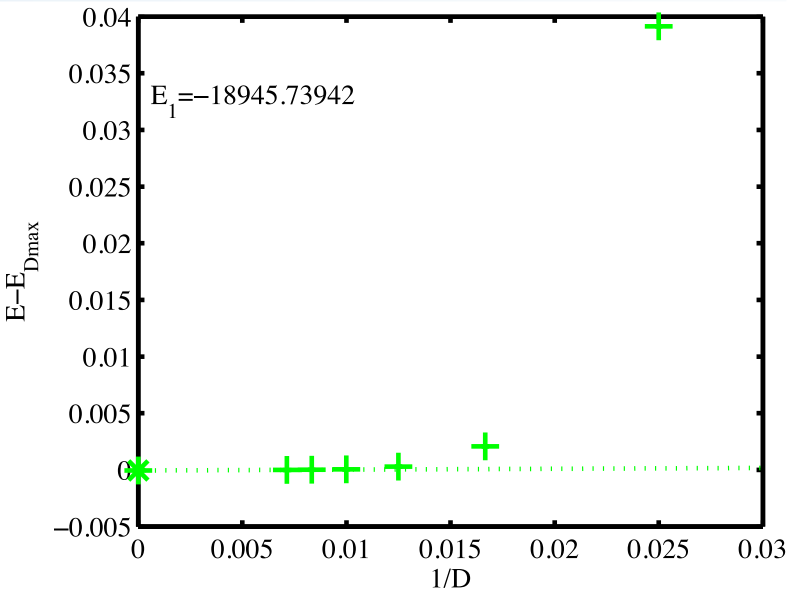
continuum limit: extrapolations

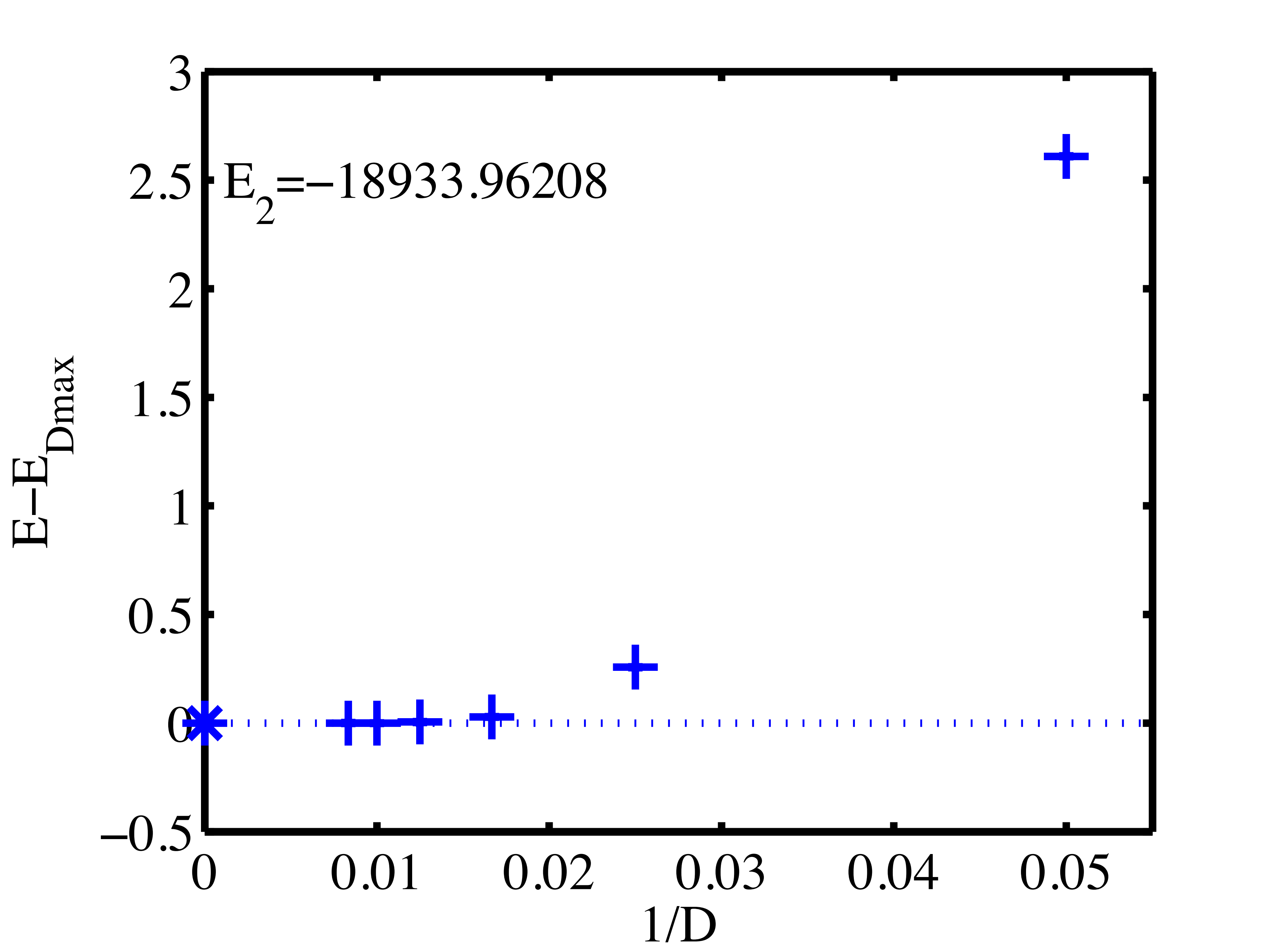
I truncation error

$m/g = 0 \quad x = 100$

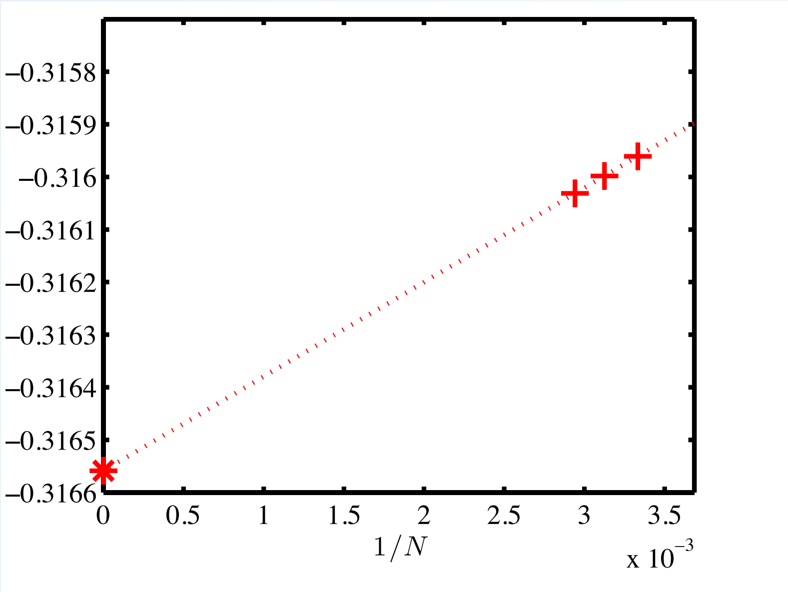
$N = 300$

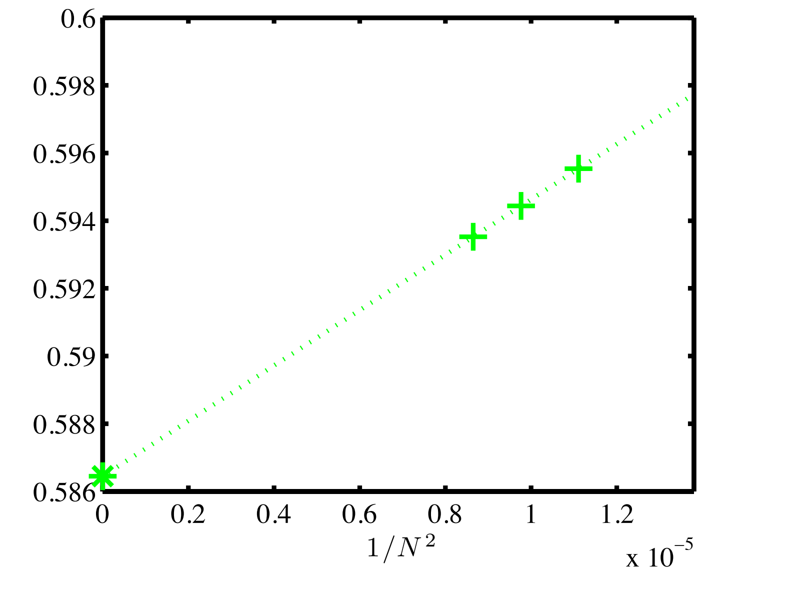


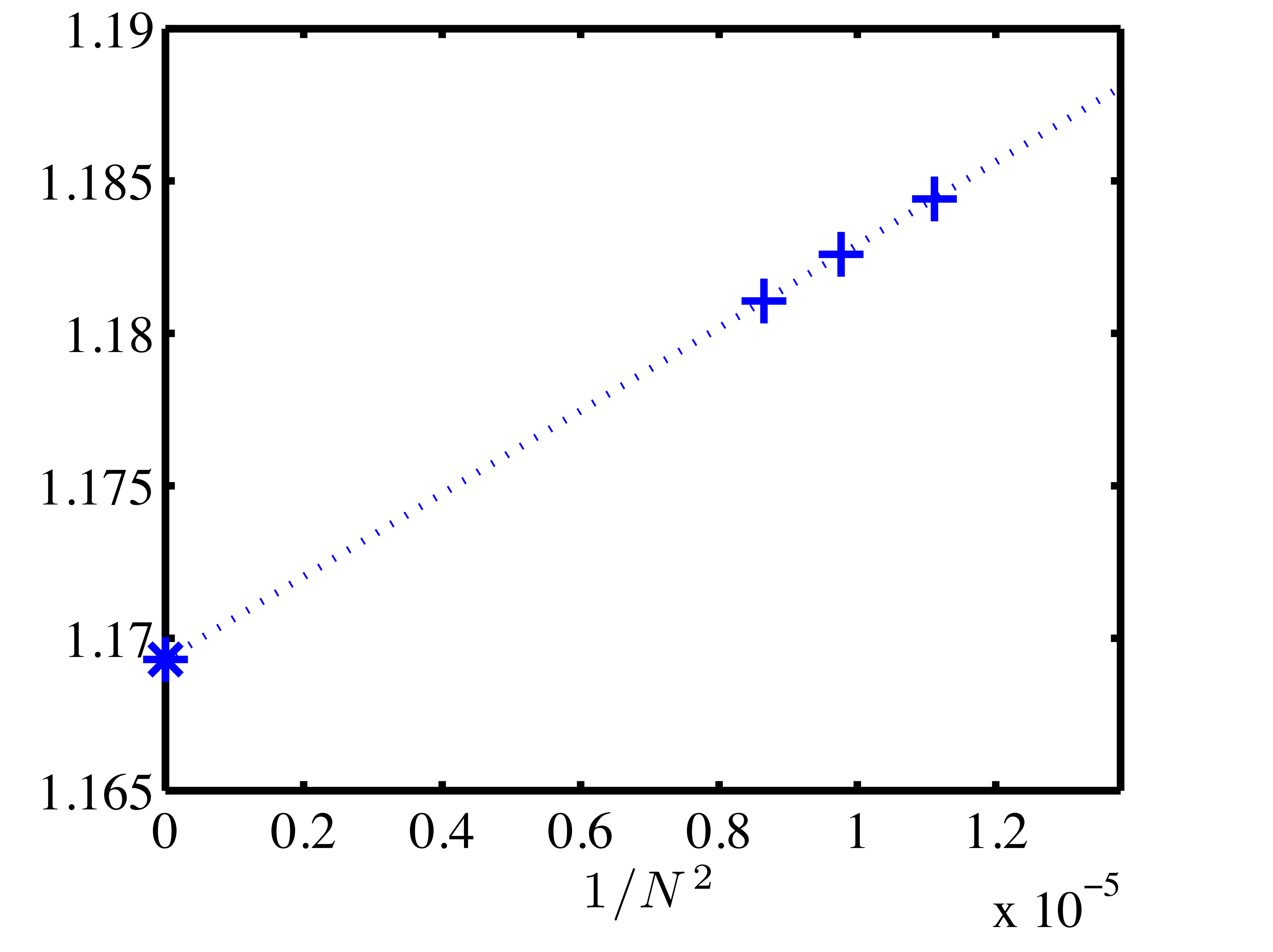




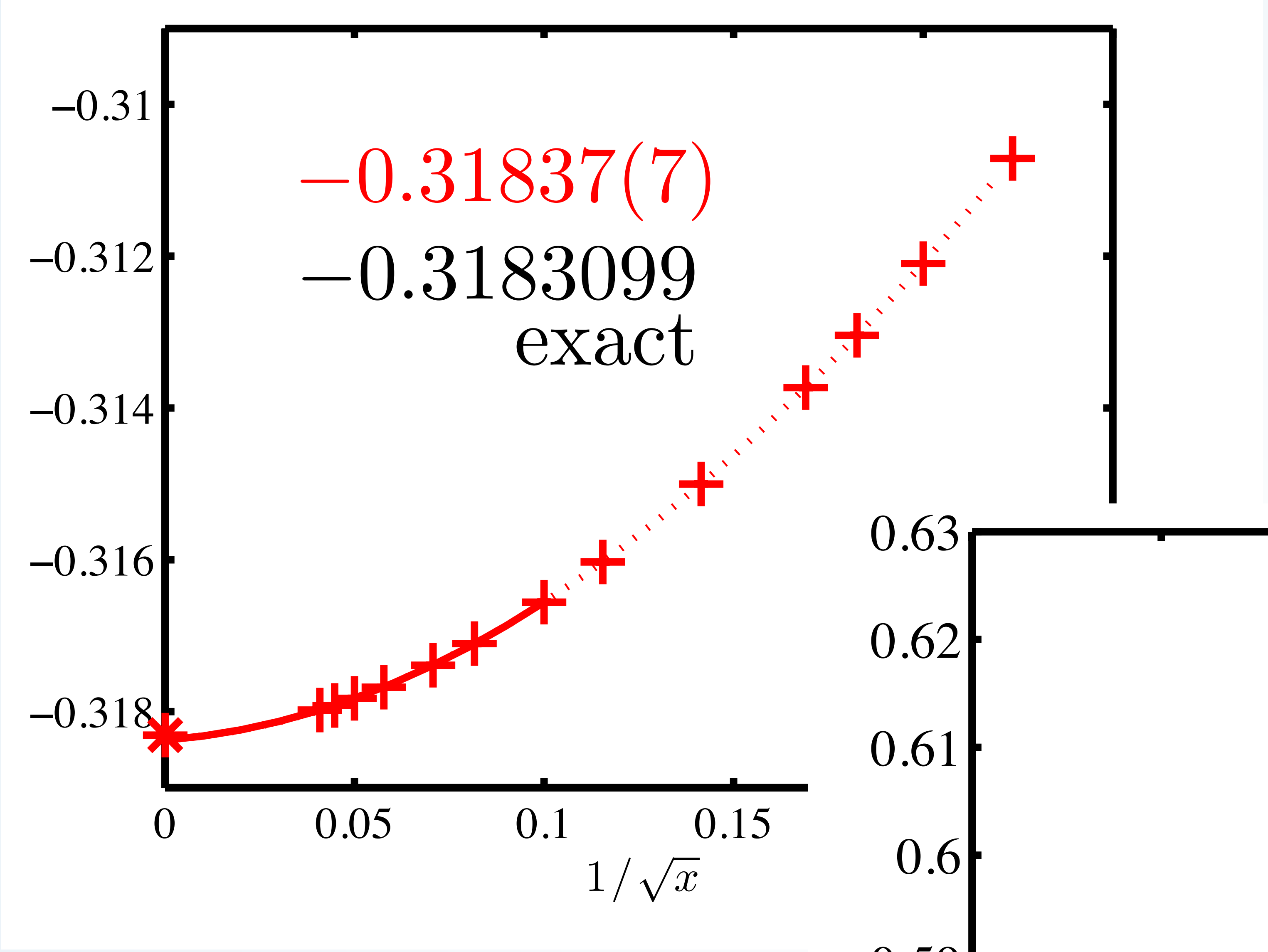
② finite-size scaling $m/g = 0 \quad x = 100$

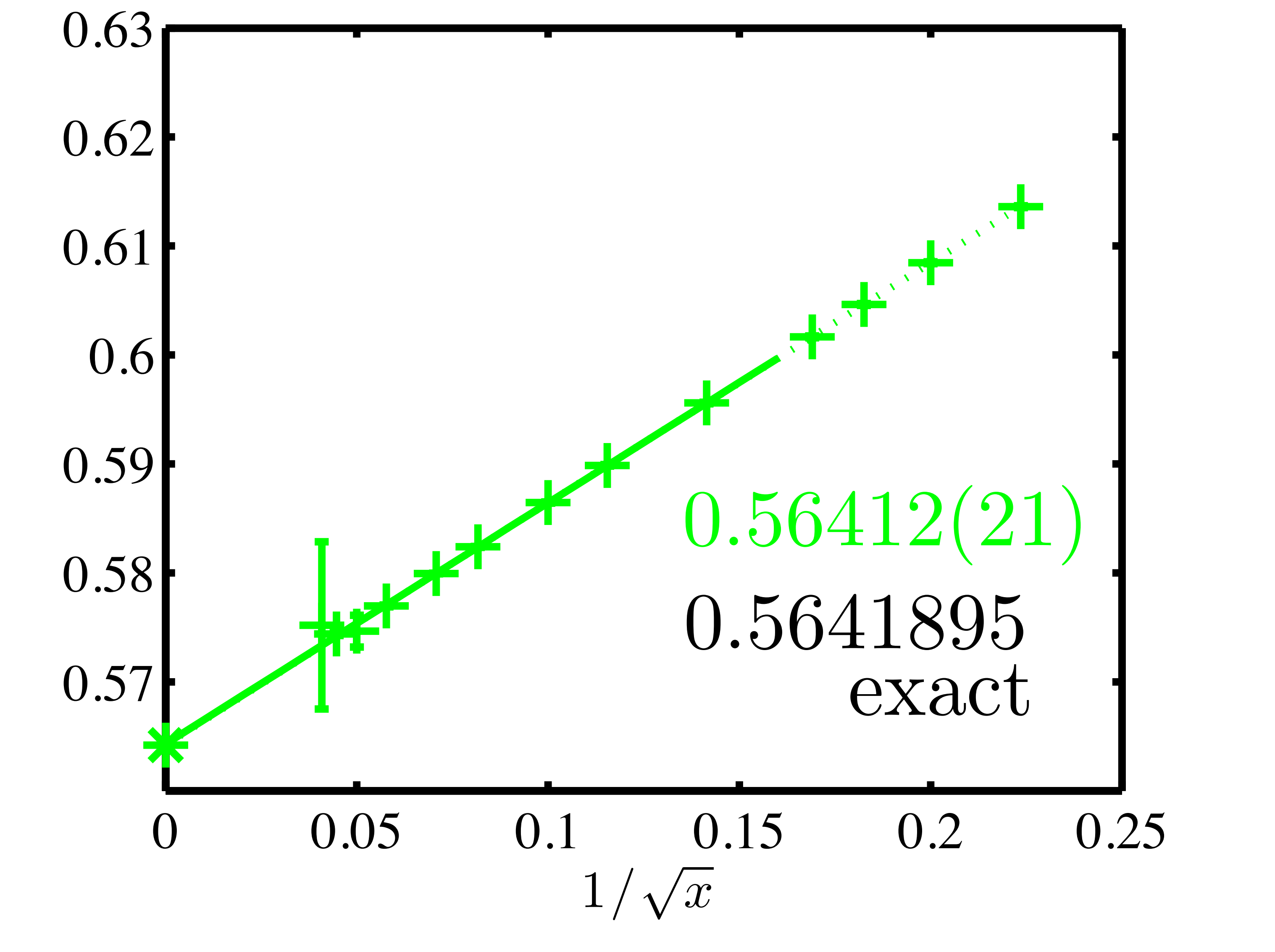




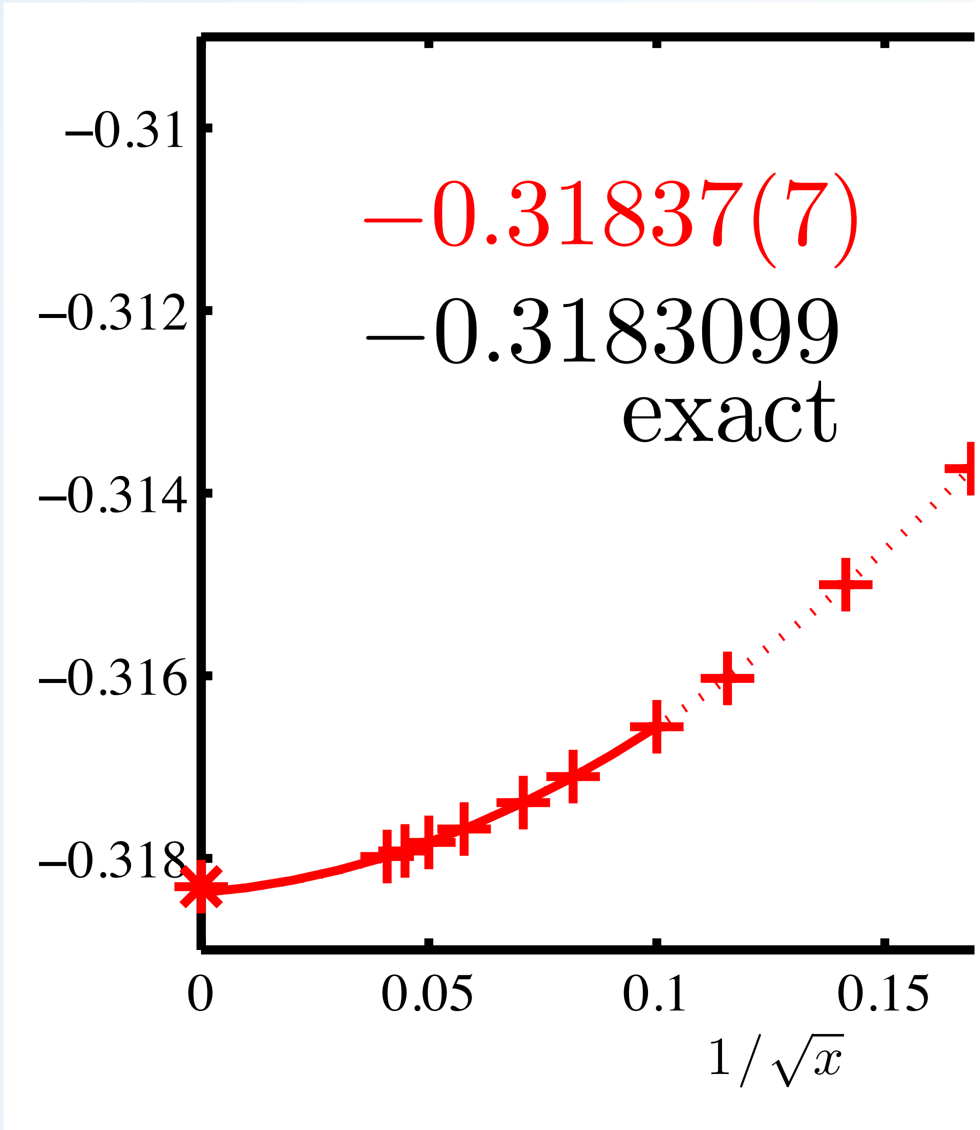


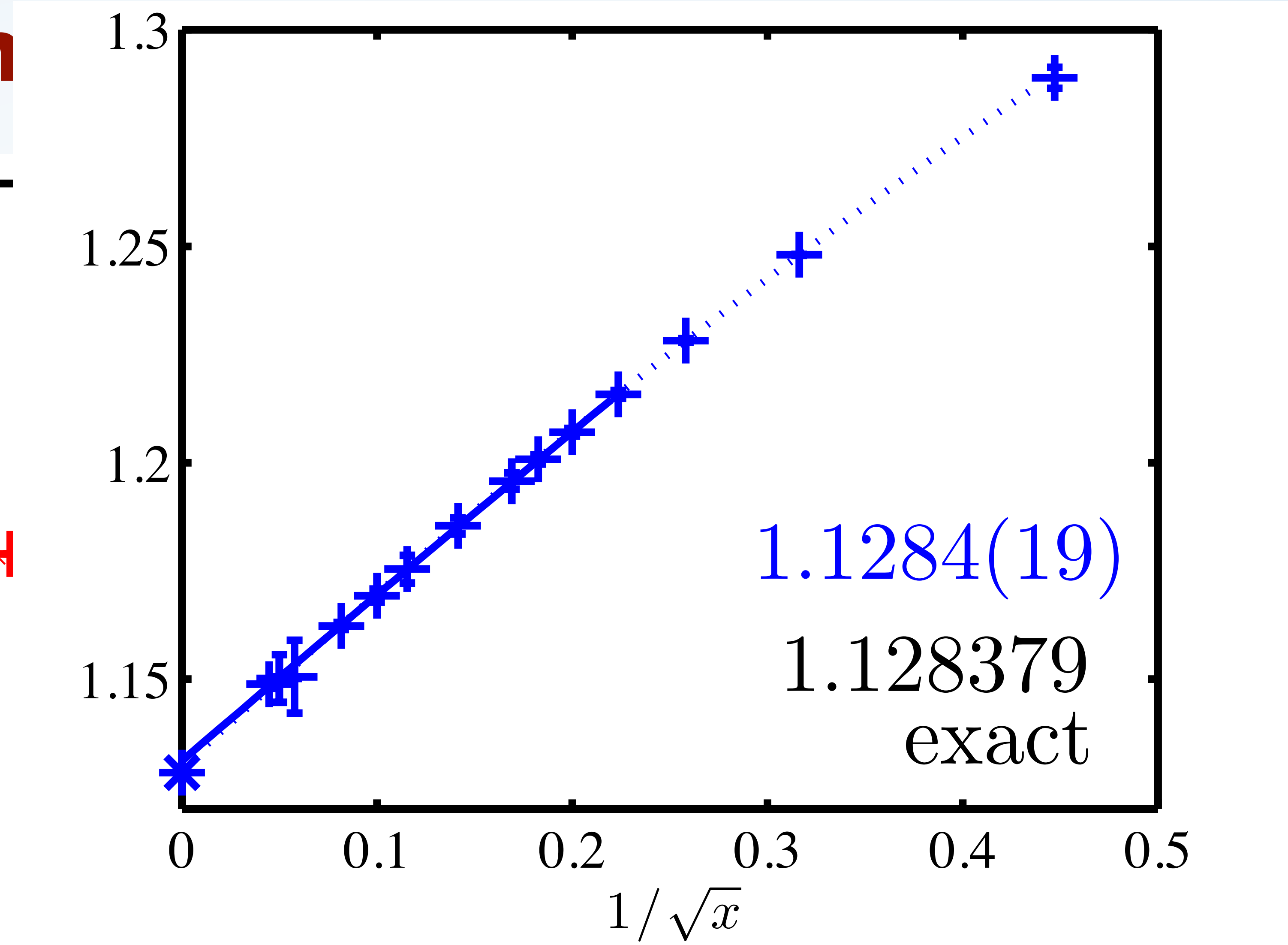
3 continuum limit $m/g = 0$

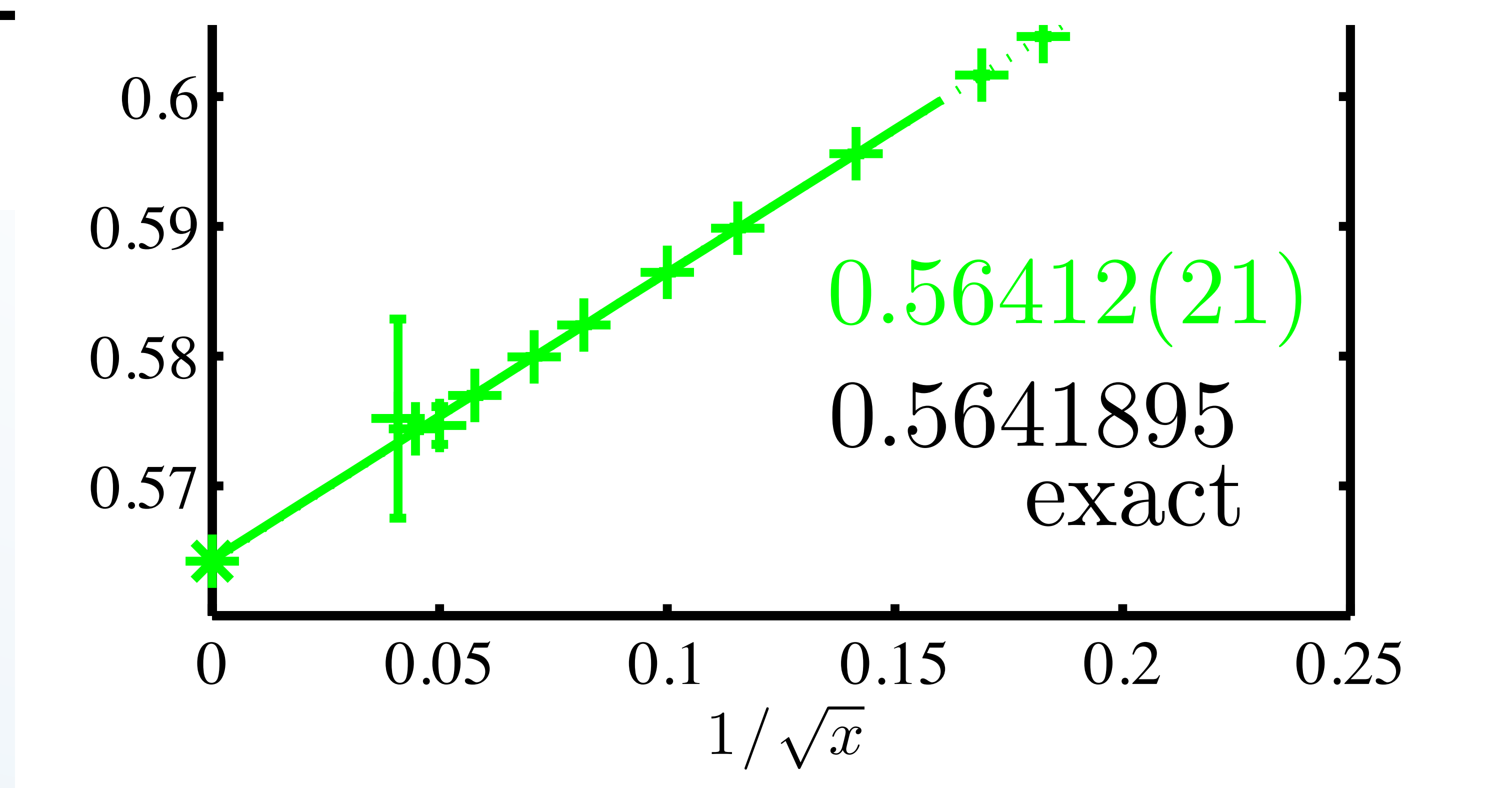




3 **continuum lin**







good agreement with exact values

main uncertainty: continuum extrapolation form unknown

systematic and conservative estimation of the errors same game for massive case

1. **truncation error**
2. **finite-size scaling**
3. **continuum limit**

m/g	DMRG	MPS with OBC [1]	gauge inv. uMPS [2]	SCE	MPS with OBC [1]	gauge inv. uMPS [2]
0	0.5641859	0.56414(26)	0.56418(2)	1,128379	1.1283(10)	-
125	0.53950(7)	0.53946(20)	0.539491(8)	1.22(2)	1.2155(28)	1.222(4)
0.25	0.51918(5)	0.51915(14)	0.51917(2)	1.24(3)	1.2239(22)	1.2282(4)
0.5	0.48747(2)	0.48748(6)	0.487473(7)	1.20(3)	1.1998(17)	1.2004(1)

better precision than any earlier numerics

[1] MCB, Cichy, Cirac, Jansen JHEP11(2013)158
[2] Buyens et al. PRL113 (2014) 091601

MPS give us access to observables: expectation values

MPS STATES ➞ OBSERVABLES

chiral condensate in the GS: order parameter for chiral symmetry breaking ($m/g=0$)

$$\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$$

in the spin language $$\frac{\sqrt{x}}{L} \sum_n (-1)^n \frac{1+\sigma_n^3}{2}$$

no exact value known for $m/g\neq 0$

only estimations

de Forcrand et al. 97
Hosotani 97

logarithmic divergence ➞ same as in non-interacting case

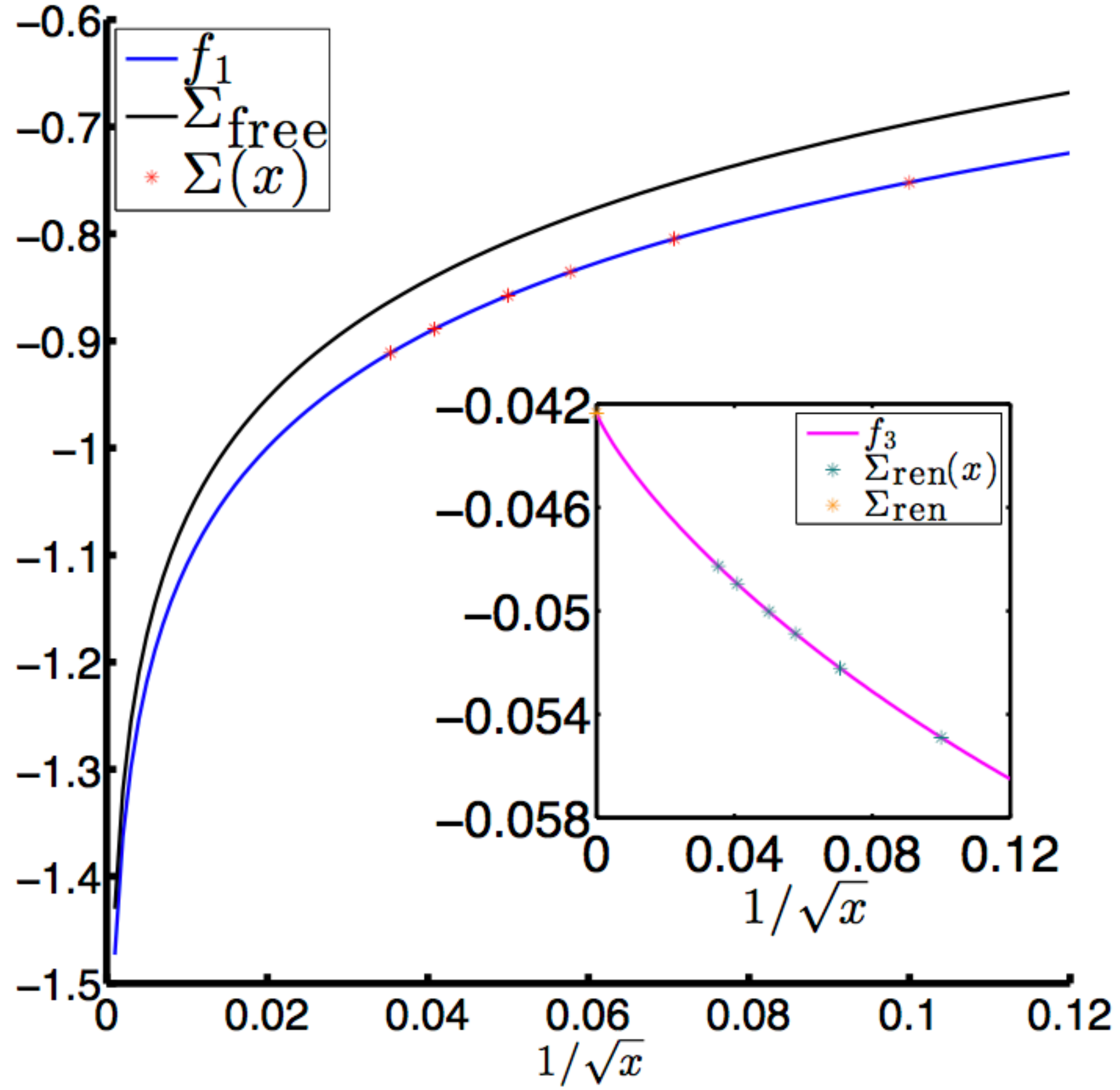
M … ES

chiral conde … er $\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$

for chiral sy …

in the spin l …

no exact va …



from Buyens et al. arXiv:1411.0020

…Forcrand et al. 97

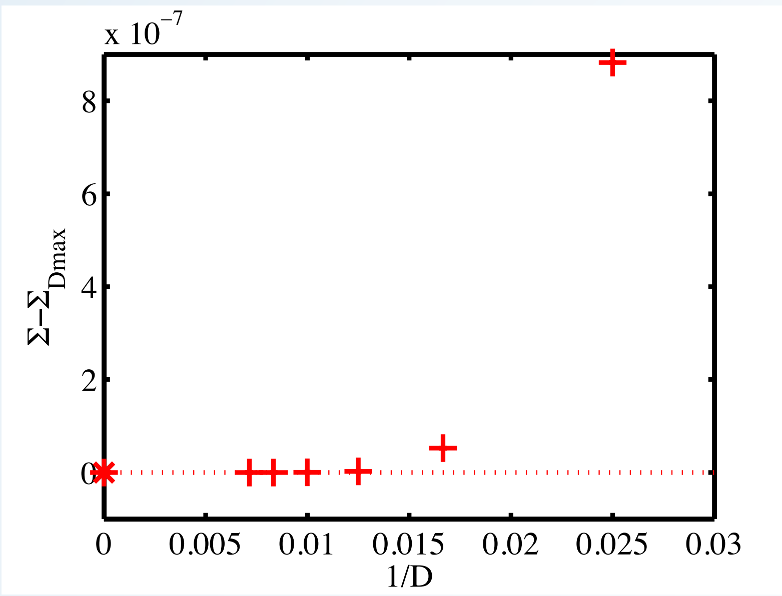
…sotani 97

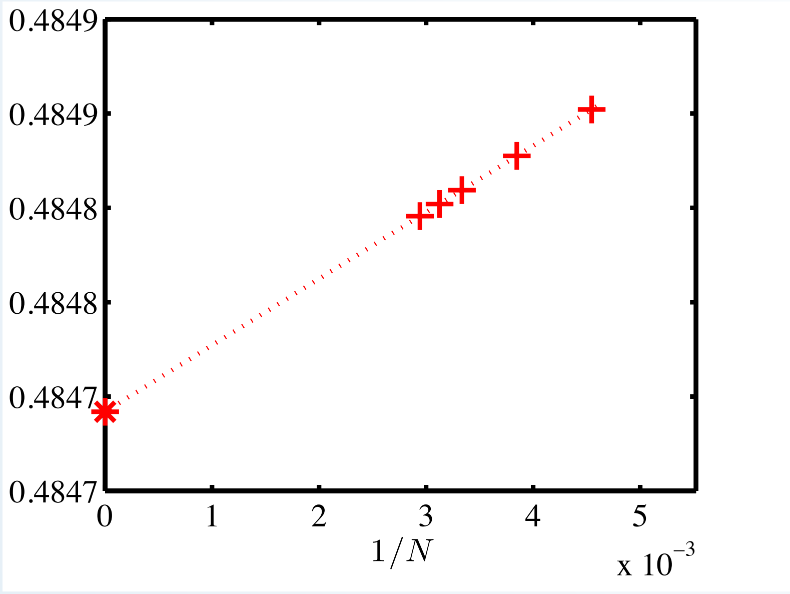
logarithmic divergence ➞ same as in non-interacting case

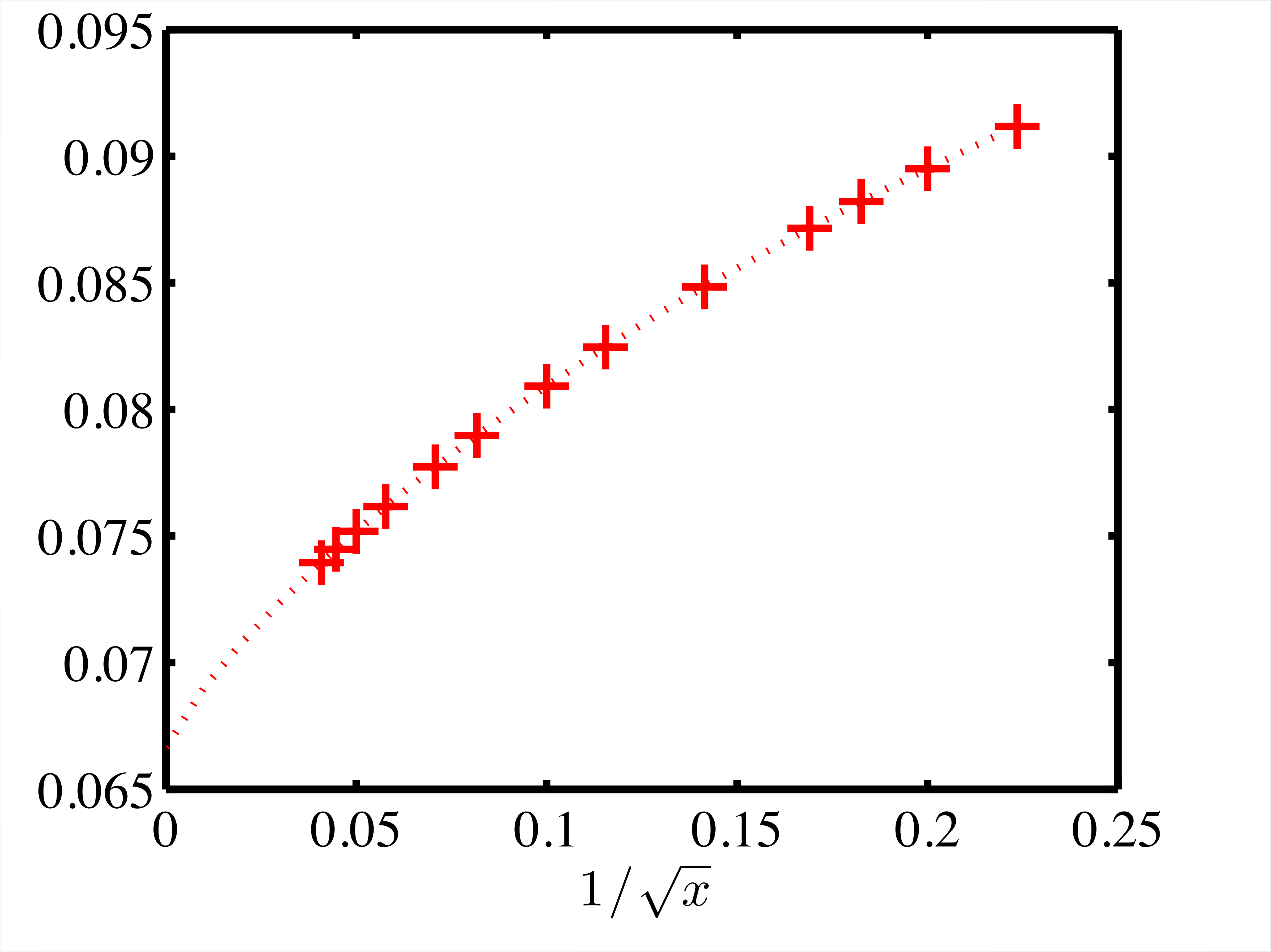
substract exact non-interacting condensate

extrapolations

$m/g = 0.25 \;\; x = 100$

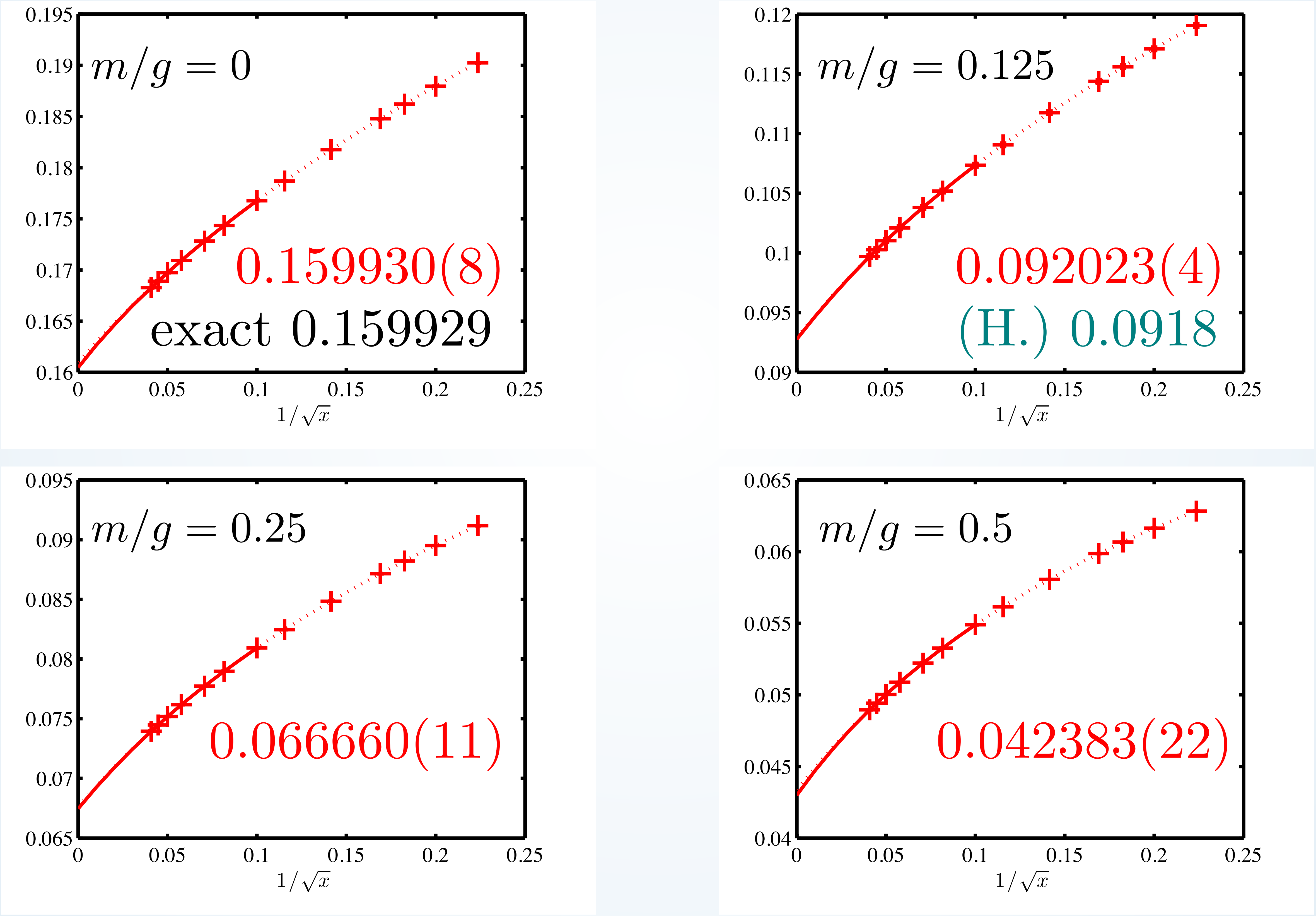






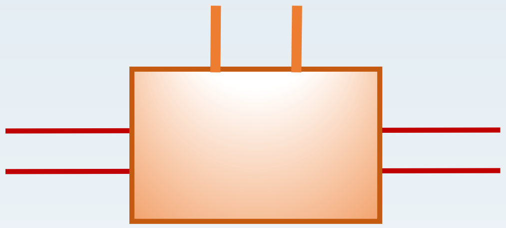
[1] MCB et al, arXiv:1310.4118
[2] Buyens et al. arXiv:1411.0020

CHIRAL CONDENSATE



uMPS: how important is the truncation of gauge dof?

symmetric MPS has block structure



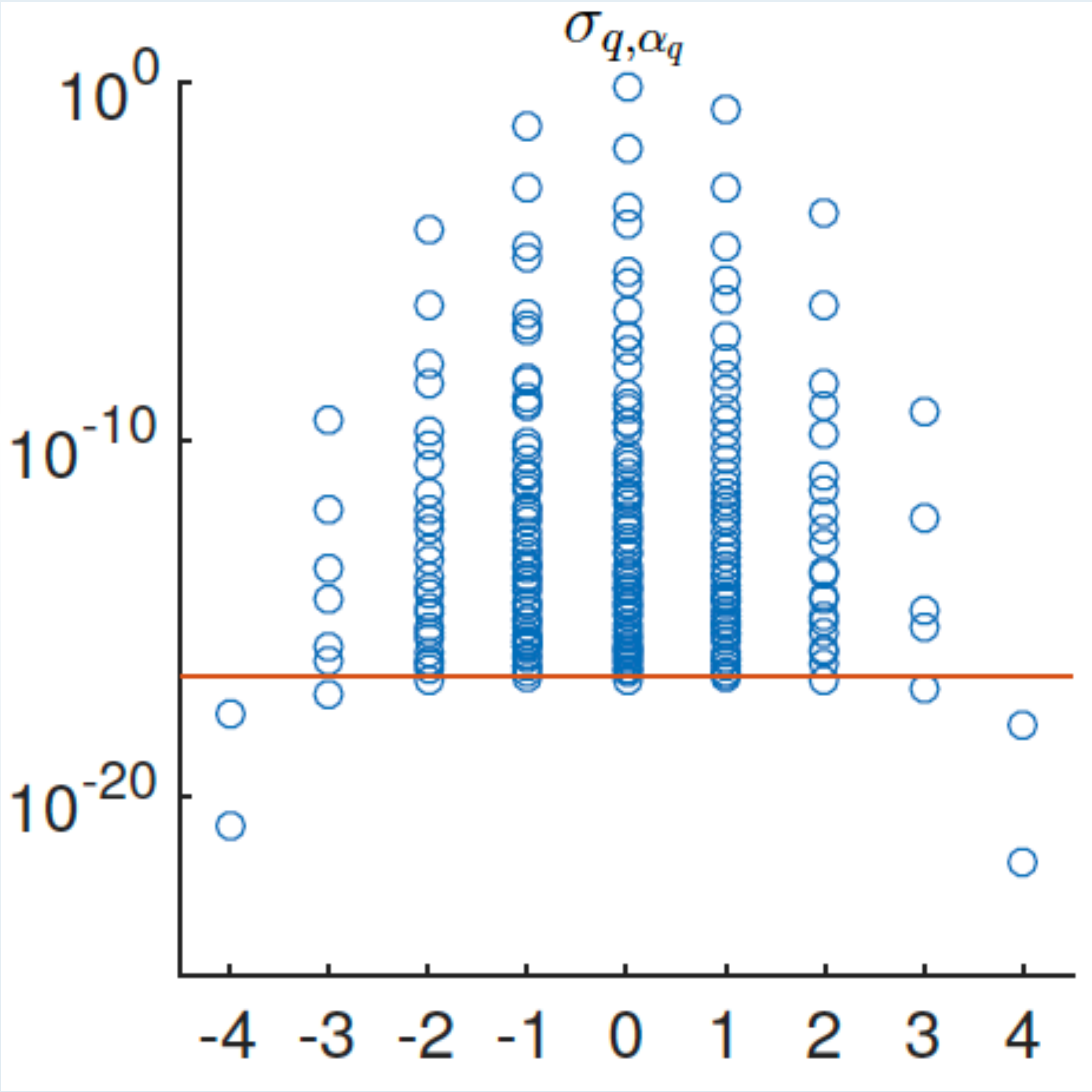
$$\ell = \ell_\beta \qquad\qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}$$

$$A_1^{1,p} = \begin{array}{c|c|c|c|c|c|}
q\backslash r & \cdots & p-1 & p & p+1 & \cdots \\
\vdots & \ddots & \vdots & \vdots & \vdots & \iddots \\ \hline
p & \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline
p+1 & \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & a_1^{q,1} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline
p+2 & \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline
\vdots & \iddots & \vdots & \vdots & \vdots & \ddots
\end{array}$$

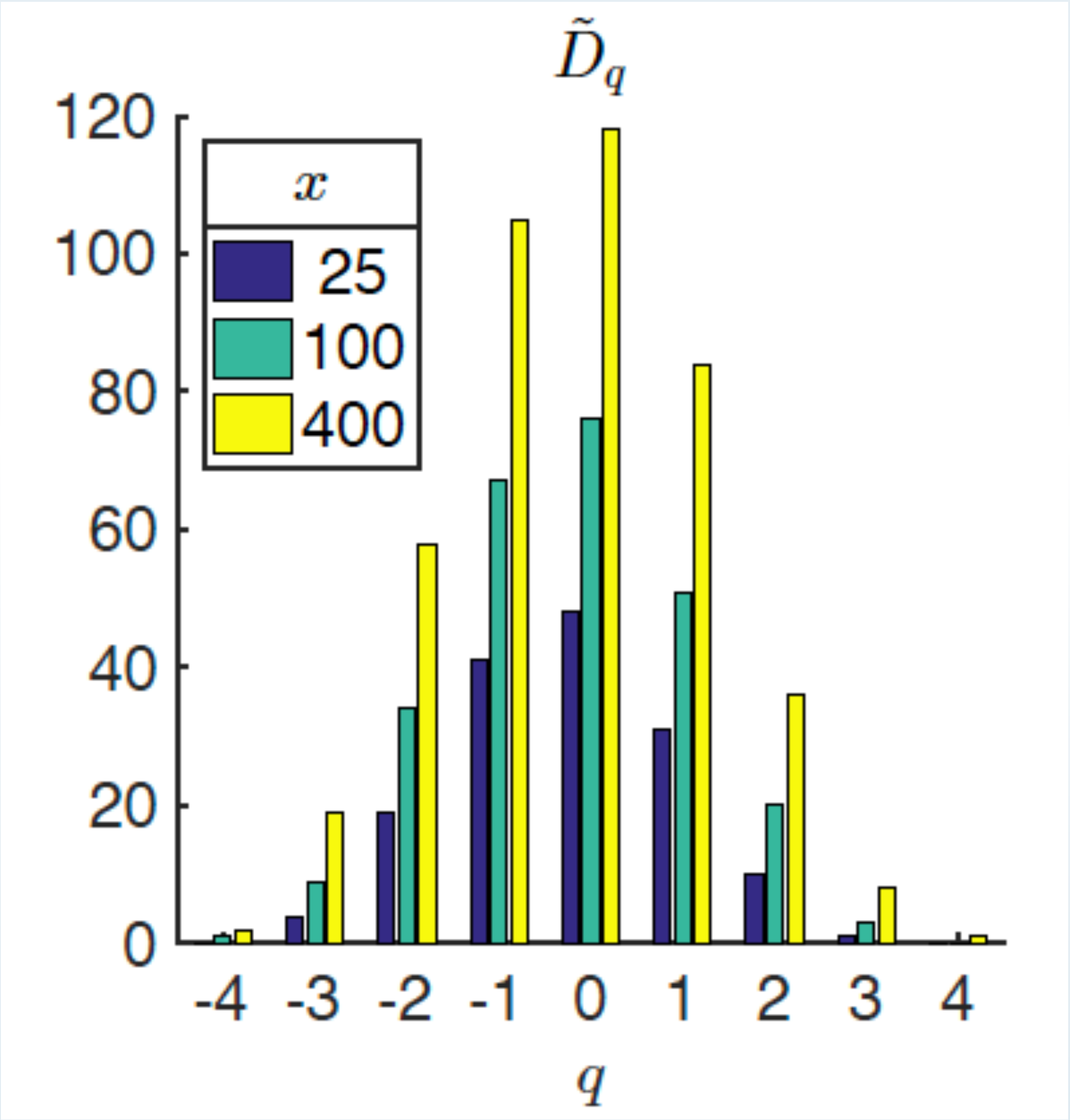
a maximum bond dimension per block

from B. Buyens

decay of Schmidt values



required D towards cont



Buyens et al PRD 95 (2017) 094509

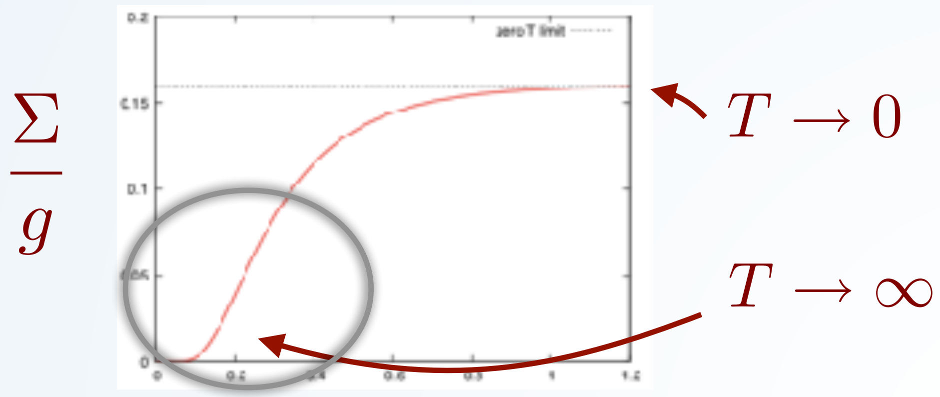
D increases as $x^{1/2}$

THERMAL STATES

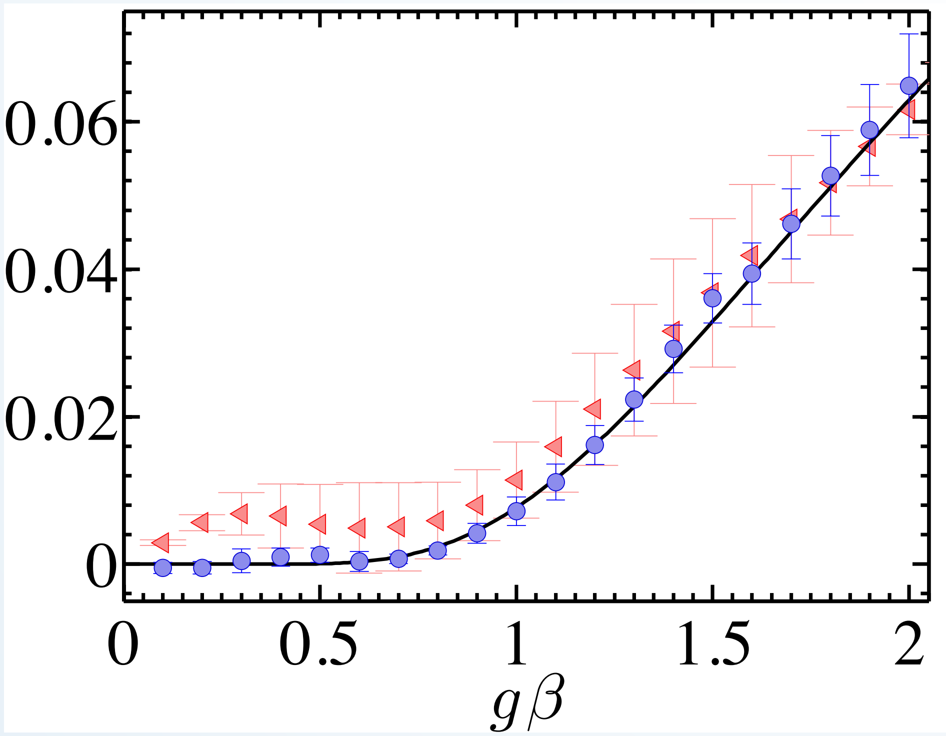
THERMAL PROPERTIES SCHWINGER

chiral condensate at finite T: analytical for m/g=0

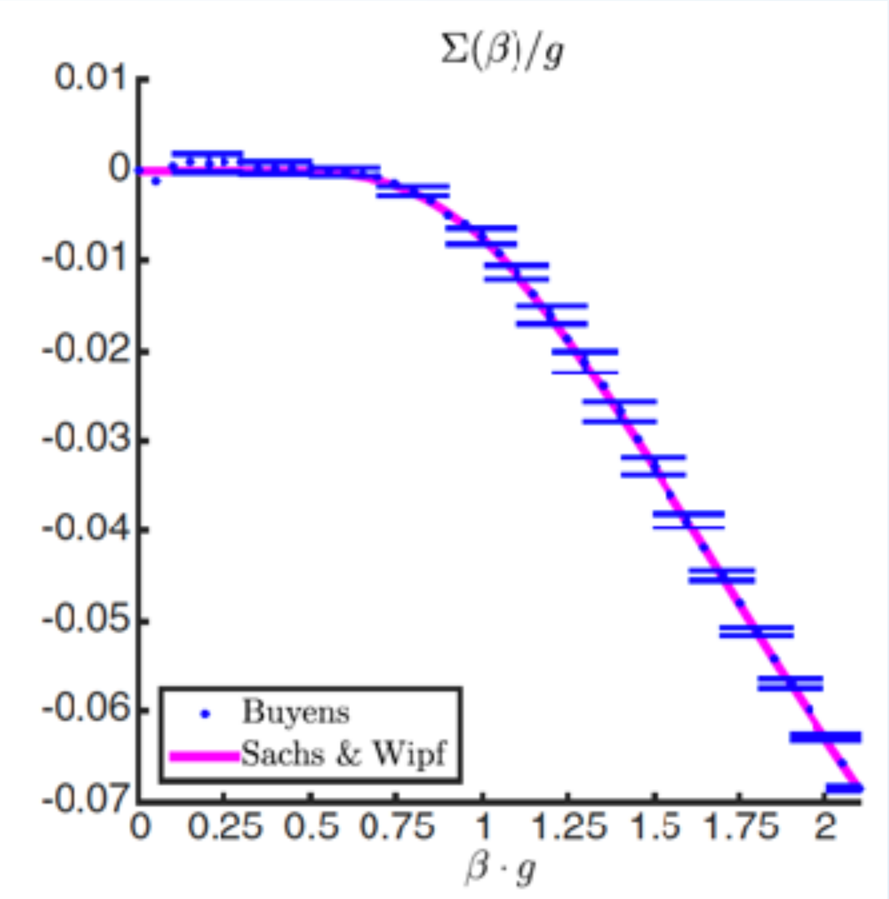
Sachs, Wipf 92



smooth
restoration of
chiral symmetry



PRD 92, 034519 (2015); PRD 93, 094512 (2016)



Buyens PRD 94, 085018 (2016)

ACTIVE RESEARCH: PEPS FOR LGT

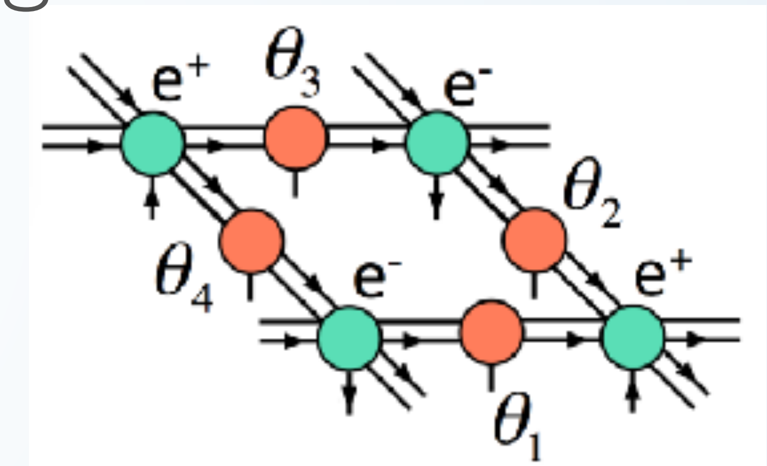
explicitly gauge invariant PEPS

restricted ansatz calculations

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015
arXiv:1807.01294

standard PEPS toolbox contains all ingredients for full variational computation

computational cost, required D



Zapp, Orús PRD 2017

restriction of the ansatz may be better strategy

e.g. fully Gaussian PEPS

Zohar, Cirac PRD 2018

CONCLUSION

Proof of feasibility of TNS for LQFT

- precisions comparable to earlier numerics massive fermions
- more adequate ansatzes possible
- particular problems where standard techniques do not work chemical potential, time evolution

Very useful for Q Simulators

- study the effects of finite dimensions
- design dynamics, observables, ...

see refs. in arXiv:1810.12838

MPQ

QUANTERA QTFLAG

THANKS

Proof of feasibility of TNS for LQFT

- precisions comparable to earlier numerics — massive fermions
- more adequate ansatzes possible
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MPQ

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