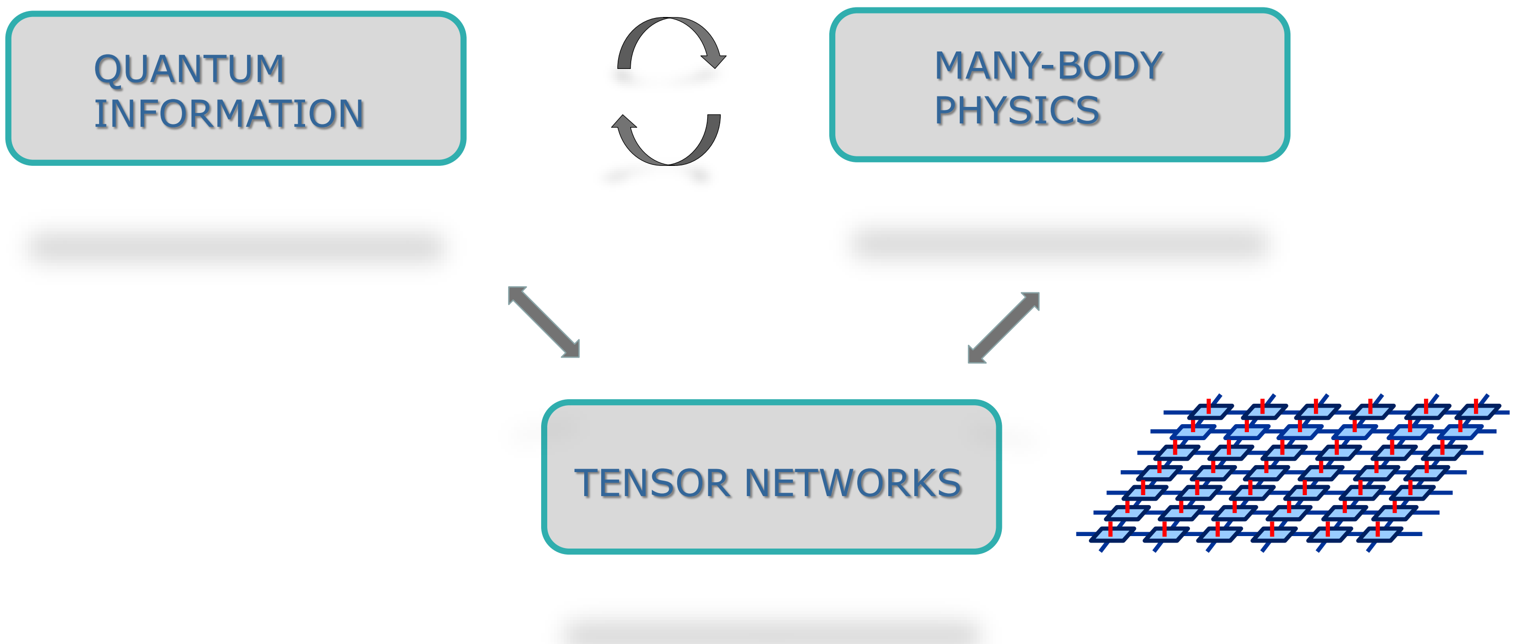
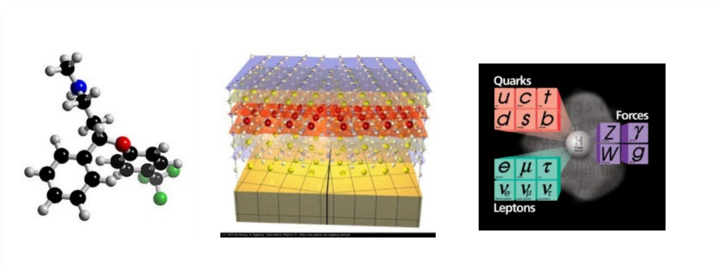
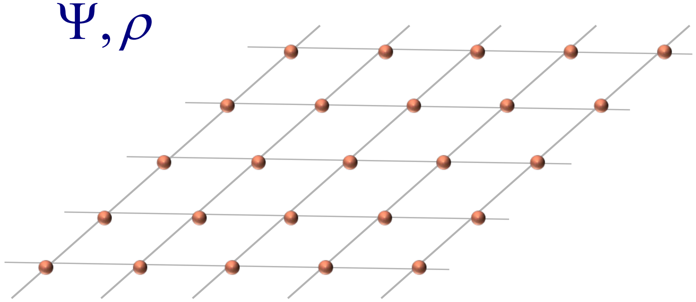

QUANTUM INFORMATION
MANY-BODY PHYSICS
TENSOR NETWORKS

QUANTUM MANY-BODY SYSTEMS



Ψ, ρ

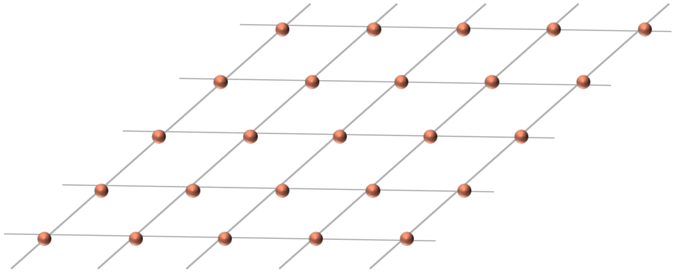


HILBERT SPACE

$$d^N$$

$$|\Psi\rangle = \sum c_{i_1, i_2, \ldots, i_N} |i_1, i_2, \ldots, i_N\rangle$$

MANY-BODY QUANTUM SYSTEM



How much entangled?

What are the consequences?

ENTANGLEMENT
in MANY-BODY QUANTUM SYSTEMS

ENTANGLEMENT

- Definition:

$$|\Phi\rangle \neq |\varphi,\psi\rangle$$

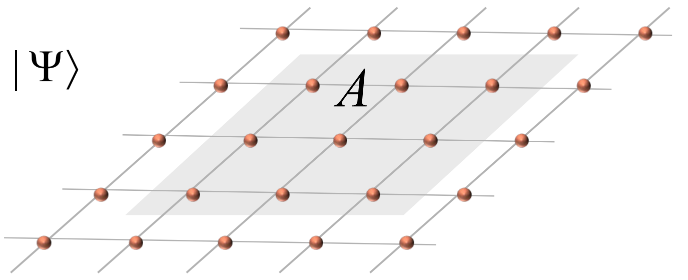
$$|\Phi\rangle = |0,0\rangle + |1,1\rangle$$

$$|\Phi\rangle = |0,\ldots,0\rangle + |1,\ldots,1\rangle$$

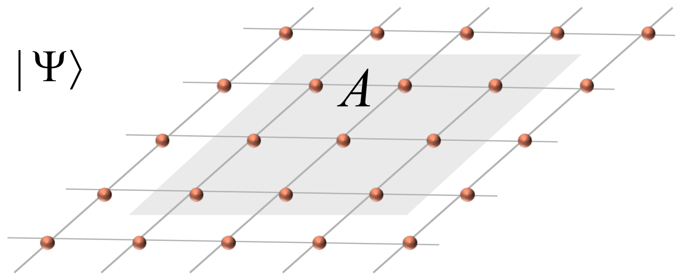
- Quantification: Entanglement entropy:

ρ

$$E(\Phi) = S(\rho) = -\mathrm{tr}(\rho \log \rho)$$



How much entangled?

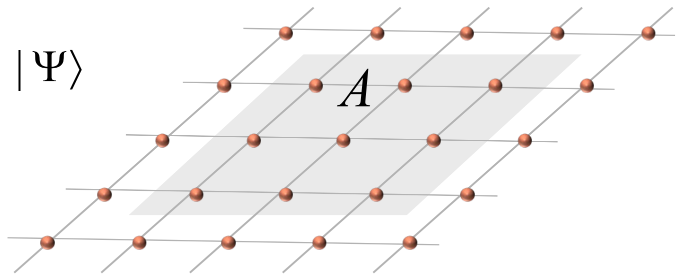


How much entangled?

- Lattice in any physical dimension and geometry
- Local Hamiltonians: $H = \sum_n h_n$
- Thermal equilibrium: $T = 0$

Very little!

ENTANGLEMENT
MANY-BODY PHYSICS



- Area law

Srednicki, Wilzek, …

- In general $E(\Psi) \sim |A|$

- T=0 (ground state)
- Local interactions

$$E(\Psi) \sim |\partial A|$$

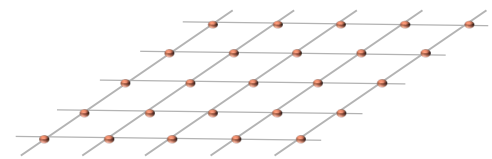
Hastings, Verstraete, …

For finite temperatures, mutual information: $I(A:\overline{A}) \sim |\partial A|$

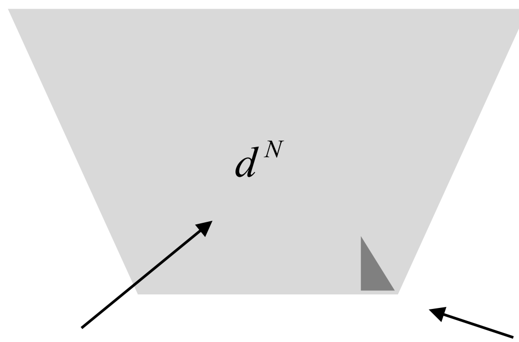
Wolf, Verstraete, Hastings, JIC, PRL 100, 070502 (2008)

- What do we learn?

$|\Psi\rangle$



EXPONENTIAL HILBERT SPACE



$$E(\Psi) \sim |A|$$

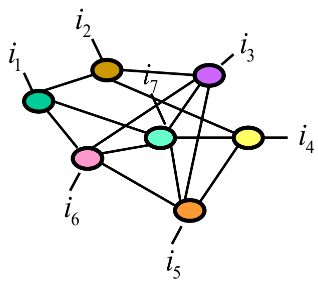
Physically relevant

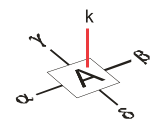
$$E(\Psi) \sim |\partial A|$$

It should be possible to find an efficient description of physical states:
Number of parameters should scale polynomically with N

TENSOR NETWORK STATES

Ψ, ρ





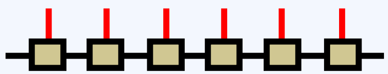
Main object:
Low-rank tensor

MPS, PEPS, MERA, …

TENSORS NETWORKS

MPS

$|\varphi\rangle$

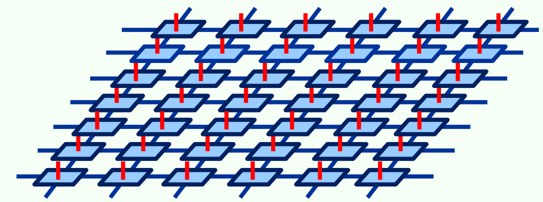


$$\psi_{s_1,s_2,\ldots,s_N} = \mathrm{tr}\left[A^{s_1}A^{s_2}\ldots A^{s_N}\right]$$

Fannes, Narchtergaele, Werner, CMP **144**, 443 (1992)

PEPS

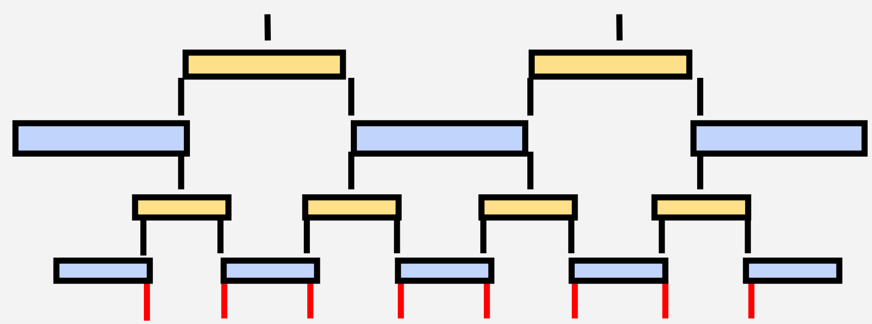
$|\varphi\rangle$



Verstraete, JIC, arxiv:0407066

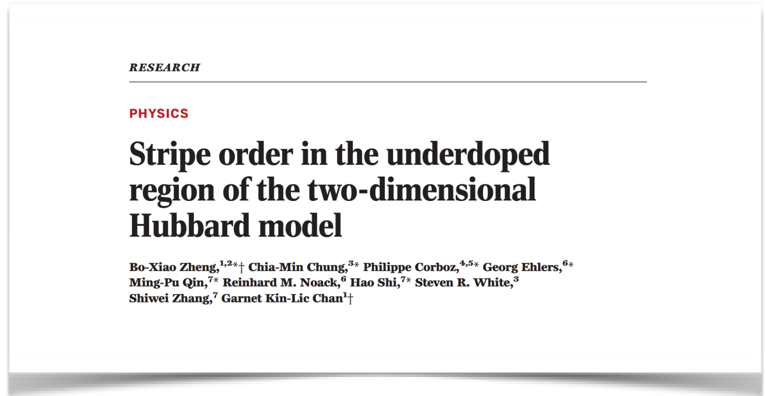
MERA

Vidal, PRL 101, 110501 (2008)



TENSORS NETWORKS

Computational Physics:



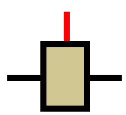
RESEARCH

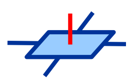
PHYSICS

Stripe order in the underdoped region of the two-dimensional Hubbard model

Bo-Xiao Zheng,[1,2]*† Chia-Min Chung,[3]* Philippe Corboz,[4,5]* Georg Ehlers,[6]* Ming-Pu Qin,[7]* Reinhard M. Noack,[6] Hao Shi,[7]* Steven R. White,[3] Shiwei Zhang,[7] Garnet Kin-Lic Chan[1]†

Analytics:



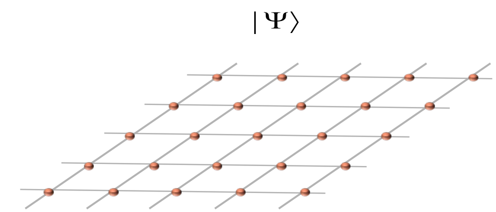


a single tensor describes the whole many-body state

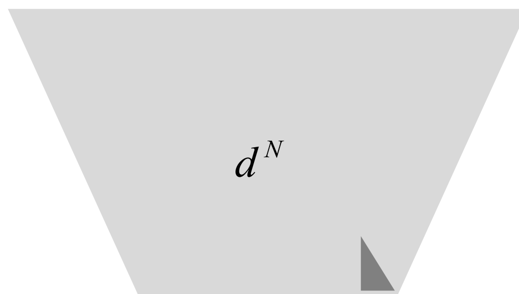
- Entanglement
- Bulk-boundary correspondence
- Symmetries: Classification of phases
- Gauging
- Topological systems
- Continuum limit

EFFICIENT DESCRIPTIONS:

- What do we learn?



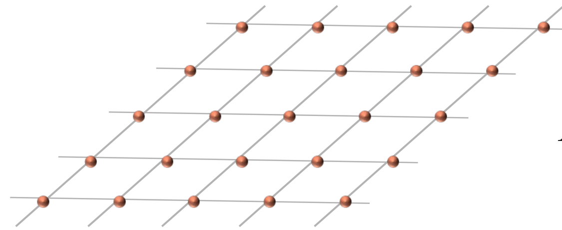
EXPONENTIAL HILBERT SPACE



It should be possible to find an efficient description of physical states:
Number of parameters should scale polynomically with N

- Zero Temperature (Gapped systems in 1D): Hastings J. Stat. Mech, P08024 (2007)
- Zero Temperature (Area law in any D): Wolf, Verstraete, Hastings, JIC, PRL 100, 070502 (2008)
- Finite Temperature (in any D): Molnar, Schuch, Verstraete, JIC, PRB**91**, 045138 (2015)

PROBLEM



$$H = \sum_n h_n$$

- We want to approximate the Gibbs state with a PEPO

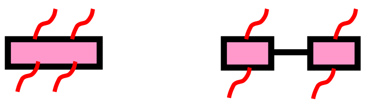
$$\rho_{PEPO} \approx \frac{e^{-\beta H}}{Z}$$

- We are interested in the scaling of the bond dimension, $D(N,\varepsilon,\beta)$
- We consider local Hamiltonians: $H = \sum_{\text{edges } n} h_n$
- Result:

$$\left\| \frac{e^{-\beta H}}{Z} - \rho_{PEPO} \right\|_1 < \varepsilon \quad \text{with a bond dimension} \quad D \approx (N/\varepsilon)^{\beta}$$

- Local operators: singular value decomposition

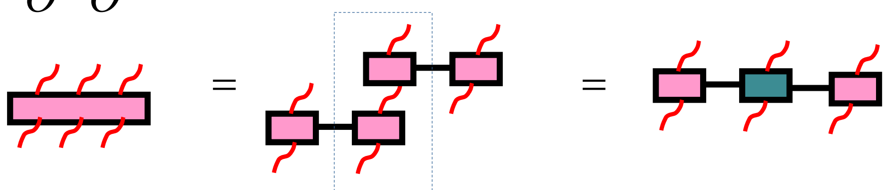
$$O^{1,2}=\sum_{\alpha=1}^{d^2} O_\alpha^1 \otimes O_\alpha^2$$



- Example: $e^{-\beta\sigma_x^1\otimes\sigma_x^2} = \cosh(\beta) 1\otimes 1 - \sinh(\beta)\sigma_x^1 \otimes \sigma_x^2$

- Concatenation:

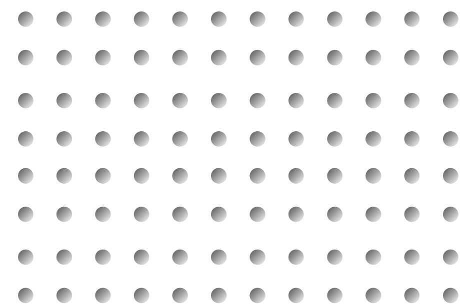
$O^{1,2}O^{2,3}$ $\qquad$ $O^{1,2}$ $\qquad$ $O^{2,3}$



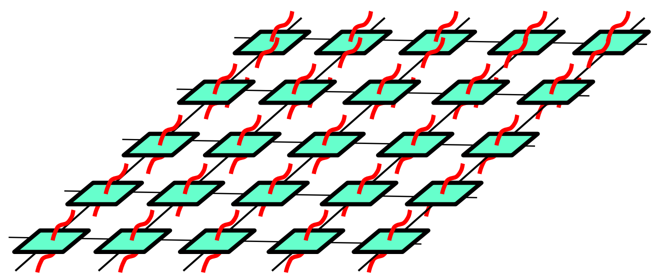
Multiplying by an operator acting on two sites, creates a link

GIBBS STATES
COMMUTING HAMILTONIANS

$$H = \sum_{\text{edges } n} h_n \qquad [h_n, h_m] = 0$$



$$e^{-\beta H} = e^{-\beta h_1} e^{-\beta h_2} \ldots e^{-\beta h_N}$$



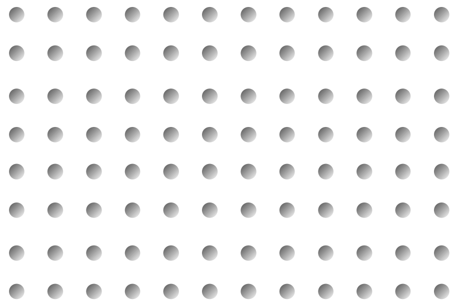
- It is an exact PEPO
- Bond dimension $D = d^2$

  - Independent of the physical dimension
  - Independent of the temperature

GIBBS STATES
GENERAL HAMILTONIANS

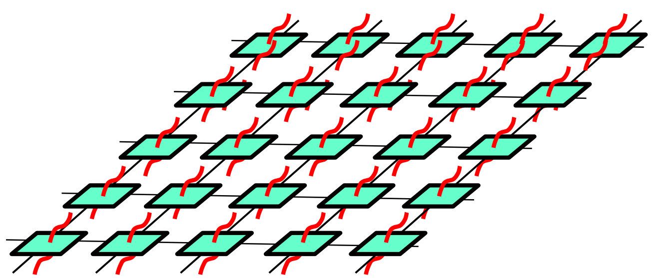
- Non-Commuting Hamiltonians:

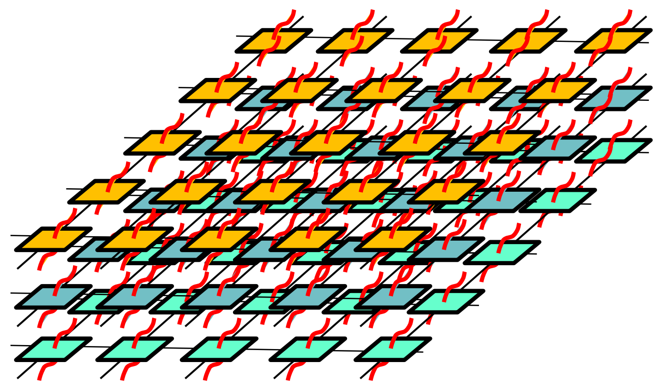
$$H = \sum_{\text{edges } n} h_n \qquad\qquad [h_n, h_m] \neq 0$$



- Trotter expansion:

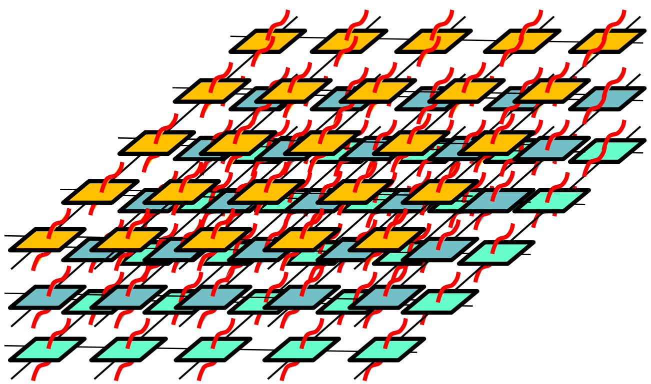
$$e^{-\beta H} = \lim_{M\to\infty} \Big(\underbrace{e^{-\beta h_1/M} e^{-\beta h_2/M} \ldots e^{-\beta h_n/M}} \Big)^M$$



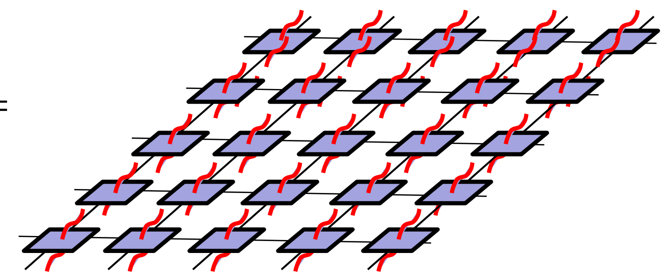


GIBBS STATES
GENERAL HAMILTONIANS

$$e^{-\beta H} = \lim_{M\to\infty}\left(e^{-\beta h_1/M} e^{-\beta h_2/M} \ldots e^{-\beta h_n/M}\right)^M$$



$=$



$$D = \left(D_0\right)^M \to \infty$$

GIBBS STATES
GENERAL HAMILTONIANS

1. Trotter approximation:

$$\frac{1}{Z}\left\| e^{-\beta H} - \left(e^{-\beta h_1/M} e^{-\beta h_2/M} \ldots e^{-\beta h_n/M} \right)^M \right\|_1 < \varepsilon$$

If $M > \beta^2 N^2 / \varepsilon$

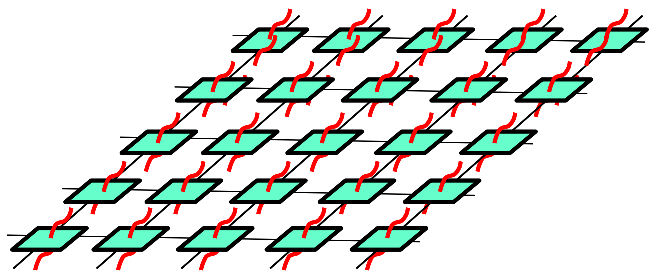
- But we still get an exponential scaling of the bond dimension:

$$D = \left(D_0\right)^M = e^{O(N^2/\varepsilon)}$$

GIBBS STATES
GENERAL HAMILTONIANS

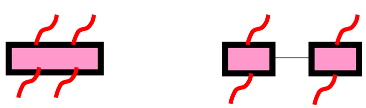
2. Compression:

$$\left(\underbrace{e^{-\beta h_1/M} e^{-\beta h_2/M} \ldots e^{-\beta h_n/M}}\right)^M$$

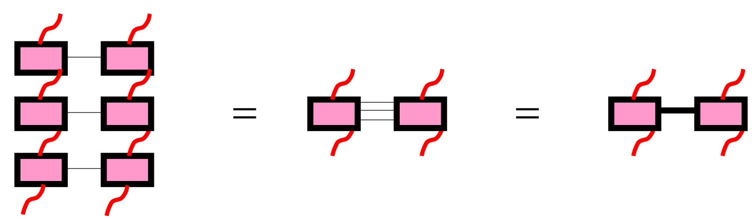


$$e^{-\beta h_{1,2}/M} \approx 1 \otimes 1 - \beta h_{1,2} / M$$

- Each product creates an „entangled bond“
- The entanglement of each bond is very small, *O(1/M)*
- The total entanglement should be *Mx(1/M),* ie *O(1)*



$$e^{-\beta h_1/M} \approx 1 \otimes 1 - \beta h_1 / M$$

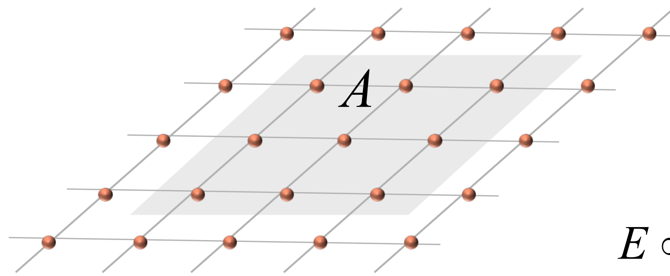


$$e^{-\beta h_1/M} e^{-\beta h'_1/M} \ldots e^{-\beta h''_1/M} \approx 1 \otimes 1 - \beta (h_1 + h'_1 + \ldots) / M + \ldots$$

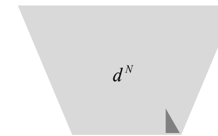
PROJECTE ENTANGLED-PAIR STATES

AREA LAW

$$|\Psi\rangle$$



$$E \propto |\partial A|$$

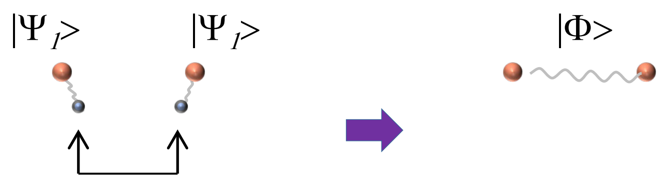


- Construction of states satisfying area law
- Translationally invariant

ENTANGLEMENT

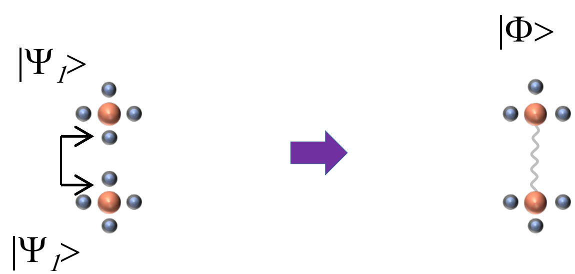
- Entanglement swapping

a)

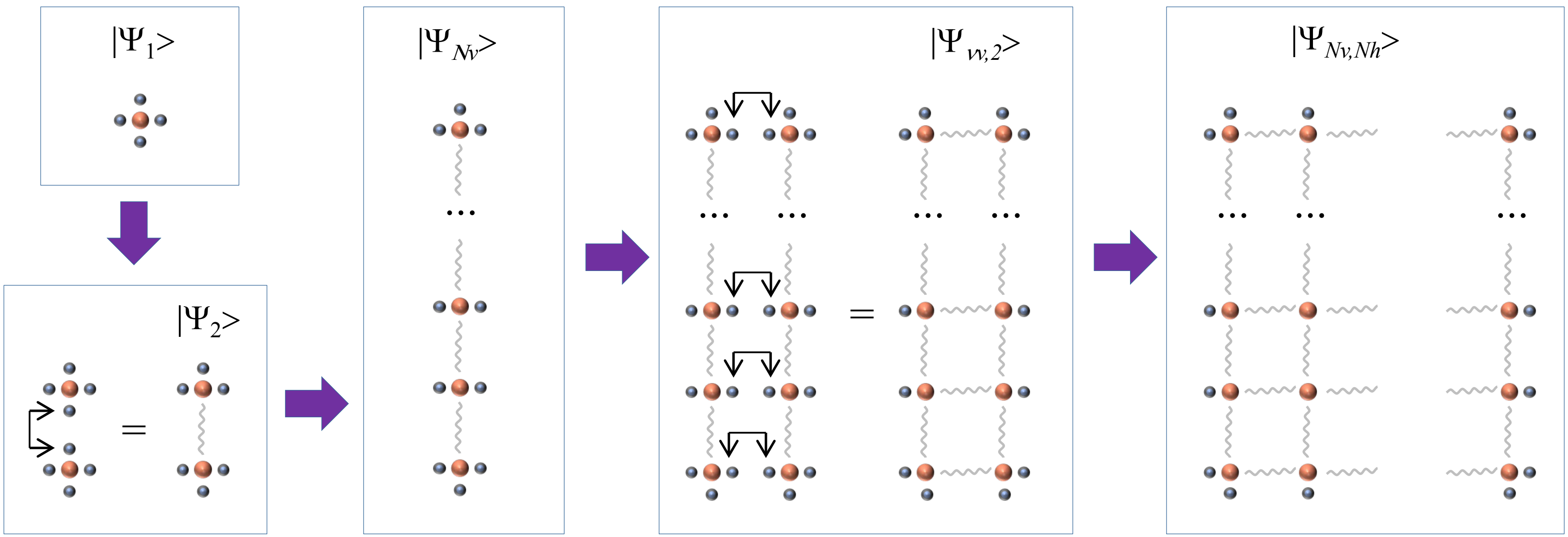


$$|\phi\rangle = \sum_{n=1}^{D} |n,n\rangle$$

b)

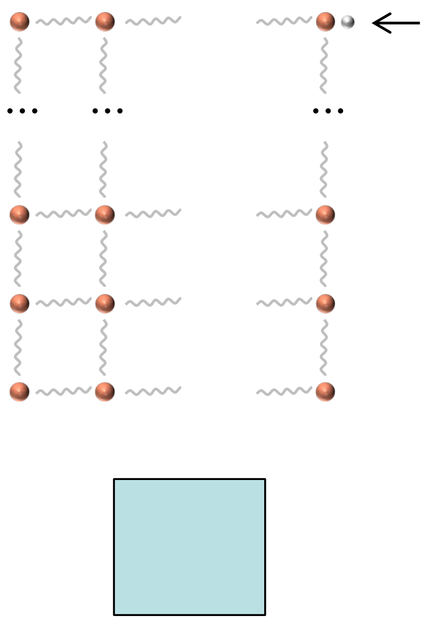


PROJECTED ENTANGLED-PAIR STATES

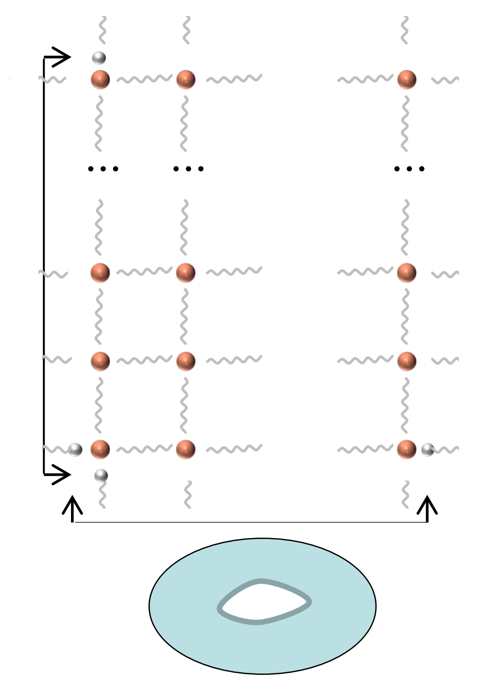


PROJECTED ENTANGLED-PAIR STATES

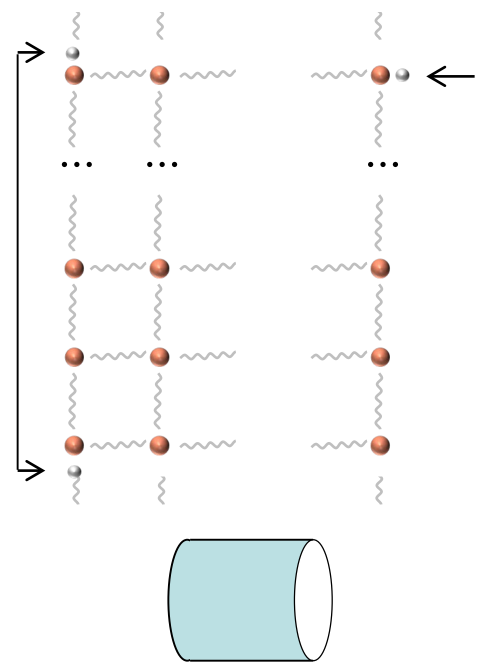
PLANE



TORUS

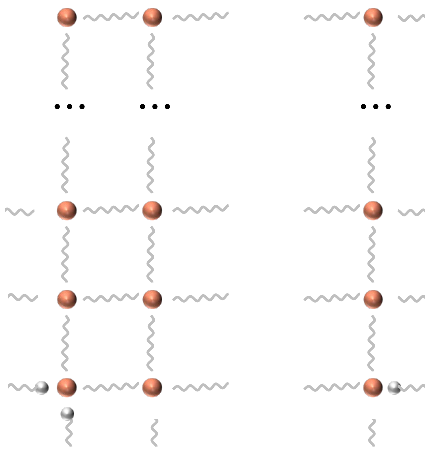


CYLINDER

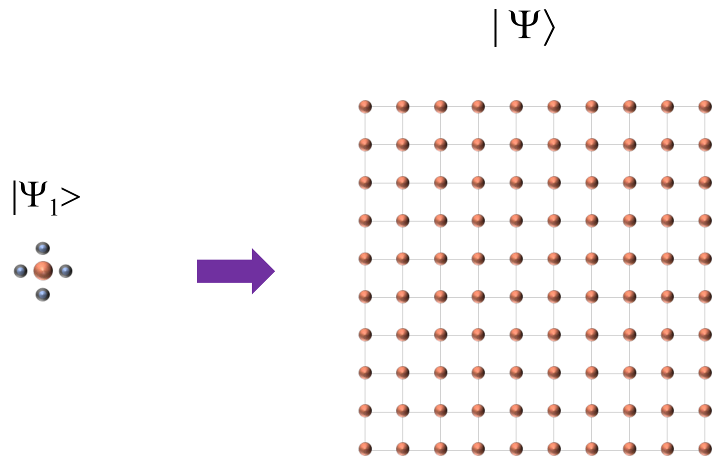


PEPS
PROPERTIES

- Fulfill are law
- Any lattice, any geometry, any dimension
- Fermions, spins, …
- Pure, mixed states
- Numerical simulations



PEPS
EFFICIENT DESCRIPTIONS

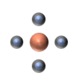


PEPS
PROPERTIES

- A single state determines the behavior

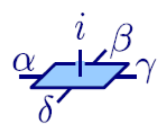
$\Rightarrow$ Tensor network

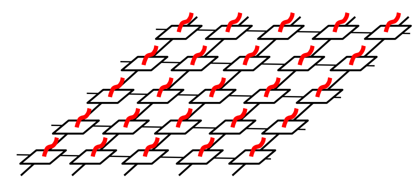
$|\Psi_1>$



$$|\Psi_1\rangle = \sum A^i_{\alpha\beta\gamma\delta} \, |i;\alpha,\beta,\chi,\delta\rangle$$

$A^i_{\alpha\beta\gamma\delta}$



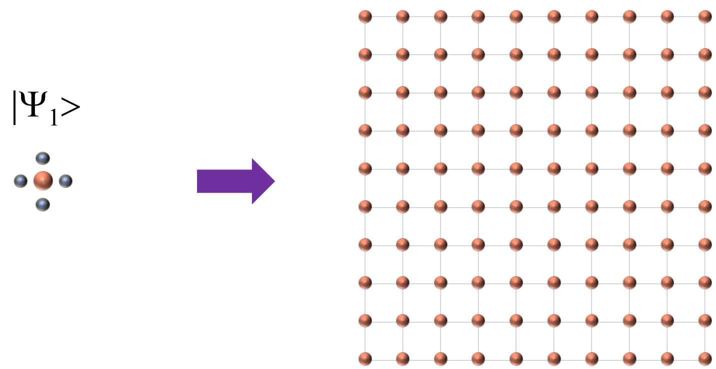


All physical properties are encapsulated in the tensor

PEPS
HAMILTONIANS

$|\Psi\rangle$

$|\Psi_1\rangle$



- Ground states

$$H|\Psi\rangle = E_0|\Psi\rangle$$

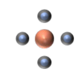
$$|\Psi\rangle \simeq |\Psi_{PEPS}\rangle$$

- Parent Hamiltonians

$$H_{PEPS}|\Psi_{PEPS}\rangle = E_0|\Psi_{PEPS}\rangle$$

PEPS
EXAMPLES

$|\Psi_1>$



PRODUCT

$$|\Psi_1\rangle = |0\rangle$$

GHZ

$$|\Psi_1\rangle = |0\rangle\,|0,0,0,0\rangle + |1\rangle\,|1,1,1,1\rangle$$

TORIC CODE

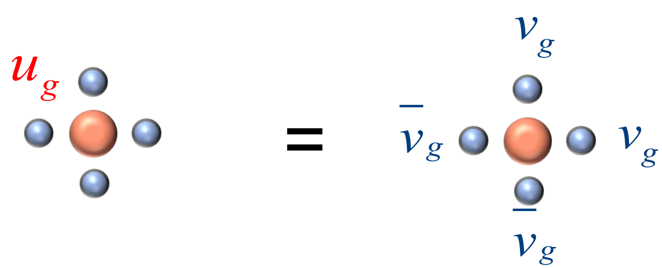
$$|\Psi_1\rangle = |0\rangle\,|\Psi^+,\Psi^+\rangle + |1\rangle\,|\Psi^-,\Psi^-\rangle$$

SYMMETRIES

GLOBAL SYMMETRIES

Perez-Garcia, Wolf, Sanz, Verstraete, JIC, PRL 100, 167202 (2008)

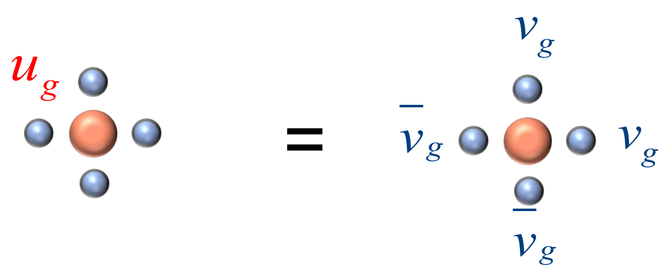
$$u_g \,|\Psi_1\rangle = v_g \otimes v_g \otimes \bar{v}_g \otimes \bar{v}_g \,|\Psi_1\rangle \qquad g \in G$$



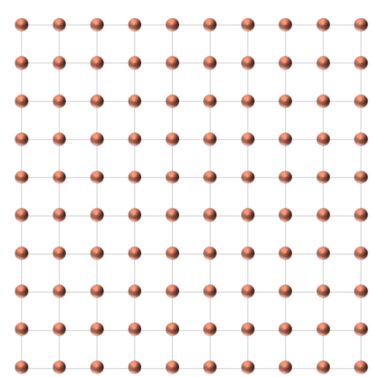
GLOBAL SYMMETRIES

Perez-Garcia, Wolf, Sanz, Verstraete, JIC, PRL 100, 167202 (2008)

$$u_g \,|\Psi_1\rangle = v_g \otimes v_g \otimes \bar{v}_g \otimes \bar{v}_g \,|\Psi_1\rangle \qquad g \in G$$



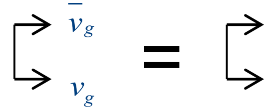
$|\Psi\rangle$



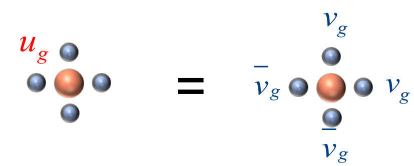
$$u_g^{\otimes N} \,|\Psi\rangle = |\Psi\rangle$$

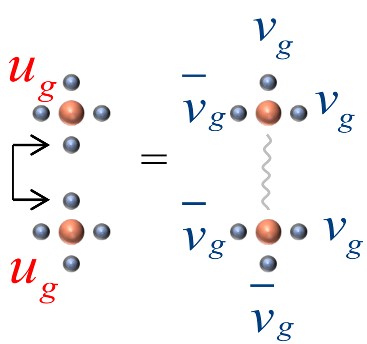
Ensures global symmetry

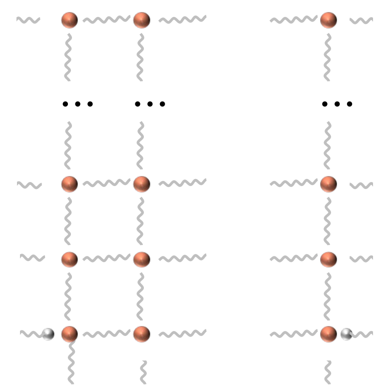
GLOBAL SYMMETRIES



$$\langle \phi | v_g \otimes \bar{v}_g = \langle \phi |$$







$$u_g^{\otimes N} | \Phi \rangle = | \Phi \rangle$$

remains invariant

And the v_g provide a (projective) representation

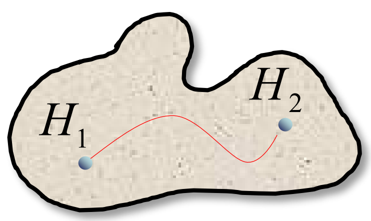
GLOBAL SYMMETRIES
CLASSIFICATION OF PHASES IN 1D

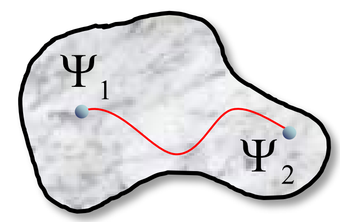
Pollman, Turner, Berg, Oshikawa, PRB 81, 064439 (2010).
Chen, Gu, Wen, PRB 83, 035107 (2011)
Schuch, Perez-Garcia, JIC, Phys. Rev. B 84, 165139 (2011)

GAPPED HAMILTONIANS

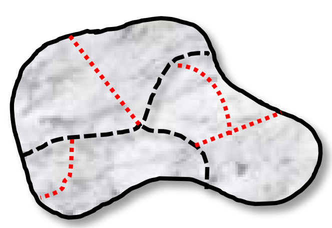
Hamiltonians

States



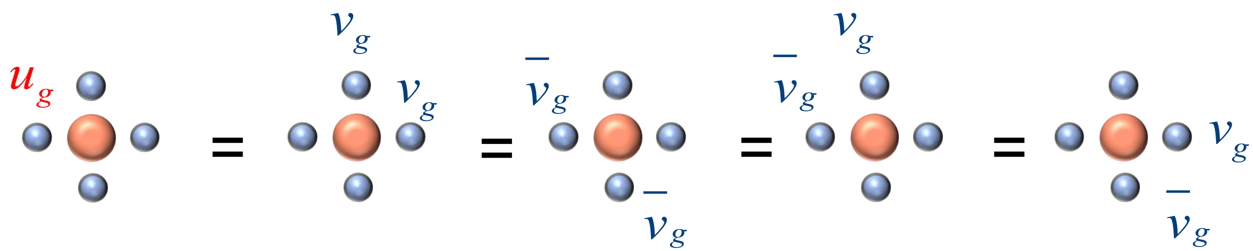


PHASES

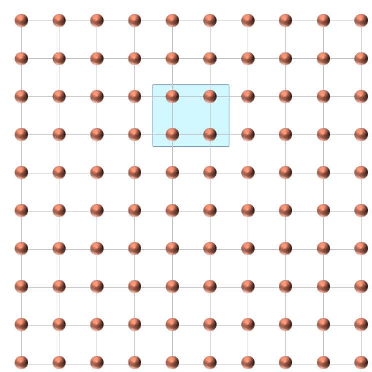


- No symmetries
- Symmetries

LOCAL (Gauge) SYMMETRIES



$|\Psi\rangle$

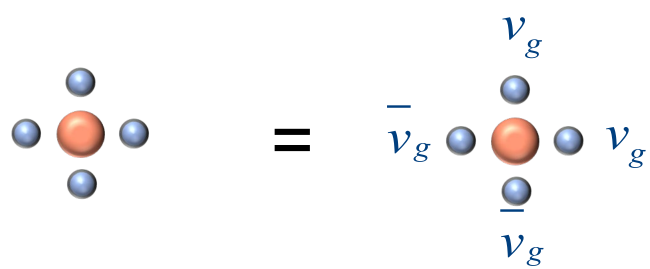


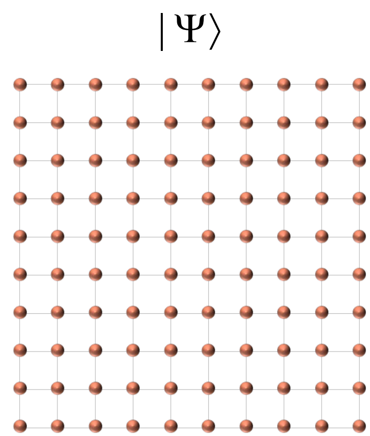
$$u_g^{\otimes 4}\,|\Psi\rangle = |\Psi\rangle$$

Ensures local gauge symmetry

AUXILIARY SYMMETRIES

Schuch, Perez-Garcia, JIC, Ann. Phys. **325**, 2153 (2010)

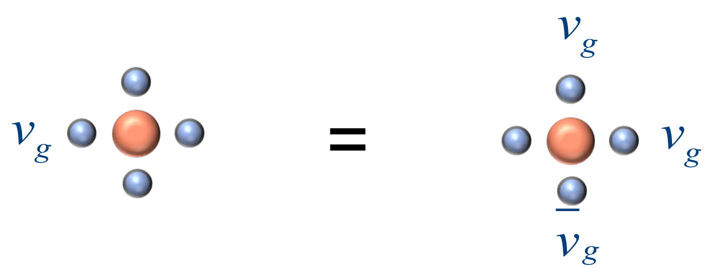


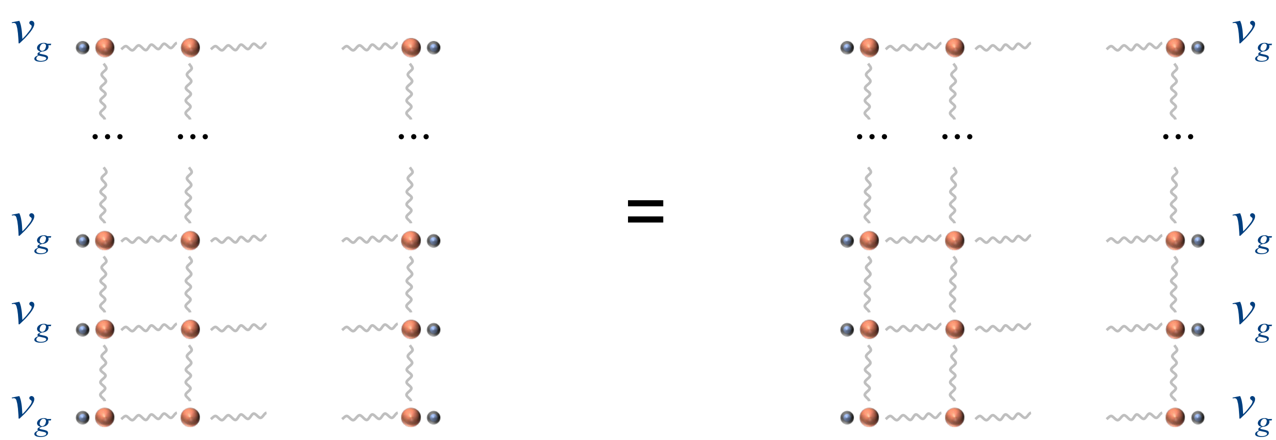


Ensures topological order

AUXILIARY SYMMETRIES

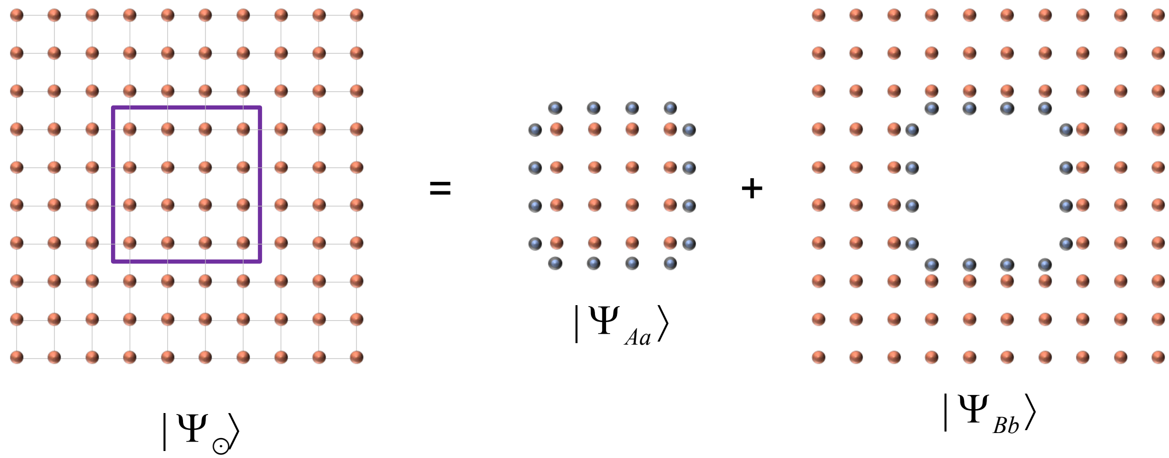
Schuch, Perez-Garcia, JIC, Ann. Phys. **325**, 2153 (2010)





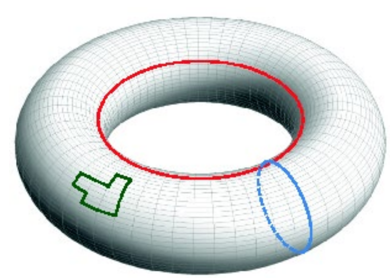
AUXILIARY SYMMETRIES
TOPOLOGICAL ORDER

Schuch, Perez-Garcia, JIC, Ann. Phys. **325**, 2153 (2010)



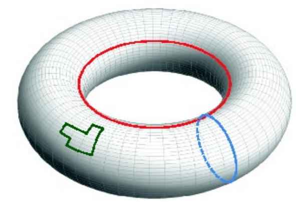
$$|\Psi_{\odot}\rangle = \langle \phi_{ab} | S | \Psi_{Aa}\rangle |\Psi_{Bb}\rangle$$

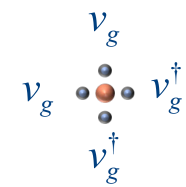
$$S = v_g^{\otimes C}$$



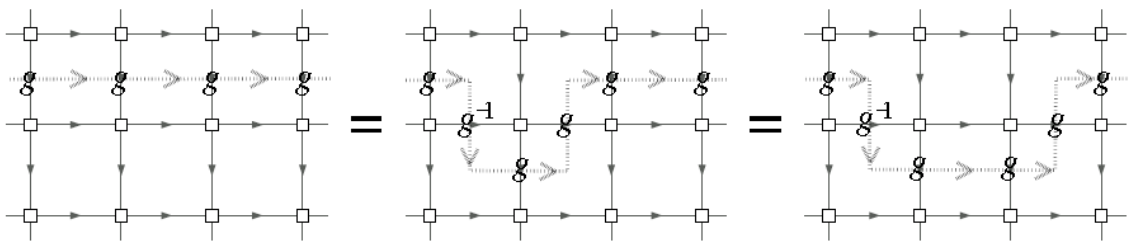
AUXILIARY SYMMETRIES
TOPOLOGICAL ORDER

Schuch, Perez-Garcia, JIC, Ann. Phys. **325**, 2153 (2010)



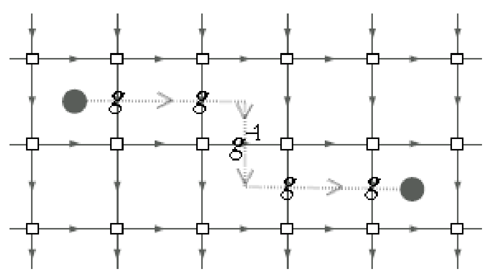


- Closed strings: Ground state



- Ground state degeneracy
- Locally indistinguishability

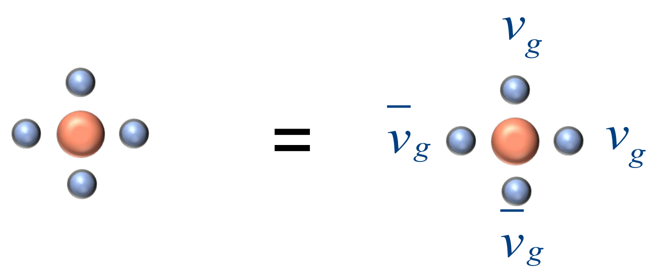
- Open strings: Excitations

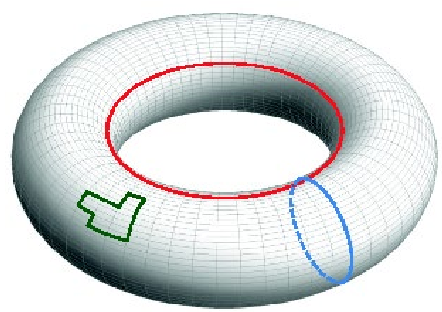


- Braiding
- Quantum computation

AUXILIARY SYMMETRIES
TOPOLOGICAL ORDER

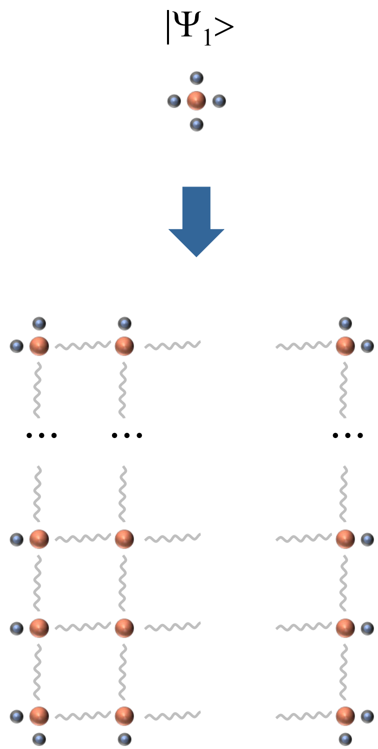
Schuch, Perez-Garcia, JIC, Ann. Phys. **325**, 2153 (2010)



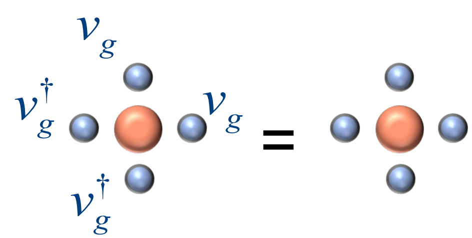


- Strings wrapping up the cylinder can be deformed and moved
- Correspond to different ground states
- Degeneracy, topological entropy (properties of the group)
- Anyons are open strings
- Brading, etc, is determined by the symmetries of the virtual particles

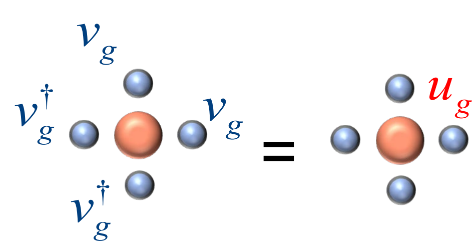
SYMMETRIES



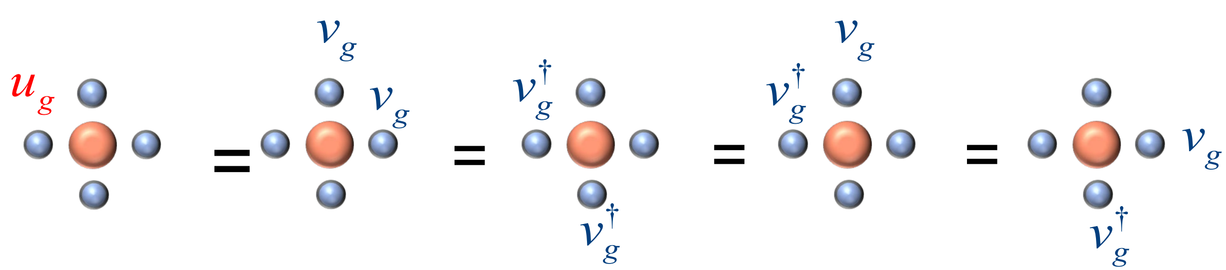
TOPOLOGY



GLOBAL SYMMETRIES



LOCAL GAUGE



BULK-BOUNDARY CORRESPONDENCE

JIC, Poilblanc, Schuch, and Verstraete, PRB **83**, 245134 (2011)
Schuch, Poilblanc, JIC, Perez-Garcia, PRB **86**, 115108 (2012),
Schuch, Poilblanc, JIC, Perez-Garcia, Phys. Rev. Lett. **111**, 090501 (2013)
Yang et al, Phys. Rev. Lett. 112, 036402 (2014)

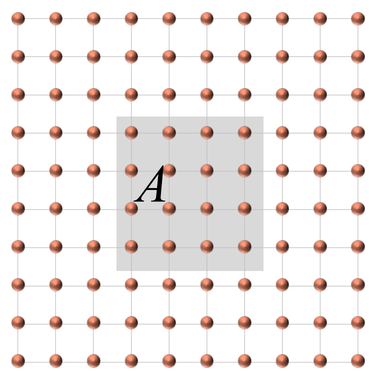
SPIN LATTICES

- Area law:

  Srednicki, Wilzek, …

$$E(\Psi) = S(\rho_A) \sim N_{\partial A}$$

\# degrees of freedom $\prec$ # particles at boundary



$$\rho_A = \mathrm{tr}\left[|\Psi\rangle\langle\Psi|\right]$$

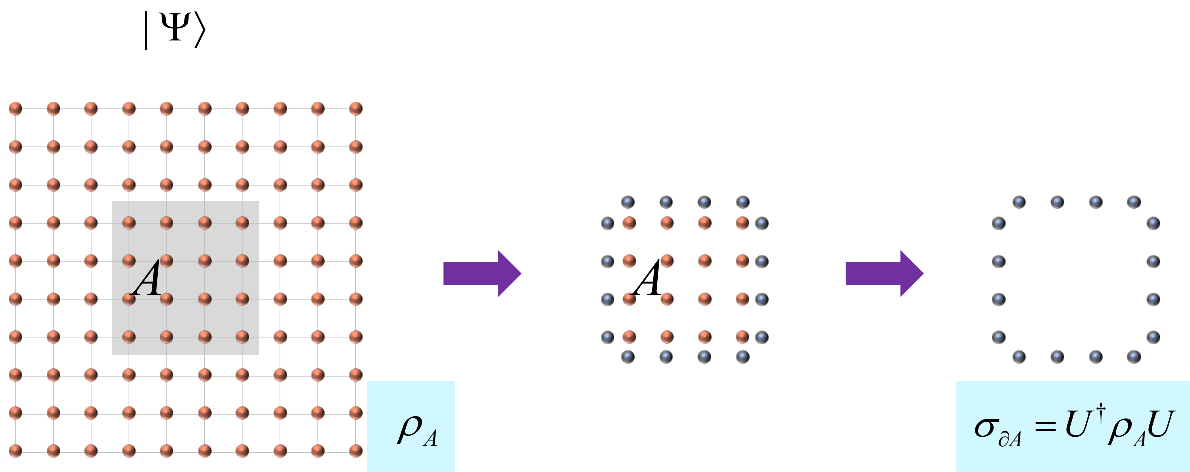
- Entanglement spectrum:

  Li and Haldane, 2008; Peschel, Kitaev and Preskill…

$$\rho_A = e^{-H_A} \qquad\qquad \sigma(H_A)$$

The low energy sector has the same structure as that for a lower dimensional theory (edge states)

BULK-BOUNDARY CORRESPONDENCE



- Isommetry between the particles in the bulk and the auxiliary ones in the boundary
- Holographic principle: all bulk properties can be determined at the boundary

- It „compresses“ the degrees of freedom
- Conserves the spectrum
- Allows to determine expectation values

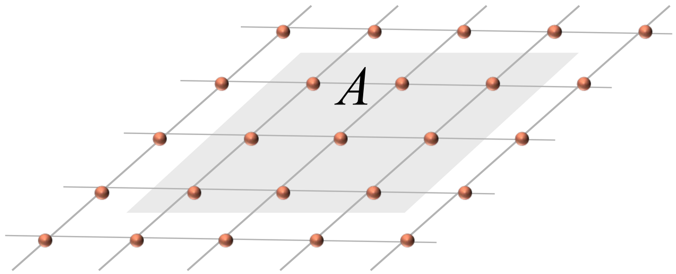
$$\mathrm{tr}\left(X_A \rho_A\right) = \mathrm{tr}\left(X_A U U^\dagger \rho_A U U^\dagger\right) = \mathrm{tr}\left(x_{\partial A} \sigma_{\partial A}\right)$$

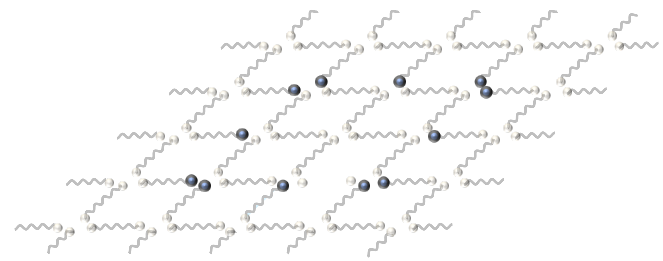
$$x_{\partial A} = U^\dagger X_A U$$

BOUNDARY HAMILTONIAN $\quad \sigma_{\partial A} = e^{-H_{\partial A}}$

- Has the same entanglement spectrum
- It can be easily determined (exactly or approximately)

PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE





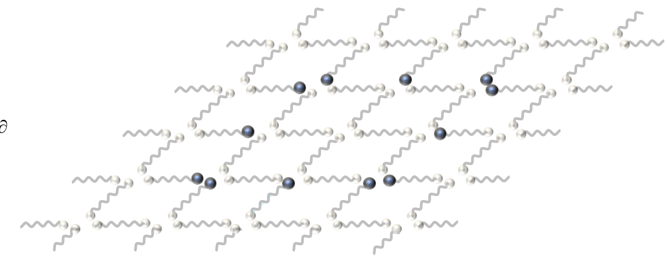
What can we say starting from the boundary Hamiltonian?
(beyond the entanglement spectrum)

- Is the Hamiltonian local?
- What are its symmetries, and how are they related to those of H?
- How do topological properties manifest themselves?
- What happens in quantum phase transitions?
- How general are those predictions?

Other approaches: Qi, Katsura, and Ludwig, 2012, Dubail, Read, and Rezayi, 2012

- Results:

$$\sigma_{\partial A} = e^{-H_{A\partial}}$$



  - Symmetries: The boundary Hamiltonian inherits the symmetries

$$u_g \, |\Psi\rangle = e^{i\theta_g} \, |\Psi\rangle \quad \Rightarrow \quad U_g H_{\partial A} U_g^\dagger = H_{\partial A}$$

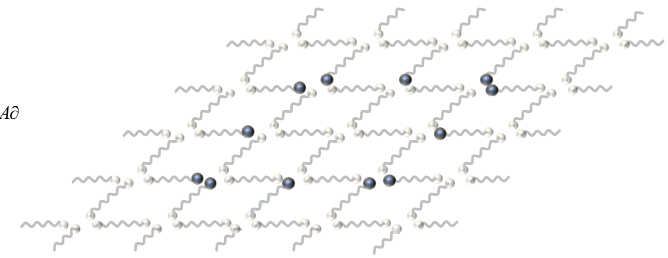
  - Locality:

    - For gapped systems, it is local
    - For critical systems, it becomes non-local

PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE

- Implications:

$$\sigma_{\partial A} = e^{-H_{A\partial}}$$



  - Entanglement spectrum corresponds to CFT theories:

    -Take a problem with su(2) symmetry and such that the boundary particles have semi-integer reps (eg, spin ½)
    - The boundary Hamiltonian is (close to) Heisenberg.

  - Quantum phase transitions:

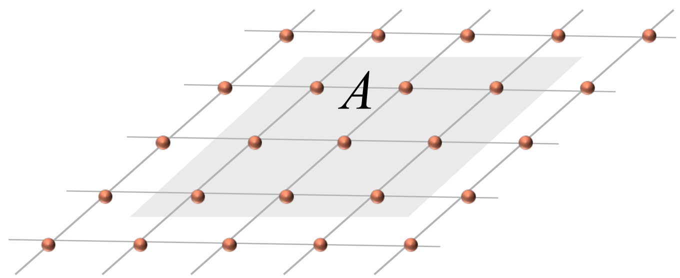
    -They are reflected in the boundary Hamiltonian

  - Contraction of PEPS:

    - One needs to express the boundary density operator as a MPO
    - Thermal states of local Hamiltonians can be (efficiently) written as MPO
    - For gapped system, the boundary density operator is such a thermal sate

PROJECTED ENTANGLED-PAIR STATES
BULK-BOUNDARY CORRESPONDENCE

Gapped topological phases in 2D

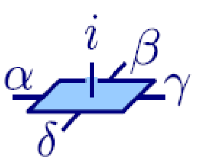


PROPERTIES

- Boundary state
- Boundary Hamiltonian

EXAMPLES

- Toric code (Kitaev)
- RVB states
- Phase transitions



- Results:
  - The boundary theory develops an extra symmetry

$$\sigma_{\partial A} = U_g \sigma_{\partial A} U_g^{\dagger}$$

- In general, the boundary operator is block diagonal $\sigma_{\partial A} = \sigma_{\partial A}^1 \oplus \sigma_{\partial A}^2 \oplus \ldots$
- The projector, P_i, on each subspace is higly non-local

  - The boundary Hamiltonian splits

$$H_{\partial A} = H_{\partial A}^{\text{topo}} + H_{\partial A}^{\text{non-universal}}$$

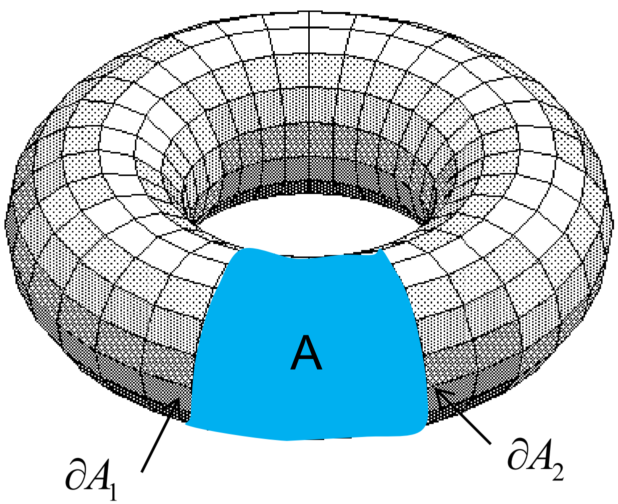
- $H_{\partial A}^{\text{topo}}$ is universal (only depends on the boundary conditions): $H_{\partial A}^{\text{topo}} = \oplus c_i P_i$
- $H_{\partial A}^{\text{non-universal}}$ is local and depends on the details of the state (but not on the boundary conditions)

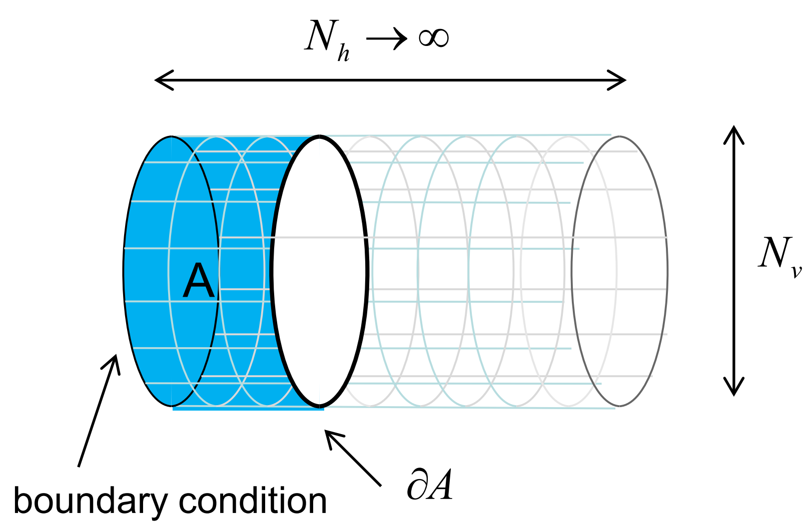
  - Phase transition

- $H_{\partial A}^{\text{non-universal}}$ becomes non-local
- It can eventually compensate the universal part $H_{\partial A}^{\text{topo}}$

BOUNDARY THEORY
TOPOLOGICAL PHASES

- Setup:
  - Cylinder or Torus

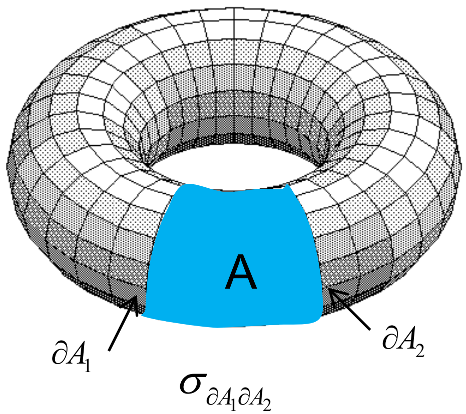




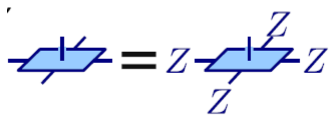
$$\sigma_{\partial A_1 \partial A_2}$$

$$\sigma_{\partial A} = \mathrm{tr}\left[X_{\partial A_1} \sigma_{\partial A_1 \partial A_2} \right]$$

BOUNDARY THEORY
TORIC CODE



- Symmetry:



$$\sigma_{\partial A_1 \partial A_2} = \left[Z^{\otimes N_v} \otimes Z^{\otimes N_v} \right] \sigma_{\partial A_1 \partial A_2} \left[Z^{\otimes N_v} \otimes Z^{\otimes N_v} \right]$$

- Projectors: $P_e = \frac{1}{2}\left(1 + Z^{\otimes N_v}\right)$ even parity

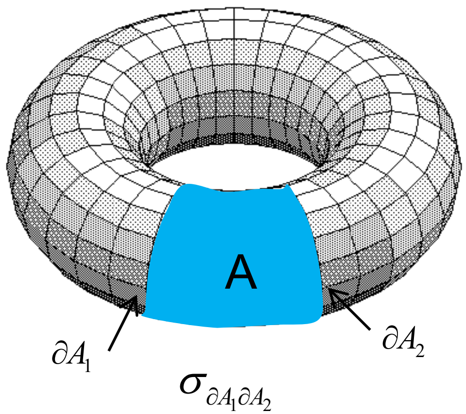
  $P_o = \frac{1}{2}\left(1 - Z^{\otimes N_v}\right)$ odd parity

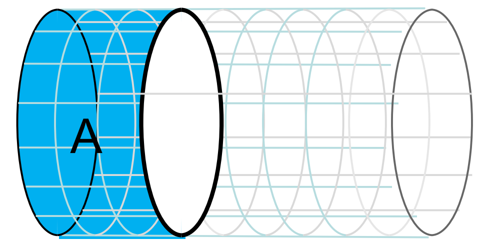
- Boundary state:

$$\sigma_{\partial A_1 \partial A_2} = P_e \otimes P_e + P_o \otimes P_o$$

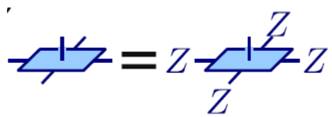
-Total even parity
- Non-local

BOUNDARY THEORY
TORIC CODE





- Symmetry:



$$\sigma_{\partial A_1 \partial A_2} = \left[Z^{\otimes N_v} \otimes Z^{\otimes N_v} \right] \sigma_{\partial A_1 \partial A_2} \left[Z^{\otimes N_v} \otimes Z^{\otimes N_v} \right]$$

- Projectors: $P_e = \frac{1}{2}\left(1 + Z^{\otimes N_v}\right)$ even parity

  $P_o = \frac{1}{2}\left(1 - Z^{\otimes N_v}\right)$ odd parity

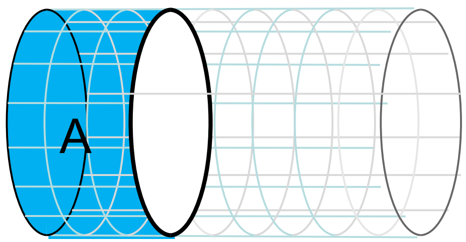
- Boundary state:

$$\sigma_{\partial A_1 \partial A_2} = P_e \otimes P_e + P_o \otimes P_o \quad \Rightarrow \quad \sigma_{\partial A} = \mathrm{tr}\left[X_{\partial A_1} \sigma_{\partial A_1 \partial A_2} \right] = p_e P_e + p_o P_o$$

-Total even parity
- Non-local

(note top EE requires $p_e = 0$)

BOUNDARY THEORY
TORIC CODE



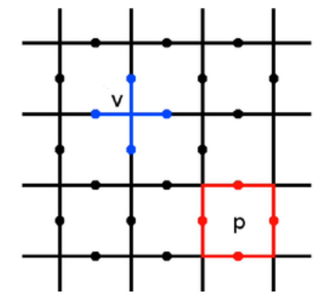
$$\sigma_{\partial A} = \mathrm{tr}\left[X_{\partial A_1} \sigma_{\partial A_1 \partial A_2} \right] = p_e P_e + p_o P_o$$

- **Symmetry:** $\sigma_{\partial A} = Z^{\otimes N_v} \sigma_{\partial A} Z^{\otimes N_v}$
- **Projectors:** $P_e = \frac{1}{2}\left(1 + Z^{\otimes N_v}\right)$ even parity

  $P_o = \frac{1}{2}\left(1 - Z^{\otimes N_v}\right)$ odd parity
- **Boundary Hamiltonian:** $H_{\partial A} = H_{\partial A}^{\text{topo}} = -\log(p_e) P_e - \log(p_o) P_o$

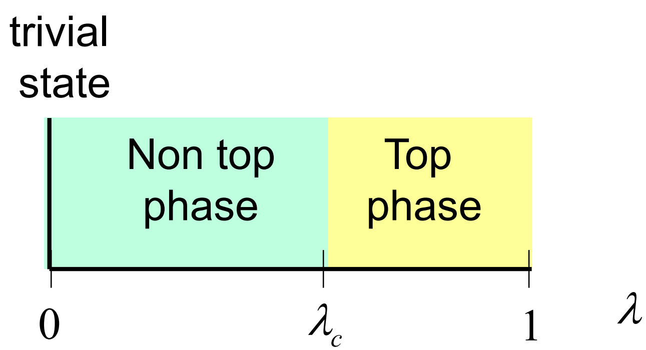
  - The values of $p_{e,o}$ depend on the chosen boundaries
  - It is highly non-local (like the projectors)
  - There is no non-universal part (it is a fixed point of the RG flow)

- Deformed Kitaev model:
  - Kitaev: $H = -\sum_p H_p - \sum_v H_v$
  - Deformed Kitav model (Castelnovo and Chamon, PRL 2008)

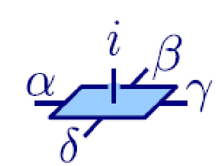


$$H(\lambda) = -\sum_p H_p(\lambda) - \sum_v H_v(\lambda)$$

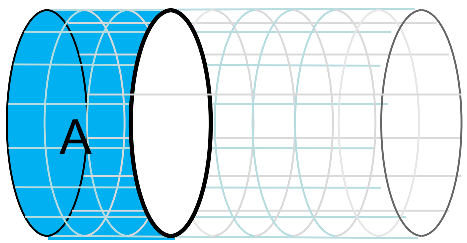
$$H_x(\lambda) = (|0\rangle\langle 0| + \lambda^{-1}|1\rangle\langle 1|)^{\otimes 4} H_x (|0\rangle\langle 0| + \lambda^{-1}|1\rangle\langle 1|)^{\otimes 4}$$

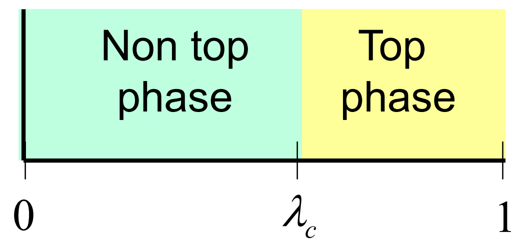


The ground state is a PEPS (with D=2) for all values of λ



- Deformed Kitaev model:





- Conjecture: $H_{\partial A} = H_{\partial A}^{\text{topo}} + H_{\partial A}^{\text{non-universal}}$

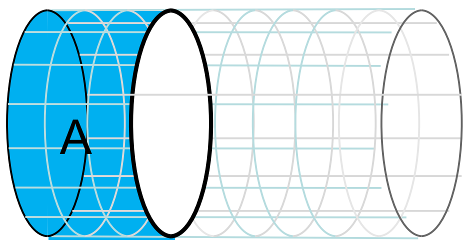
  where $H_{\partial A}^{\text{topo}} = -\log(p_e)P_e - \log(p_o)P_o$ is the one calculated for λ=1 (toric code)

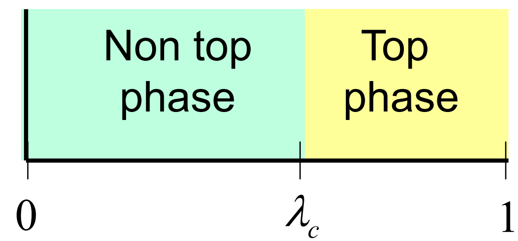
- Calculation:

  1. Fix the boundary condition (determines $p_{e,o}$).
  2. Determine the boundary state: $\sigma_{\partial A}$
  3. Compute the boundary Hamiltonian: $H_{\partial A} = -\log\left(\sigma_{\partial A}\right)$
  4. Substract the universal topo Hamiltonian: $H_{\partial A}^{\text{non-universal}} = H_{\partial A} - H_{\partial A}^{\text{topo}}$
  5. Determine the „interaction lenght of the non-universal part“

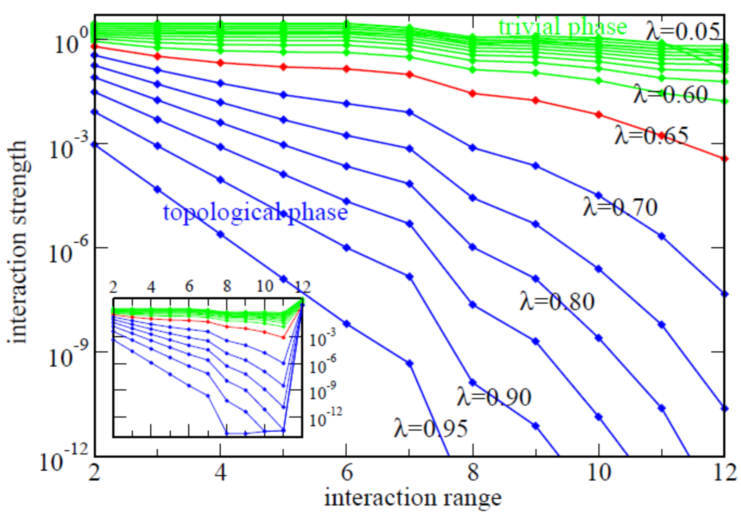
BOUNDARY THEORY
PHASE TRANSITIONS

- Deformed Kitaev model:





$$H_{\partial A}^{\text{non-universal}} = H_{\partial A} - H_{\partial A}^{\text{topo}}$$

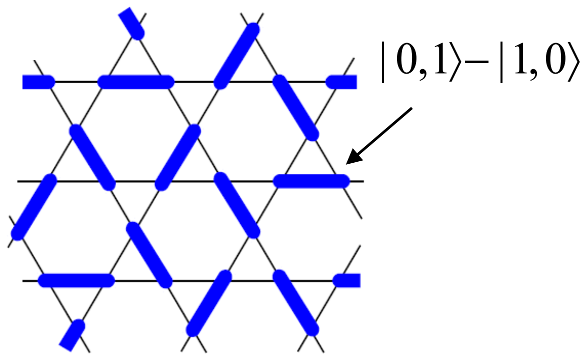


- Lattice: $\infty \times 12$
- Topological pase:
  Interaction lenght decreases exponentially
- Non-Topological pase:
  Long-range interactions

BOUNDARY THEORY
PHASE TRANSITIONS

- RVB states in a Kagome lattice:

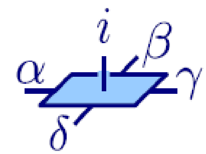
VBS



$$|RVBS\rangle = \sum |VBS\rangle$$

(Anderson)

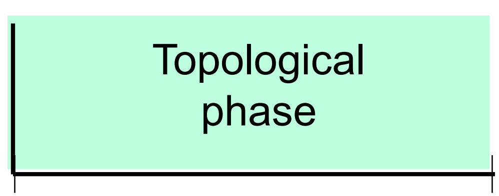
$$A^{i_n}_{\alpha\beta\gamma\delta}$$



$$A^0_{0222} = A^0_{2022} = -A^0_{2212} = -A^0_{2221} = 1$$

$$A^1_{1222} = A^1_{2122} = -A^1_{2202} = -A^1_{2220} = 1$$

- We can find a local parent Hamiltonian, H.
- We can find a deformation: $A^{i_n}_{\alpha\beta\gamma\delta}(\theta) \Rightarrow |\Psi(\theta)\rangle$

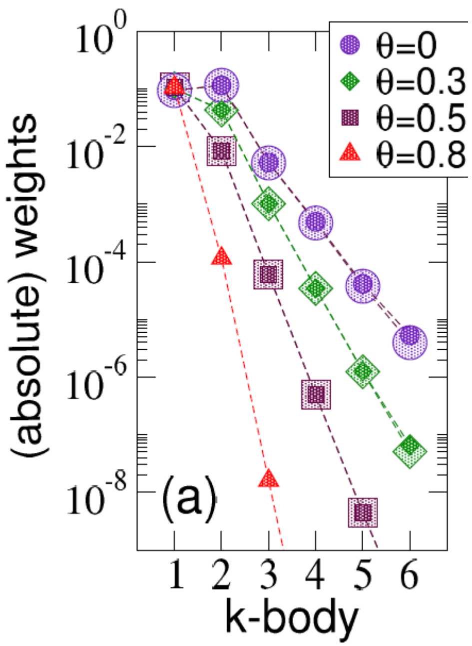


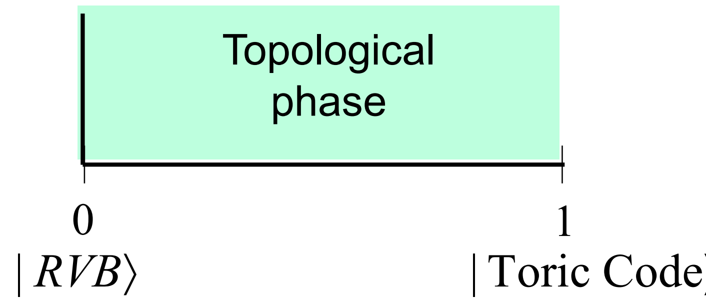
0 $\quad$ 1

$|RVB\rangle$ $\quad$ $|\text{Toric Code}\rangle$

BOUNDARY THEORY
PHASE TRANSITIONS

- RVB states in a Kagome lattice:





- We calculate the non-universal part.

$$H_{\partial A}^{\text{non-universal}} = H_{\partial A} - H_{\partial A}^{\text{topo}}$$

- It is local: no phase transition

BOUNDARY THEORY
EDGES OF A SYSTEM

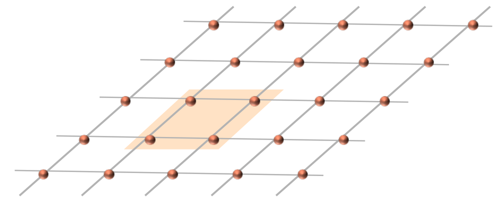
- PEPS: Parent Hamiltonians

$$H\,|\,\Psi\rangle = 0$$

$$H = \sum h_n \leftarrow$$

Finite range interaction

$$h_n \geq 0$$



$$h_n\,|\,\Psi\rangle = 0$$

BOUNDARY THEORY
EDGES OF A SYSTEM

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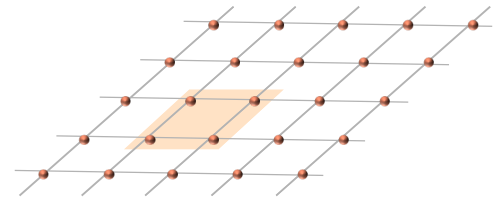
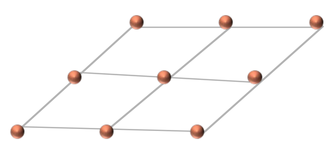
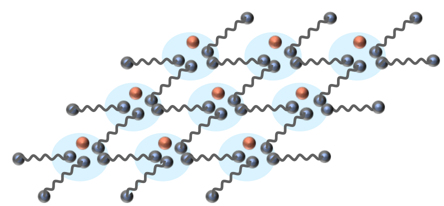


$$h_n\,|\,\Psi\rangle = 0$$

- Open boundary conditions:





  - Ground state degeneracy: $D^{\partial A}$

BOUNDARY THEORY
EDGES OF A SYSTEM

- PEPS: Parent Hamiltonians

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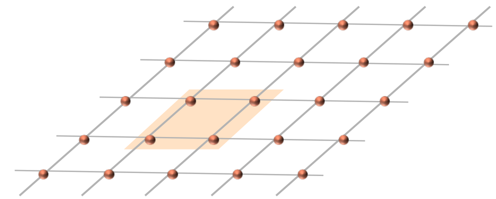

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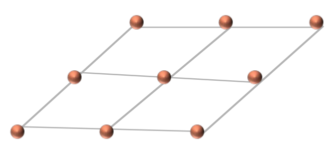

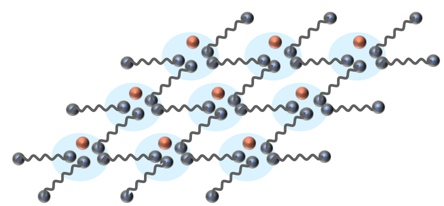

  - Ground state degeneracy: $D^{\partial A}$
  - It will be lifted by any perturbation in the bulk: $H' = \sum\left(h_n + \varepsilon v_n\right)$

$$H' = \sum\left(h_n + \varepsilon v_n\right) \sim \varepsilon \mathbf{P}_0 \sum v_n \mathbf{P}_0 \quad \Rightarrow h_{\text{Edge}}$$

Edge modes: their properties depend on the perturbation