Tensor Networks and the Many-Body Problem

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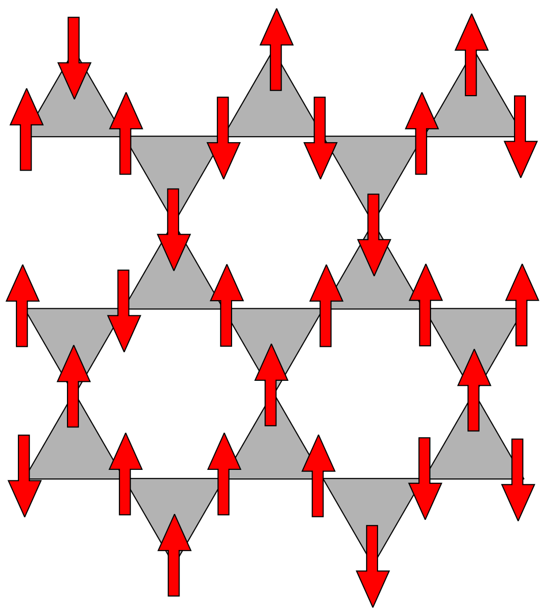
Slides & Tutorials:
http://go.tum.de/475840

Quantum Dynamics: From Electrons to Qbits
ICTP, August 2022

Efficient Simulation of Quantum Many-Body Systems



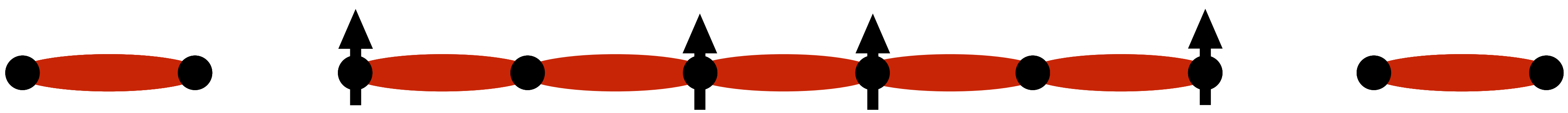
$$H = \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

What is the nature of the many body ground state $|\psi_0\rangle$?

Efficient Simulation of Quantum Many-Body Systems

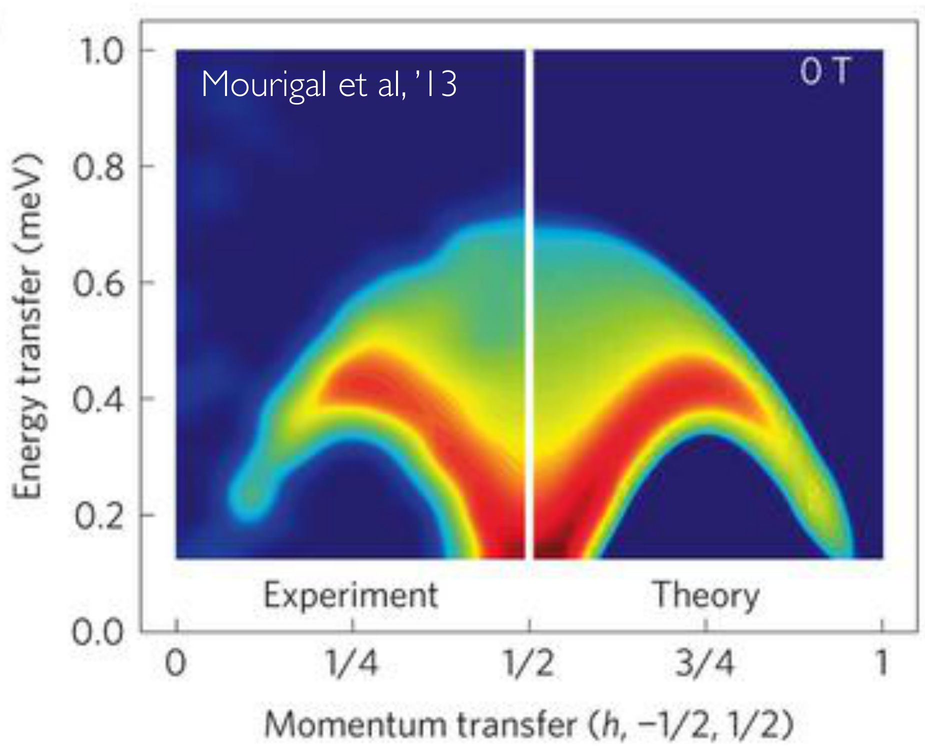
1D quantum spin liquid: Fractional **spinon excitations** in the antiferromagnetic S=1/2 Heisenberg chain

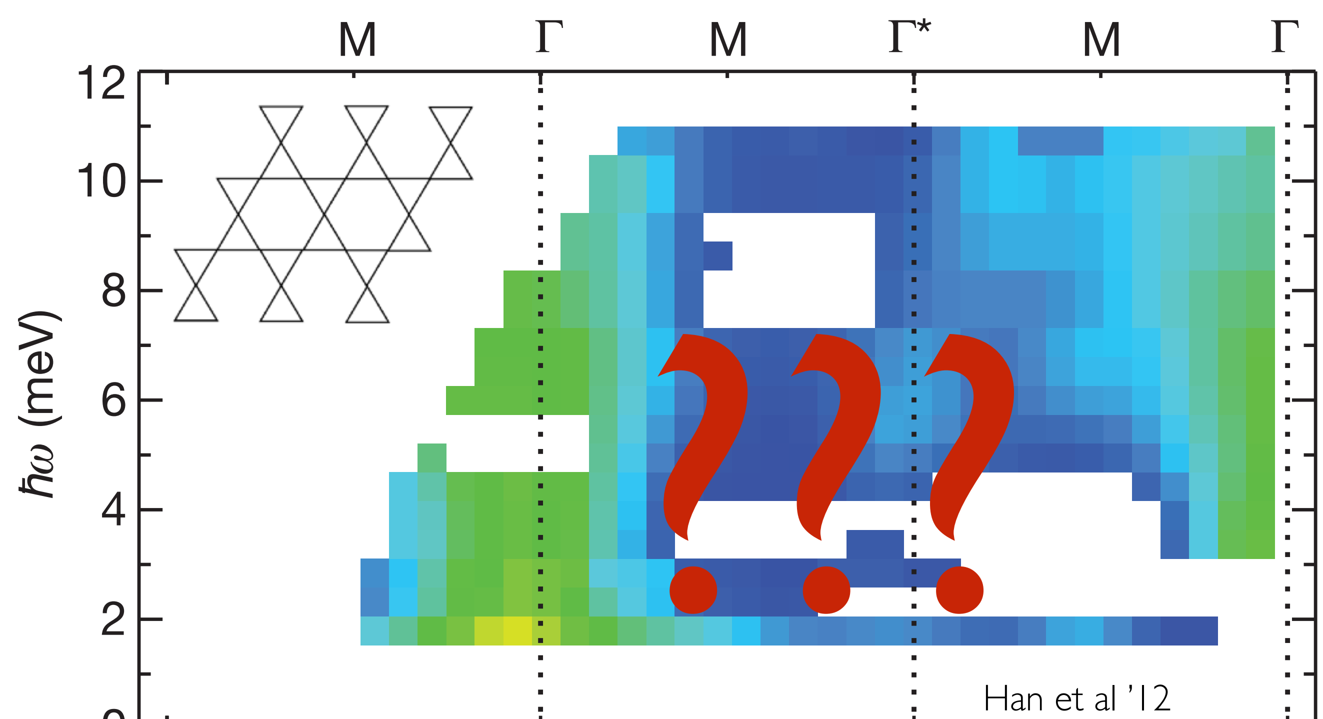
Copper Sulphate



$S = 0$

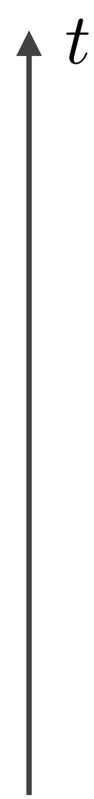
Dynamical structure factor:



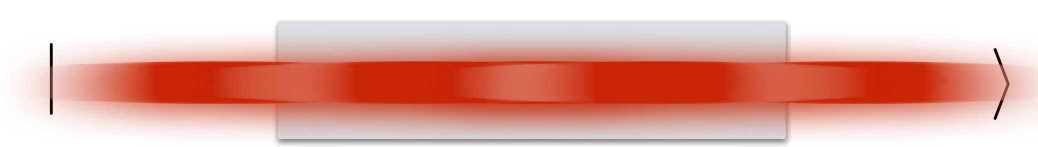


Simulating Quantum Thermalization

Investigate whether/how closed quantum many-body systems thermalize:

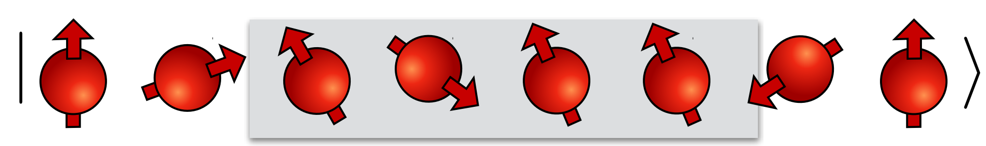


$$\rho_{\text{Block}} = \rho_{\text{Thermal}}$$



$$U_t = \exp(-itH)$$

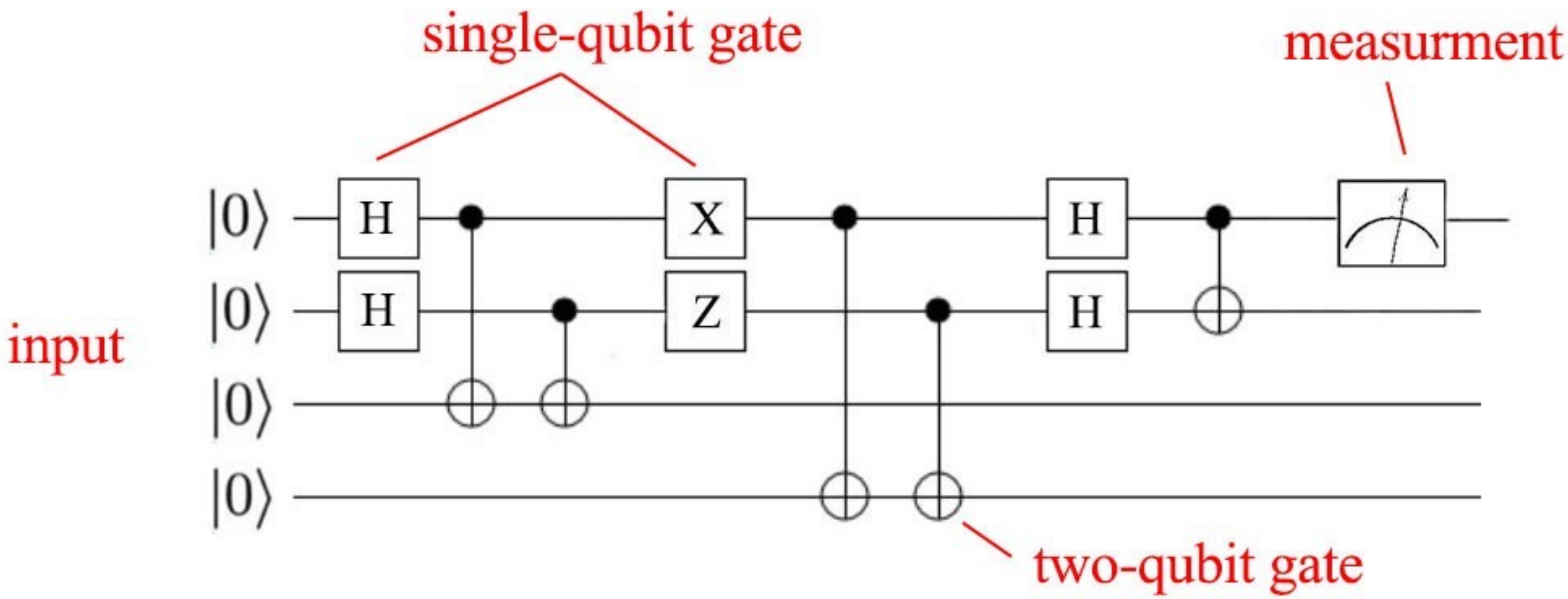
[non-integrable]



Closed quantum system

[Srednicki, Deutsch, Rigol]

Simulating a Quantum Computer



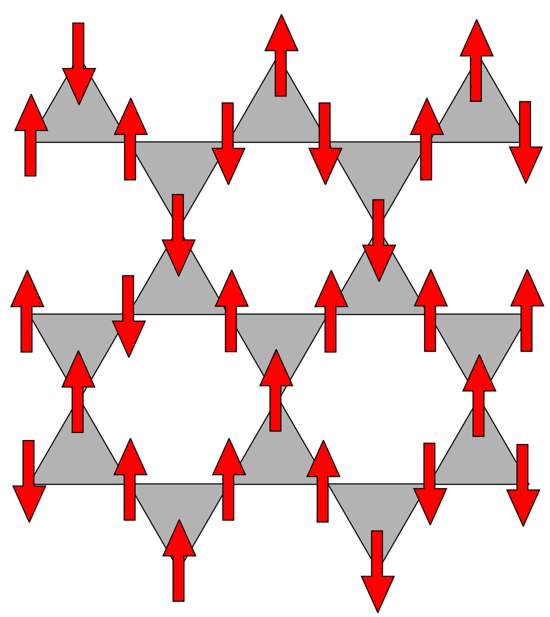
$|\psi_i\rangle$ $|\psi_f\rangle$

Quantum Many-Body Systems

The Hilbert space of a quantum many body system grows exponentially!

$$|\psi\rangle = \sum_{j_1, j_2, \dots, j_L} \psi_{j_1, j_2, \dots, j_L} |j_1\rangle |j_2\rangle \dots |j_L\rangle$$

For N spin-1/2 particles: **2^N states**



10 spins dim=1‘024
20 spins dim=1‘048‘576
30 spins dim=1’073‘741‘824
40 spins dim=1‘099‘511‘627‘776

Matrix-Product States

Many-body Hilbert space

$$|\psi\rangle = \sum_{j_1,j_2,\ldots,j_L} \psi_{j_1,j_2,\ldots,j_L} |j_1\rangle|j_2\rangle\ldots|j_L\rangle \ , \ j_n = 1\ldots d$$

Matrix-Product States: Reduction of #variables $d^L \rightarrow Ld\chi^2$

$$\psi_{j_1,j_2,\ldots,j_L} \approx \sum_{\alpha_1,\alpha_2,\ldots,\alpha_{L-1}} A^{j_1}_{\alpha_1} A^{j_2}_{\alpha_1,\alpha_2} \ldots A^{j_L}_{\alpha_{L-1}} \qquad \alpha_j = 1\ldots\chi$$

Once we have an MPS representation, we can calculate (almost) everything exactly!

Tensor Networks and the Many-Body Problem

I) Entanglement and Matrix-Product States 👈

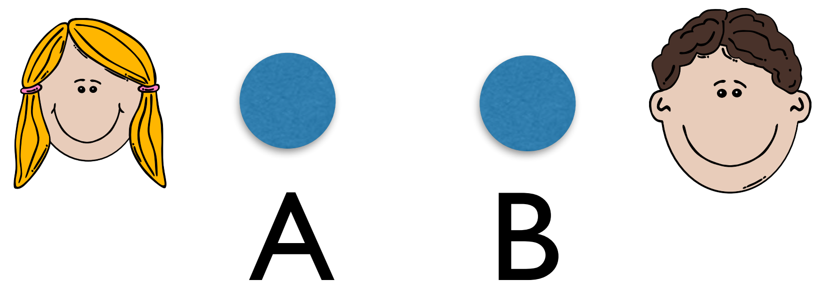
II) Time Evolving Block Decimation

III) Density-Matrix Renormalization Group

IV) Extracting topological invariants

V) (isometric Tensor-Network States)

Bipartite Entanglement



Product state (=non-entangled):

$$|\psi\rangle = \frac{1}{2}\Big(|\uparrow\rangle_A + |\downarrow\rangle_A\Big)\Big(|\uparrow\rangle_B + |\downarrow\rangle_B\Big)$$

Entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(|\uparrow\rangle_A|\downarrow\rangle_B + |\downarrow\rangle_A|\uparrow\rangle_B\Big)$$

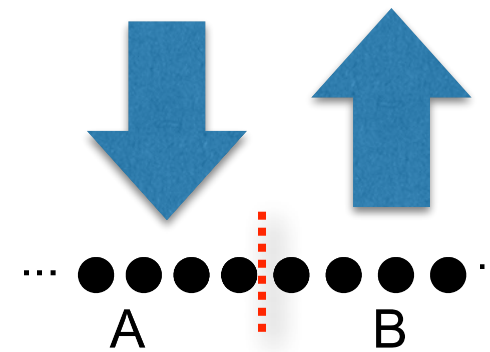
[Einstein, Rosen, Podolsky '35]

Bipartite Entanglement

Quantum state in d^L dimensional Hilbert space

$$|\psi\rangle = \sum_{j_1, j_2, \ldots, j_L} \psi_{j_1, j_2, \ldots, j_L} |j_1\rangle |j_2\rangle \ldots |j_L\rangle \ , \ j_n = 1 \ldots d$$

Decompose a state into a superposition of product states (**Schmidt decomposition**)



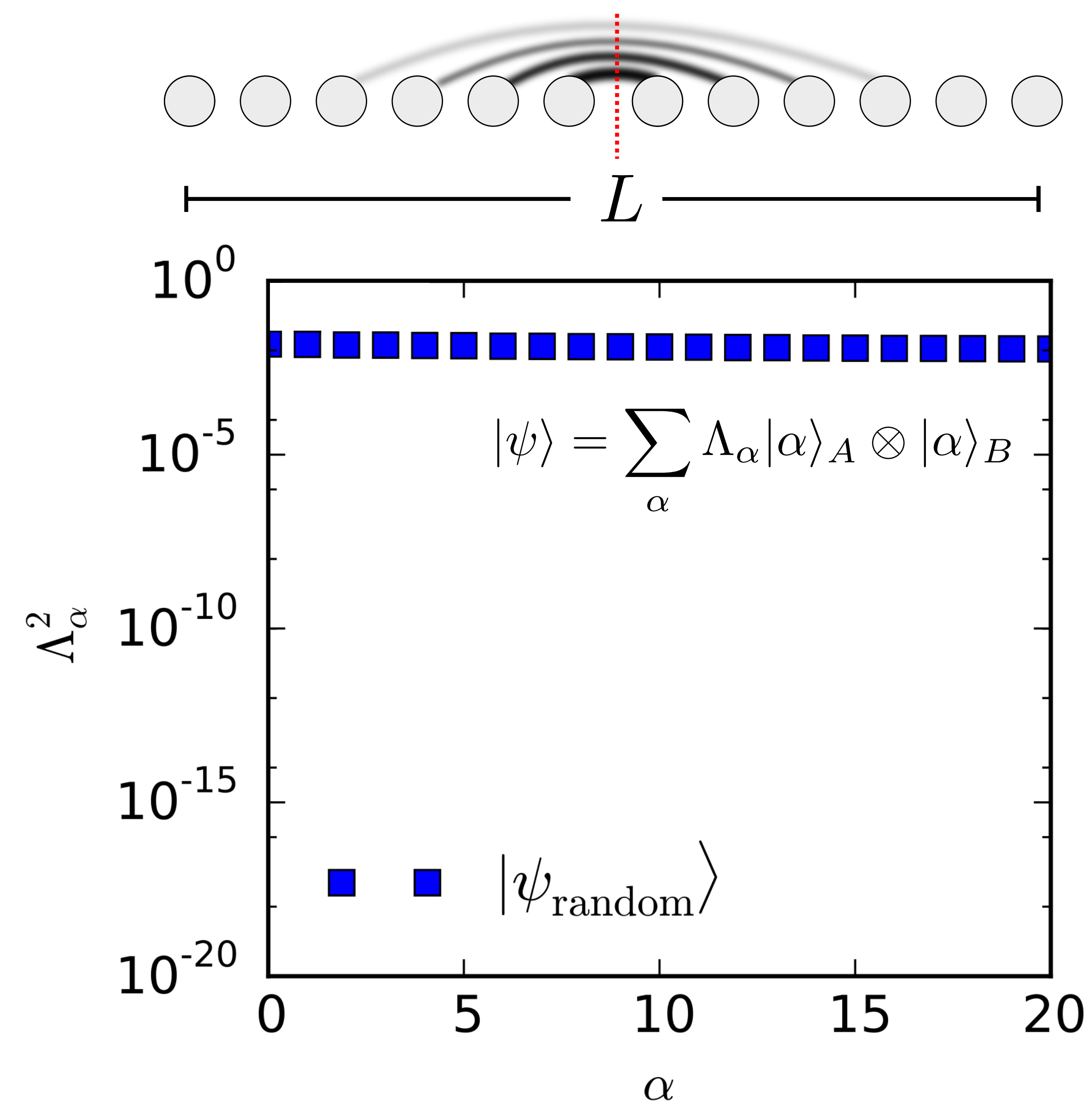
$$|\psi\rangle = \sum_{i,j} C_{i,j} |i\rangle_A \otimes |j\rangle_B = \sum_\alpha \Lambda_\alpha |\alpha\rangle_A \otimes |\alpha\rangle_B, \ \langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}$$

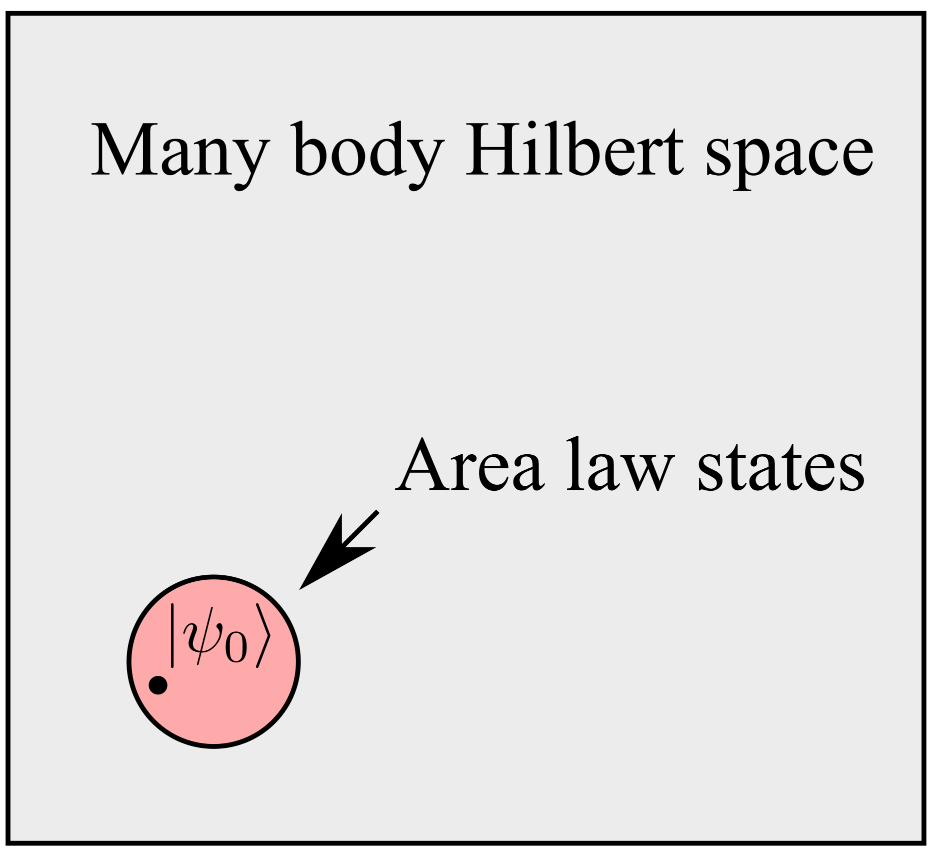
Entanglement entropy as a measure for the amount of entanglement $S = -\sum_\alpha \Lambda_\alpha^2 \log \Lambda_\alpha^2$

Bipartite Entanglement

Area law for ground states of local (gapped) Hamiltonians in one dimensional systems

$S(L) = \text{const.}$ [Srednicki '93, Hastings '07]





All ground states live in a tiny corner of the Hilbert space!

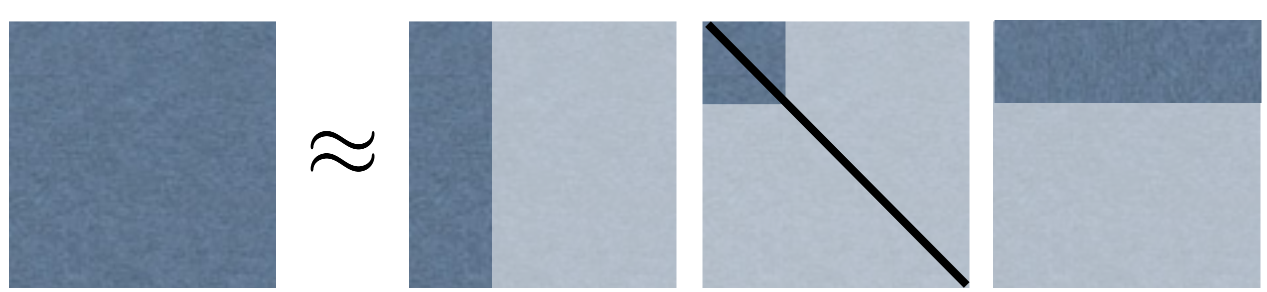
Tutorial: (A) SVD Compression

Example: $$|\psi\rangle = \sum_{i=1}^{N_A}\sum_{j=1}^{N_B} C_{ij}|i\rangle_A|j\rangle_B = \sum_\gamma \lambda_\gamma|\phi_\gamma\rangle_A|\phi_\gamma\rangle_B$$

Matrix can represent an image (array of pixel)

$$C = \begin{pmatrix} 0.23 & \cdots & 0.56 \\ \vdots & \ddots & \vdots \\ 0.22 & \cdots & 0.34 \end{pmatrix} =$$

Reconstruction of the matrix (image) from a small number of Schmidt states (SVD):



Tutorial: (A) SVD Compression

http://go.tum.de/475840

I) Analyse the Schmidt spectra of the images
and how much they can be compressed

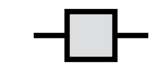
II) How does the number of singular values needed
to represent the image on its size?

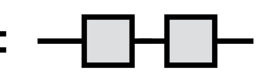
III) How does a product state look like as an image?

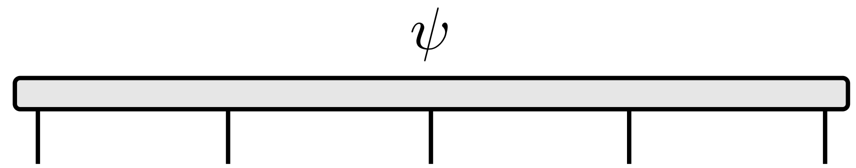
Matrix-Product States

Coefficients in the many-body wave function:

Order-L tensor: diagrammatic representation

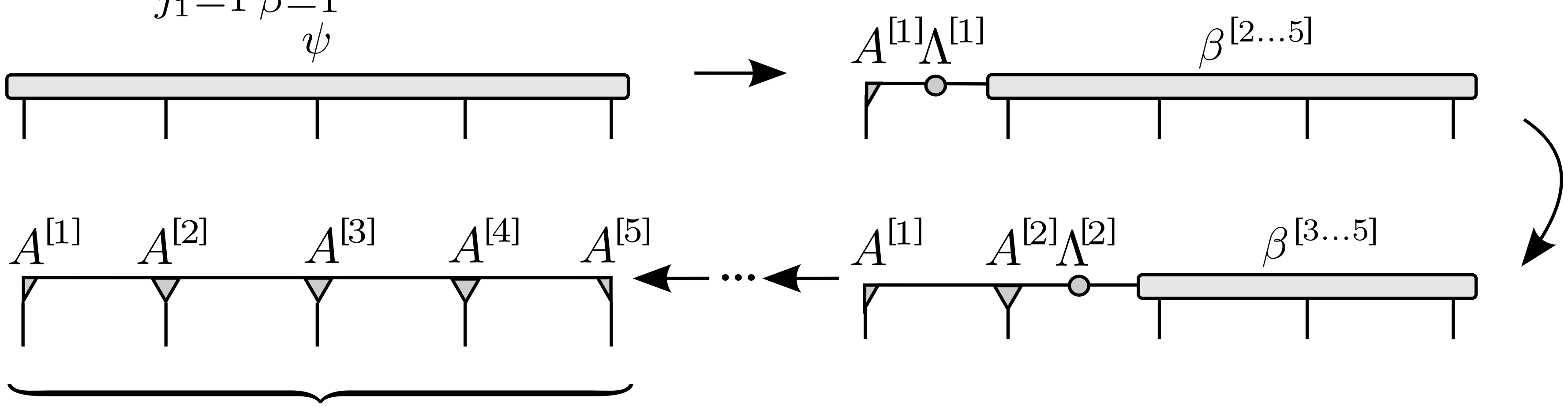
$A_{i,j} =$ 

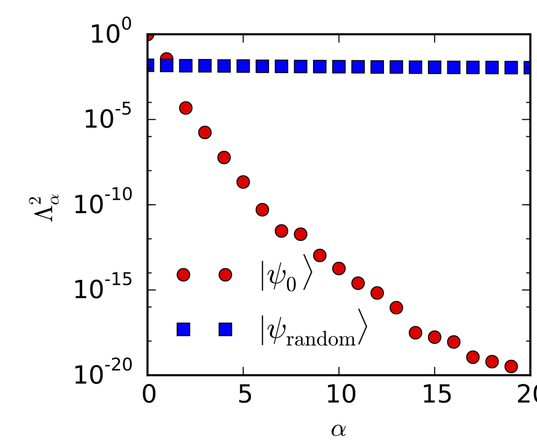
$\sum_k A_{i,k} B_{k,j} =$ 

$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

Successive Schmidt decompositions

$$|\psi\rangle = \sum_{j_1=1}^{d} \sum_{\beta=1}^{d} A_{\beta}^{[1]j_1} \Lambda_{\beta}^{[1]} |j_1\rangle |\beta\rangle_{[2,\dots L]}$$





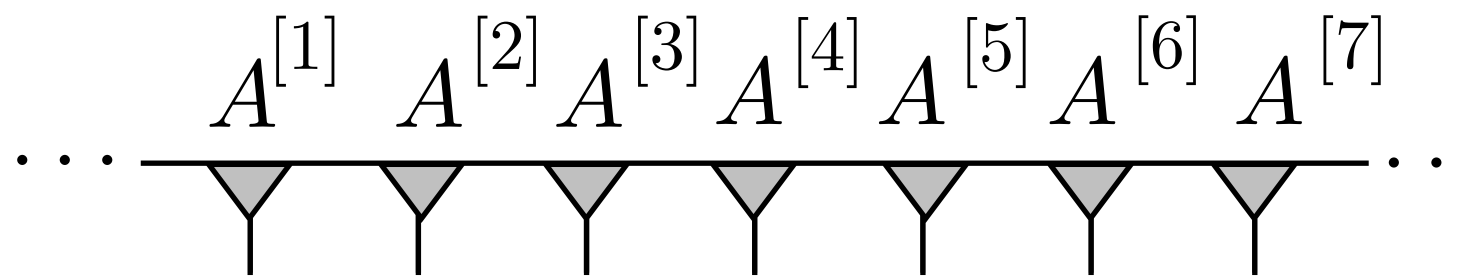
$$\sum_{\alpha_1,\alpha_2,\dots,\alpha_{L-1}} A_{\alpha_1}^{j_1} A_{\alpha_1,\alpha_2}^{j_2} \dots A_{\alpha_{L-1}}^{j_L}$$

MPS are tailored to describe 1D systems with an area law!

MPS and the Canonical Form

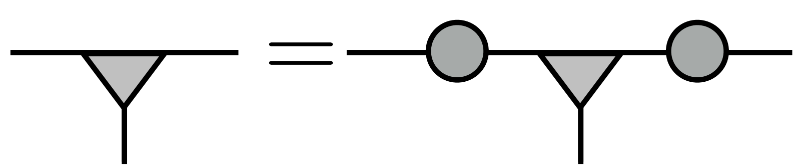
From now on: Leave out site indices!

$|\psi\rangle$:



MPS is not unique

$$\tilde{A}^{i_n} = X A^{i_n} X^{-1}$$



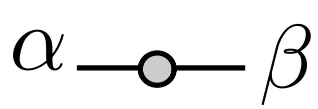
➡ $\tilde{A}^{i_n}$ describes the same state!

MPS and the Canonical Form

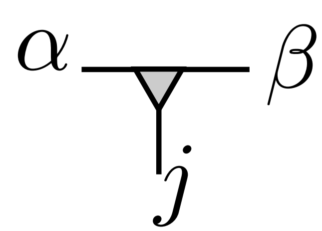
Choose a convenient representation in **Canonical Form**:
Bond index corresponds to Schmidt decomposition! [Vidal '03]

$$|\psi\rangle = \sum_{\alpha=1}^{\chi} \Lambda_\alpha |\alpha\rangle_L \otimes |\alpha\rangle_R \quad \text{with} \quad \langle\alpha|\alpha'\rangle = \delta_{\alpha\alpha'}$$

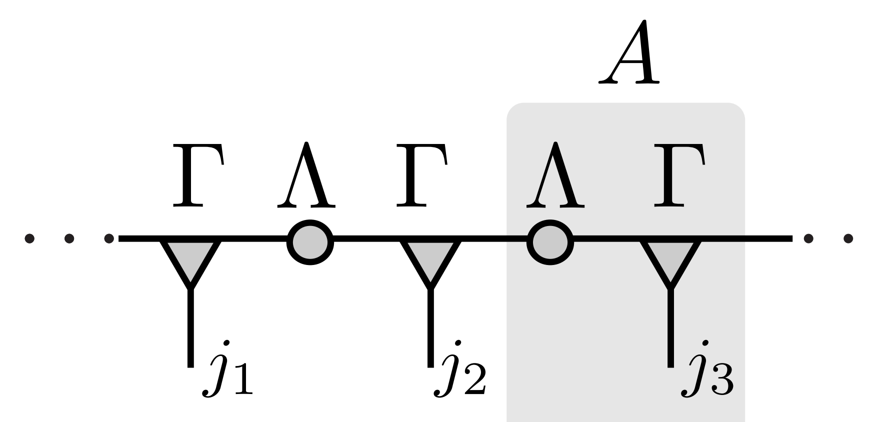
Write tensor $A^{i_n}_{\alpha\beta}$ as product of

$\Lambda_{\alpha\beta} =$ 

: Diagonal matrix with Schmidt values

$\Gamma^j_{\alpha\beta} =$ 

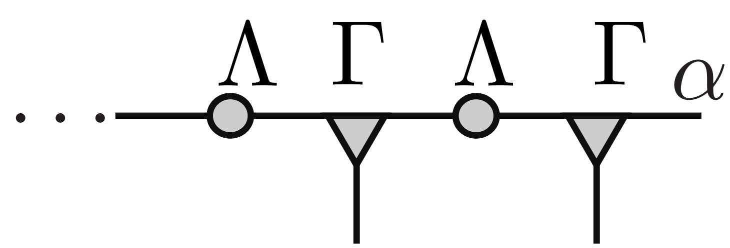
: Tensor relating to Schmidt basis

$\psi_{\ldots,j_1,j_2,j_3,\ldots} =$ 

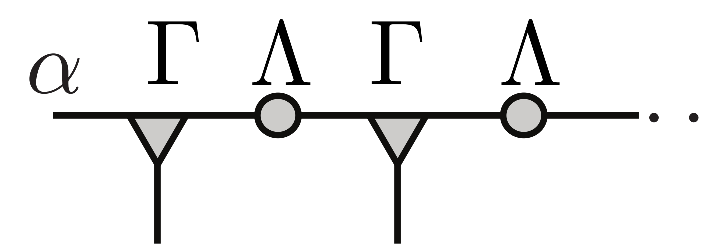
MPS and the Canonical Form

Schmidt states in terms of the MPS:

$$|\alpha\rangle_L =$$

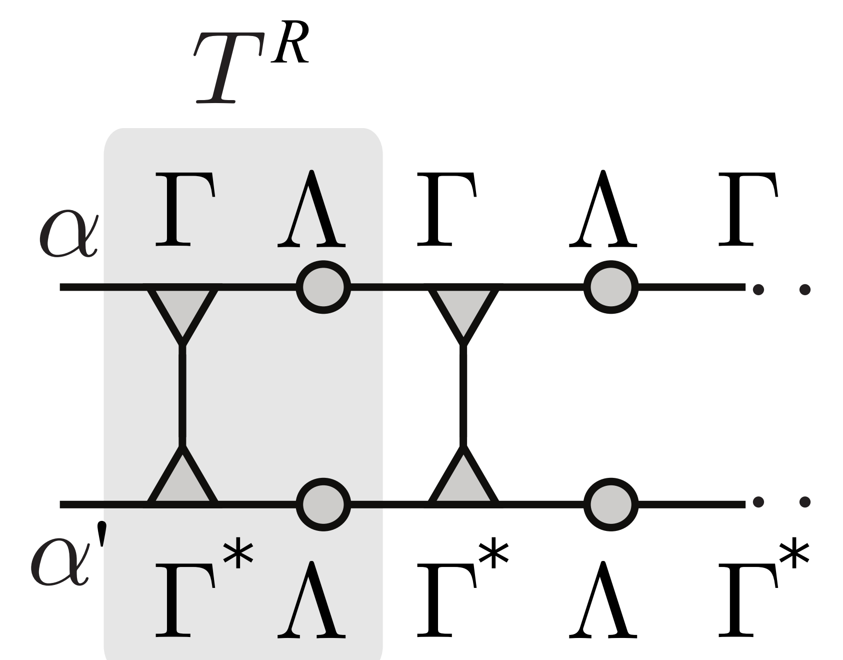


$$|\alpha\rangle_R =$$



Orthogonality:

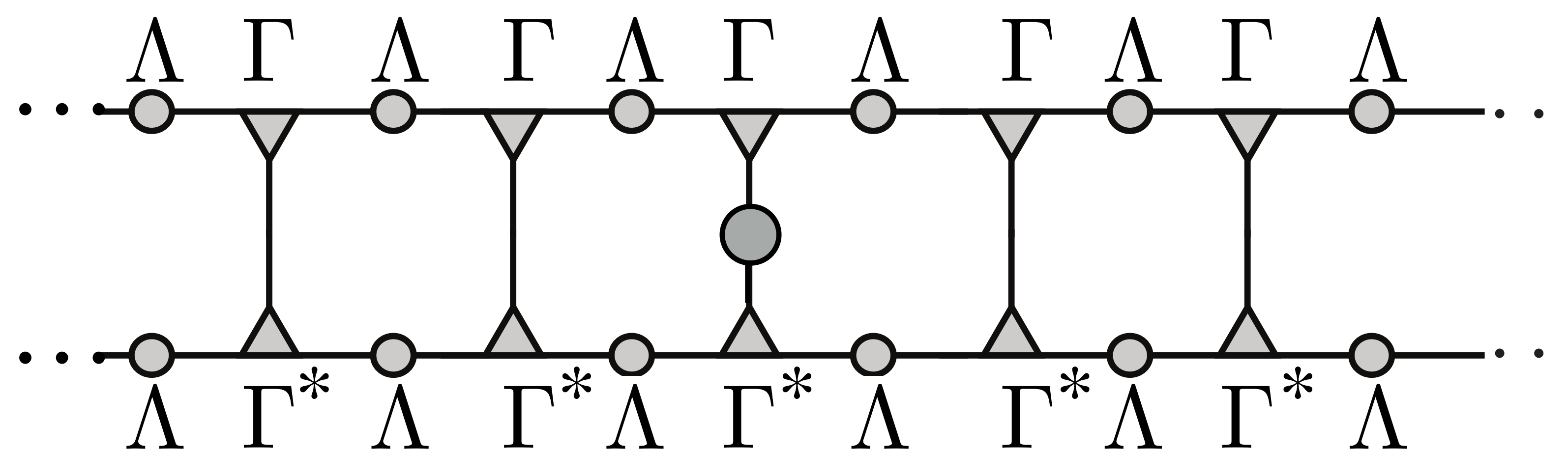
$$\langle\alpha'|\alpha\rangle_R = \delta_{\alpha'\alpha} \equiv$$



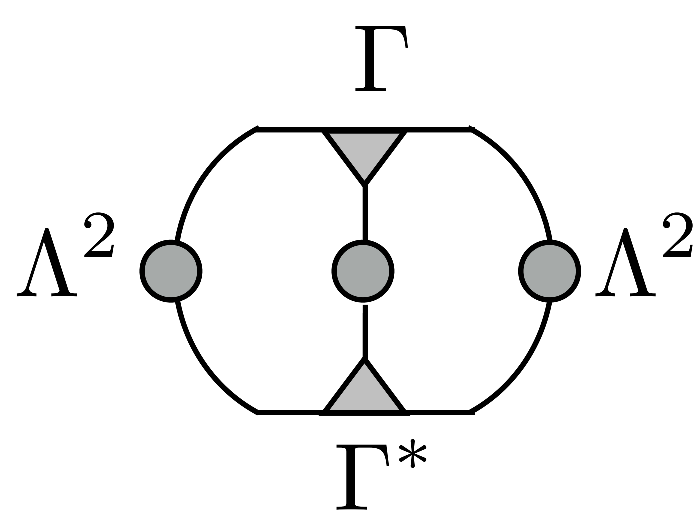
MPS and the Canonical Form

Efficient evaluation of **expectation values**:

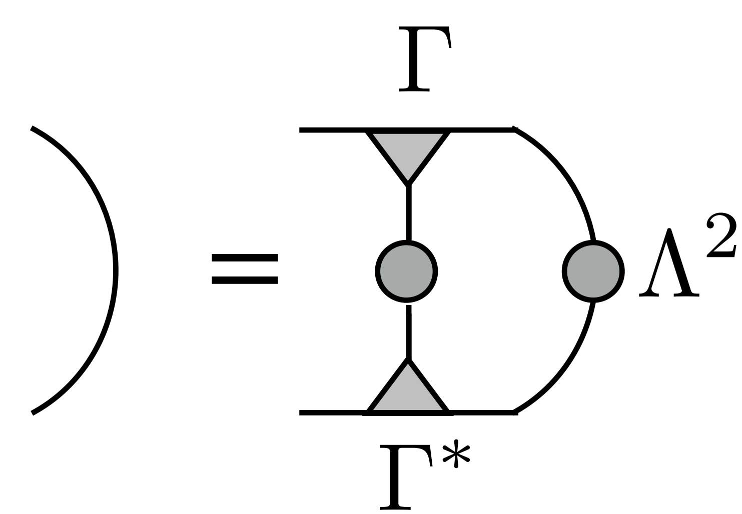
$$\langle\psi|O_n|\psi\rangle =$$



$$=$$



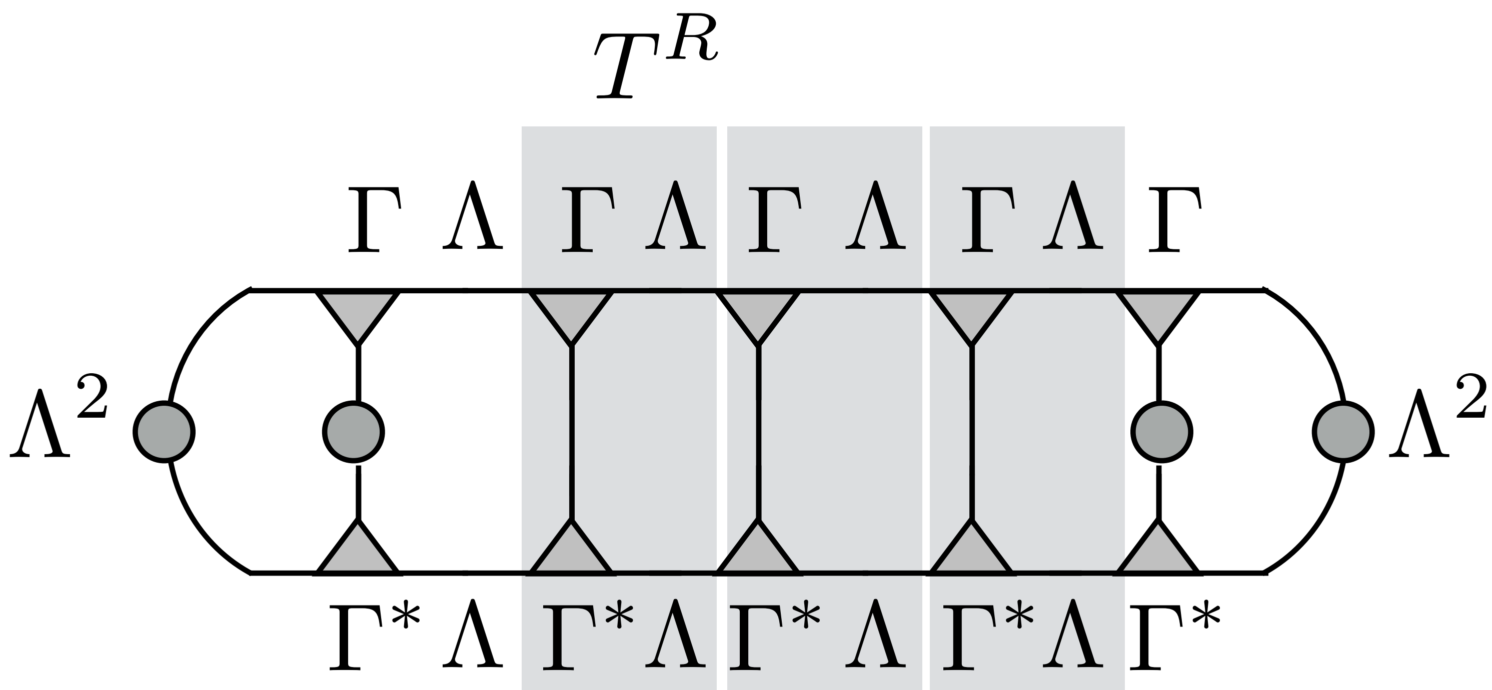
using



MPS and the Canonical Form

Efficient evaluation of **correlation functions**:

$$\langle\psi|O_m O_n|\psi\rangle =$$



Tensor Networks and the Many-Body Problem

I) Entanglement and Matrix-Product States

II) Time Evolving Block Decimation

III) Density-Matrix Renormalization Group

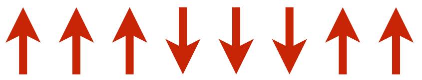
IV) Extracting topological invariants

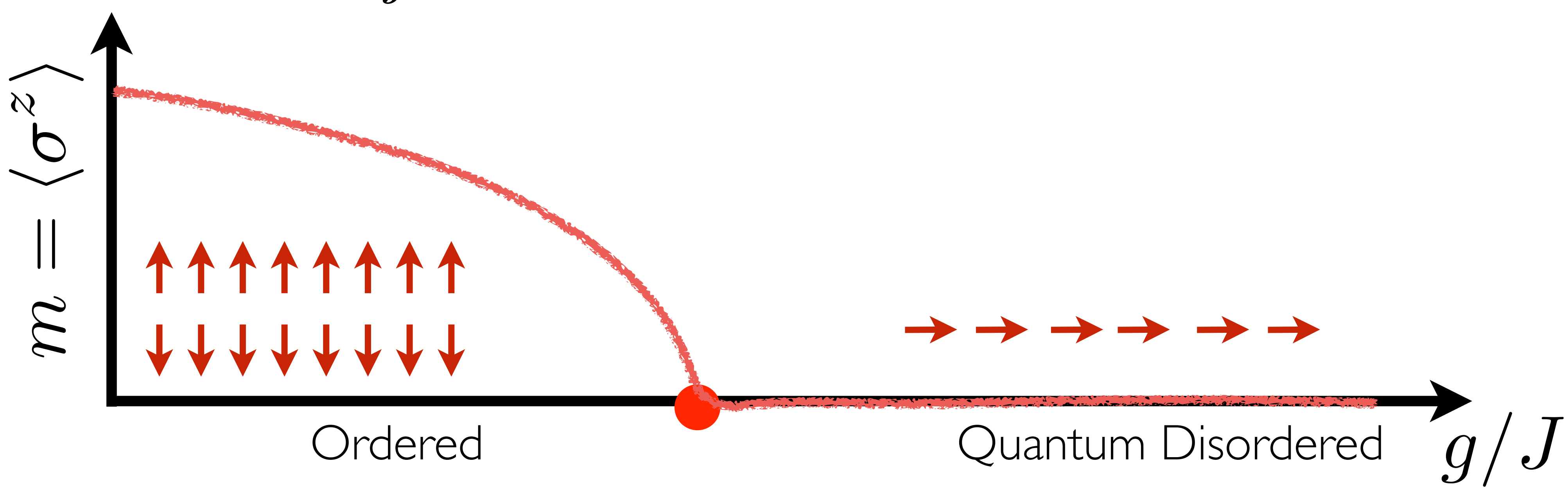
V) (isometric Tensor-Network States)

Transverse Field Ising Model

Quantum phases at $T = 0$: Transverse field Ising model with $\mathbb{Z}_2$ symmetry [Elliott et al.'70]

$$H = -\sum_j (J\sigma^z_j \sigma^z_{j+1} + g\sigma^x_j)$$





Entanglement of the ground state depends on g/J.

Time Evolving Block Decimation

Assume we have a Hamiltonian of the form

$$H = \sum_j h^{[j,j+1]}$$

Time evolution in real time

$$|\psi_t\rangle = \exp(-iHt)|\psi_{t=0}\rangle$$

Time evolution in imaginary time

$$|\psi_0\rangle = \lim_{\tau\to\infty} \frac{\exp(-H\tau)|\psi_i\rangle}{||\exp(-H\tau)|\psi_i\rangle||}$$

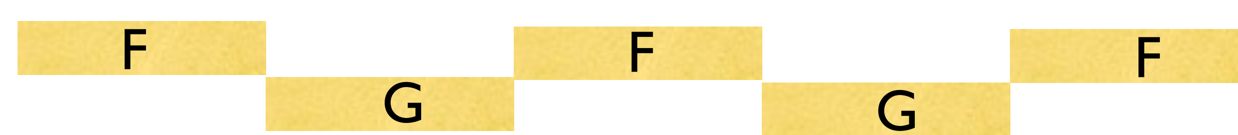
Time Evolving Block Decimation

Consider a Hamiltonian $H = \sum_j h^{[j,j+1]}$ [Vidal '03]

Decompose the Hamiltonian as $\mathrm{H{=}F{+}G}$

$$F \equiv \sum_{\text{even } j} F^{[j]} \equiv \sum_{\text{even } j} h^{[j,j+1]}$$

$$G \equiv \sum_{\text{odd } j} G^{[j]} \equiv \sum_{\text{odd } j} h^{[j,j+1]}$$



We observe $[F^{[r]}, F^{[r']}] = 0$ $([G^{[r]}, G^{[r']}] = 0)$
but $[G, F] \neq 0$

Time Evolving Block Decimation

Apply Suzuki-Trotter decomposition of order p

$$\exp\left(-i(F+G)\delta t\right) \approx f_p\left[\exp(-F\delta t), \exp(-G\delta t)\right]$$

with $f_1(x,y) = xy$, $f_2(x,y) = x^{1/2} y x^{1/2}$, etc.

Two chains of two-site gates

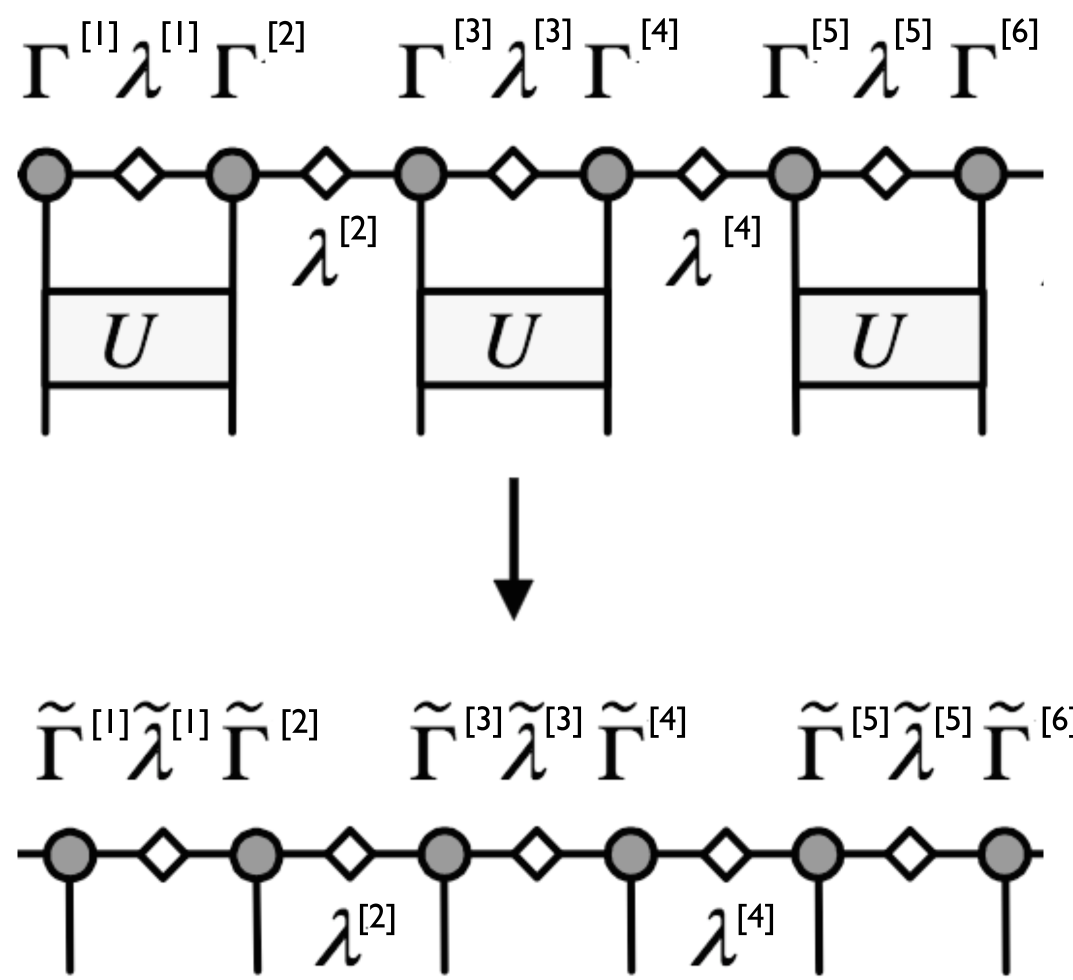
$$U_F = \prod_{\text{even } r} \exp(-iF^{[r]}\delta t)$$

$$U_G = \prod_{\text{odd } r} \exp(-iG^{[r]}\delta t)$$

Each gate affects the state only locally

Time Evolving Block Decimation

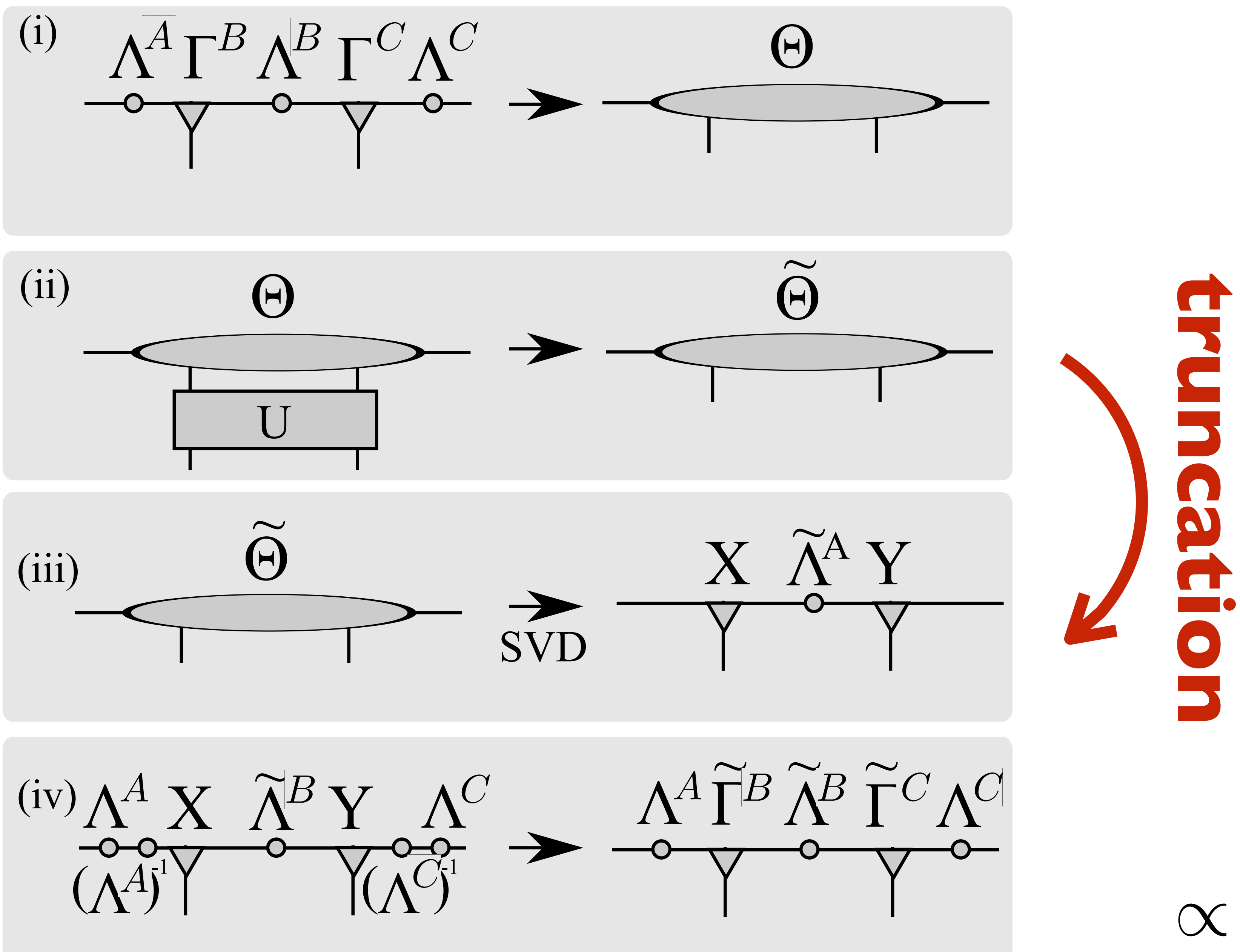
Time Evolving Block Decimation algorithm (TEBD)



How do we get the original form back?

Time Evolving Block Decimation

Time Evolving Block Decimation (TEBD) algorithm [Vidal '03]



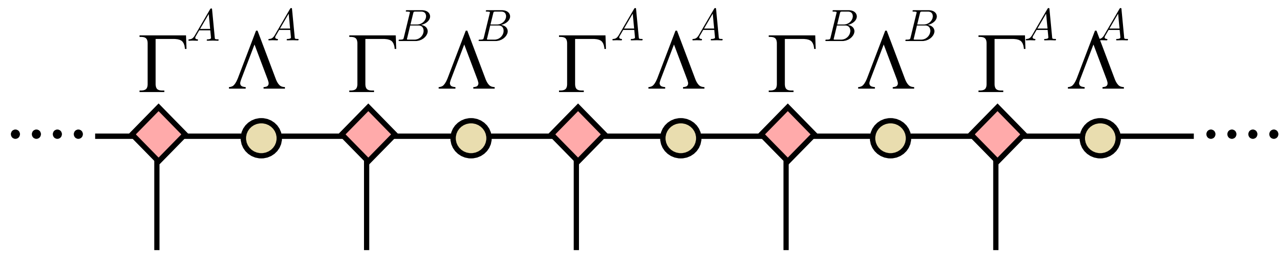
$\propto d^3 \chi^3$

Translationally invariant, infinite systems!

Assume that $|\psi\rangle$ is translational invariant and $L = \infty$:
infinite TEBD (iTEBD)

Partially break translational symmetry to simulate the action of the gates

$$\Gamma^{[2r]} = \Gamma^A,\ \lambda^{[2r]} = \lambda^A,\ \Gamma^{[2r+1]} = \Gamma^B,\ \lambda^{[2r+1]} = \lambda^B$$



Tutorial: (B) TEBD

http://go.tum.de/475840

I) Check the convergence of the code for different parameters (number of iterations, χ, $d\tau$).

II) Evaluate the phase diagram of the transverse field Ising model by plotting the magnetization, correlation length)

III) Check the stability of the transition by adding new terms to the Hamiltonian: $\sum_i \sigma_i^x \sigma_{i+1}^x$ and $\sum_i \sigma_i^z$

IV) Plot the entanglement growth following a global quench.

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III) Density-Matrix Renormalization Group 👉

IV) Extracting topological invariants

V) (isometric Tensor-Network States)

Density Matrix Renormalization Group (DMRG)

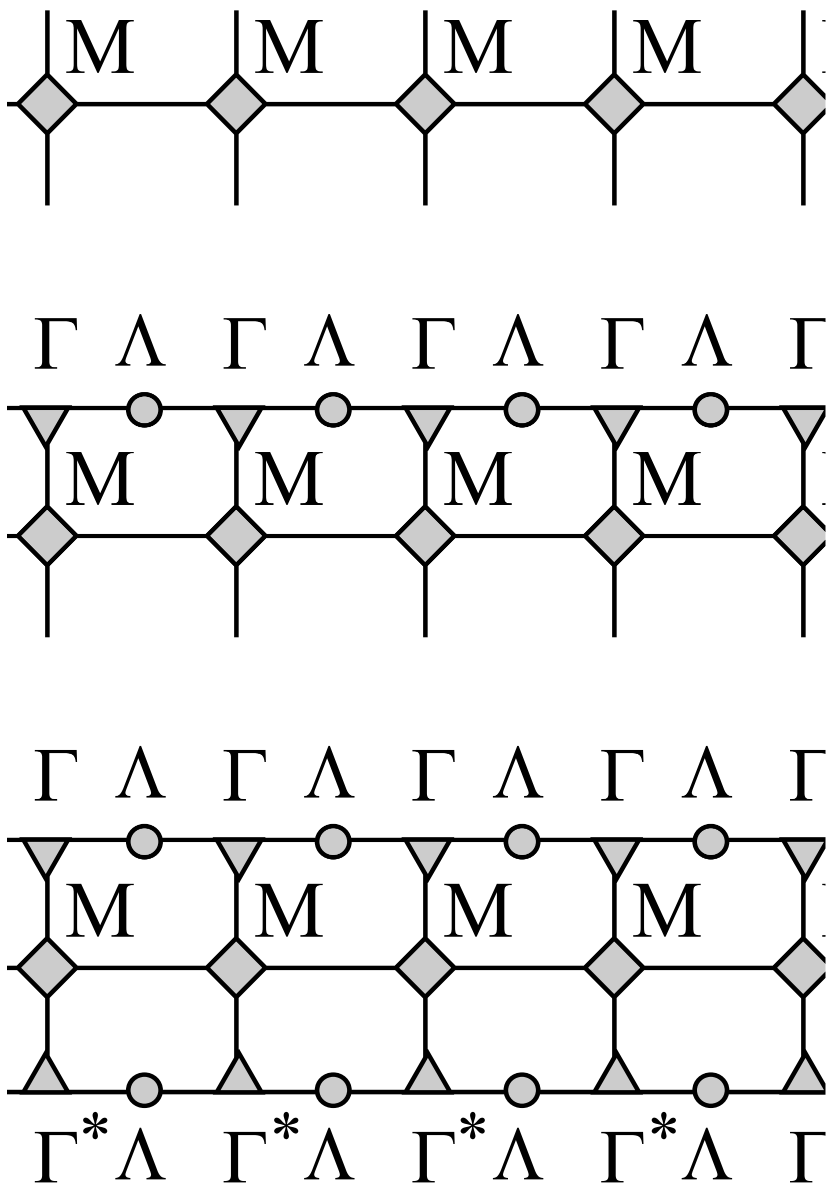
DMRG: Find the MPS representation of the ground state of a given Hamiltonian variationally
[White '92]

- Much faster convergence than TEBD
- Long range interactions can be easily implemented
- Similarly to TEBD, it allows to work directly in the thermodynamic limit

Density Matrix Renormalization Group (DMRG)

Matrix-Product Operators (MPO): Generalization of MPS

$$\mathscr{O}_{i_1 i_2 \ldots i_L,\ i'_1, i'_2 \ldots i'_L} =$$



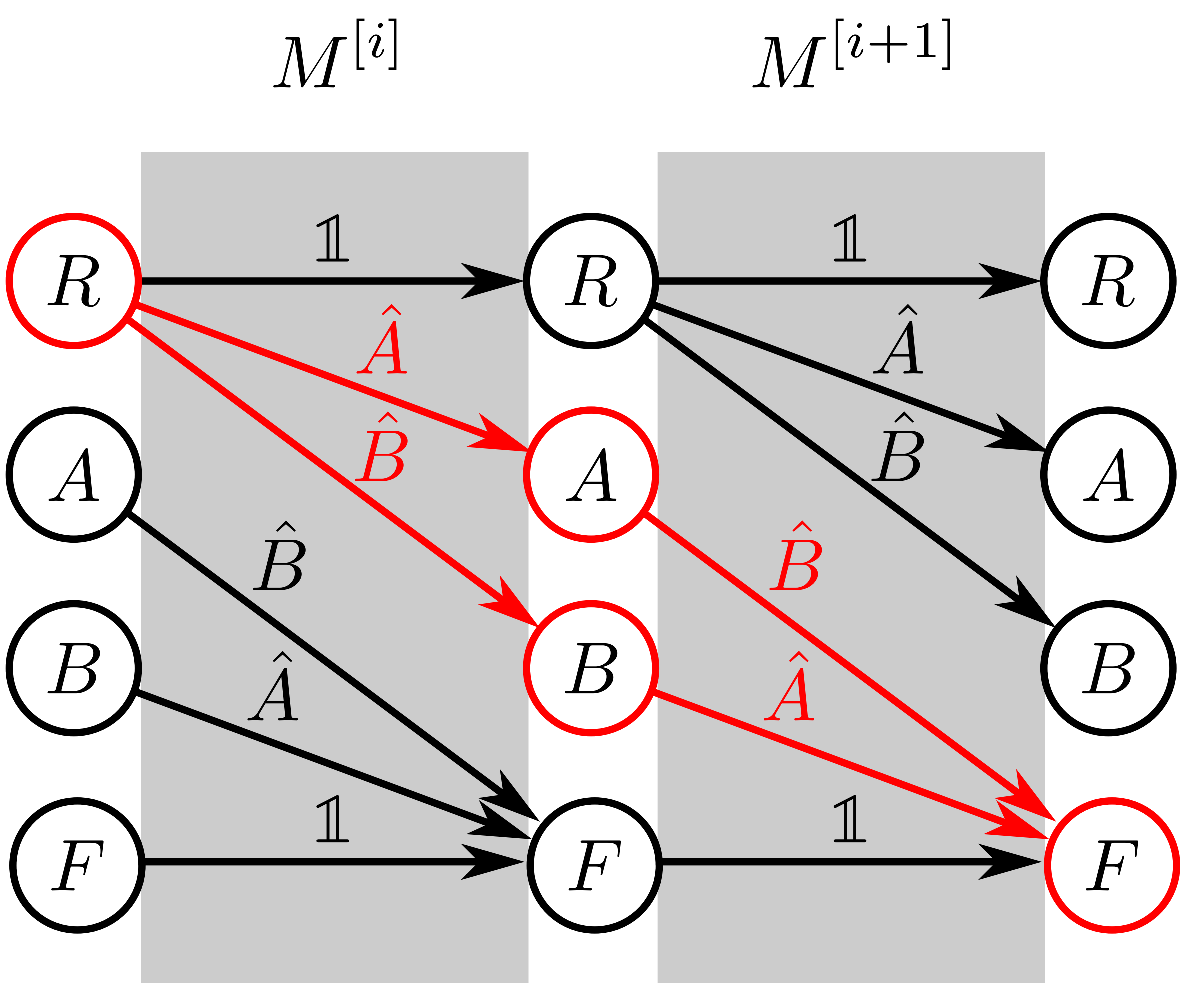
Density Matrix Renormalization Group (DMRG)

MPO representation of $\hat{O} = \sum_i \left(\hat{A}_i \hat{B}_{i+1} + \hat{B}_i \hat{A}_{i+1}\right)$

$$
\begin{aligned}
&= \hat{A} \otimes \hat{B} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \\
&\quad + \mathbb{1} \otimes \hat{A} \otimes \hat{B} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} + \cdots \\
&\quad + \hat{B} \otimes \hat{A} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} \\
&\quad + \mathbb{1} \otimes \hat{B} \otimes \hat{A} \otimes \mathbb{1} \otimes \cdots \otimes \mathbb{1} + \cdots
\end{aligned}
$$

Density Matrix Renormalization Group (DMRG)

MPO representation of $$\hat{O} = \sum_i \left(\hat{A}_i \hat{B}_{i+1} + \hat{B}_i \hat{A}_{i+1} \right)$$

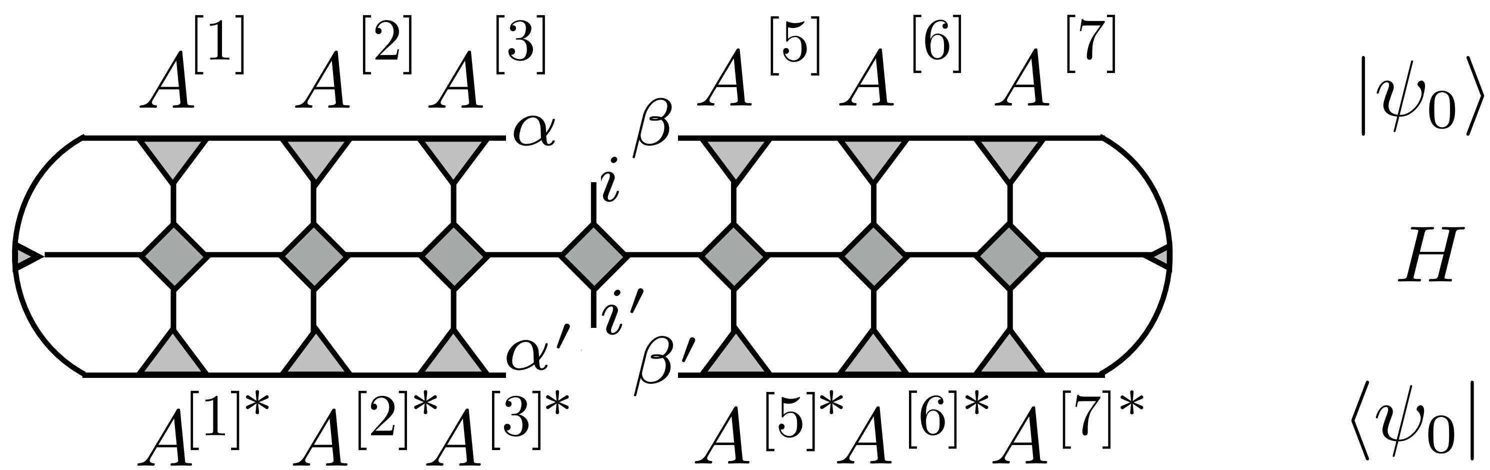


$$\begin{array}{c} \\ M = \end{array} \begin{array}{c} \begin{array}{cccc} R & A & B & F \end{array} \\ \begin{pmatrix} \mathbb{1} & \hat{A} & \hat{B} & 0 \\ 0 & 0 & 0 & \hat{B} \\ 0 & 0 & 0 & \hat{A} \\ 0 & 0 & 0 & \mathbb{1} \end{pmatrix} \end{array} \begin{array}{c} \\ R \\ A \\ B \\ F \end{array}$$

Density Matrix Renormalization Group (DMRG)

Find the **ground state** iteratively

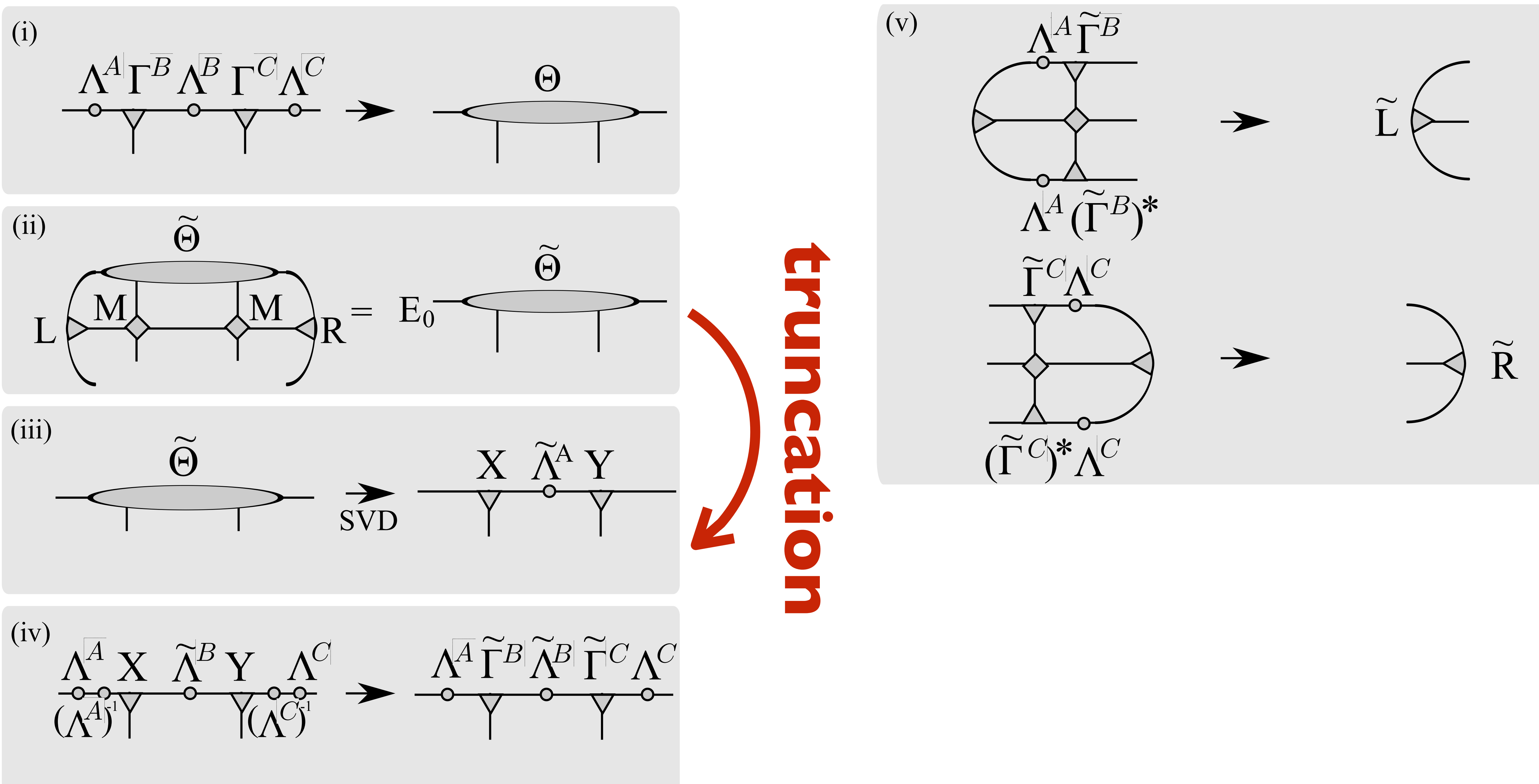
$$H_{\alpha i \beta;\alpha' i' \beta'} =$$



by locally minimizing energy of $H_{\alpha i \beta;\alpha' i' \beta'}$ (e.g., Lanczos)

Much faster convergence than TEBD + allows for long range interactions!

Density Matrix Renormalization Group (DMRG)



Tensor Network Python (TeNPy)

What is TeNPy

- Python 3 library for simulations with tensor network
  https://github.com/tenpy/tenpy
- Object oriented, modular structure, and easy to install
- HTML documentation
  https://tenpy.readthedocs.io/en/latest/
- (in)finite DMRG, TEBD; TDVP

Johannes Hauschild

Tensor Network Python (TeNPy)

Example: DMRG

Example

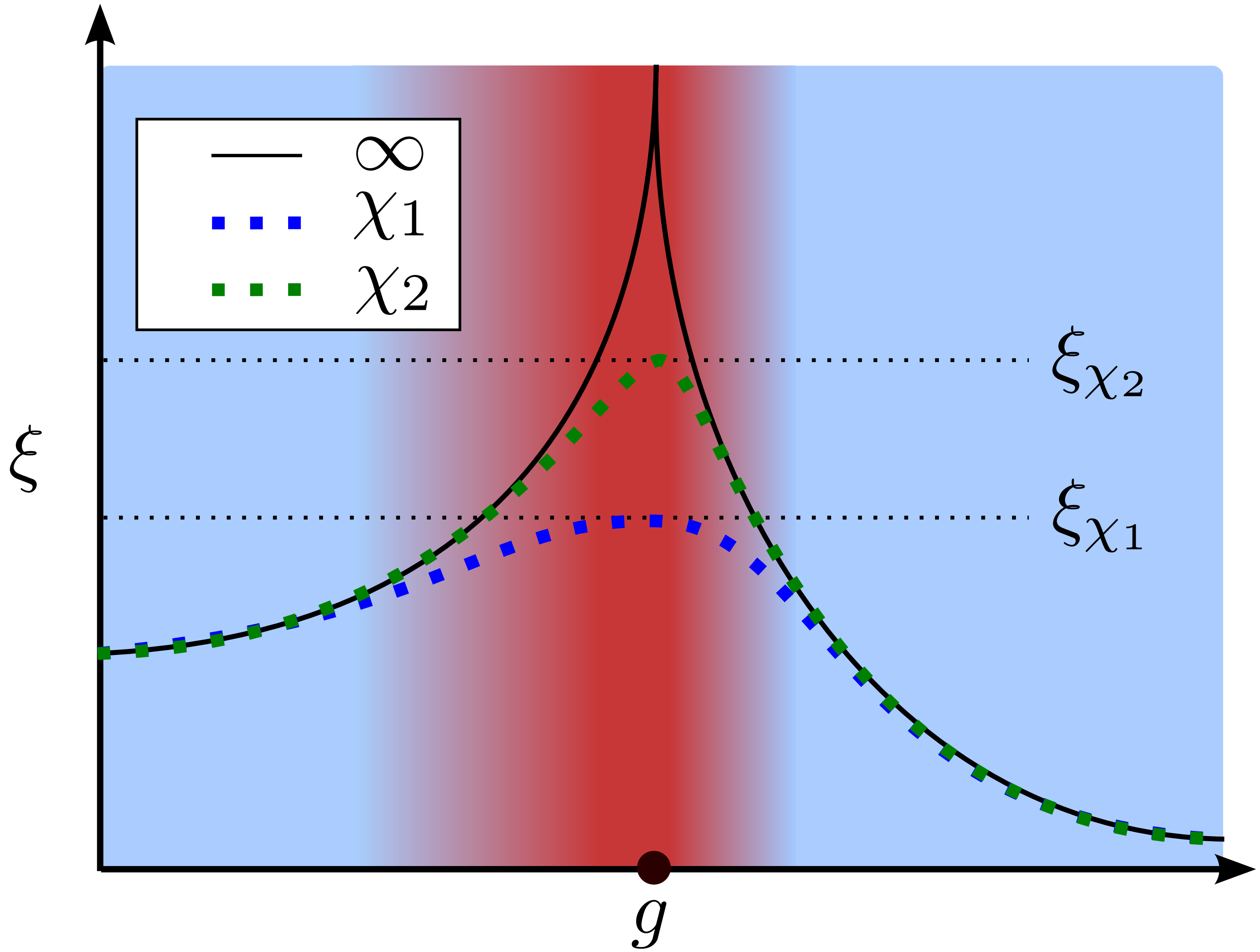
```
from tenpy.networks.mps import MPS
from tenpy.models.tf_ising import TFIChain
from tenpy.algorithms import dmrg

M = TFIChain({'L': 16, 'J': 1., 'g': 1.5})
psi = MPS.from_product_state(M.lat.mps_sites(),
                             ['up']*16, 'finite')
dmrg_params = {'trunc_params': {'chi_max': 30,
                                'svd_min': 1.e-10}}
dmrg.run(psi, M, dmrg_params) # find ground state
print("E =", sum(M.bond_energies(psi)))
print("final bond dimensions: ", psi.chi)
```

Hauschild and FP, arxiv:1805.00055

Finite Entanglement Scaling

Finite entanglement scaling: Entanglement and correlation length are always finite in an MPS



[FP, Mukerjee, Turner, and Moore, PRL 102, 255701 (2009)]

Tutorial: (C) TeNPy DMRG

http://go.tum.de/475840

I) Get familiar with the TeNPy interface: https://tenpy.github.io

II) Find the phase transition by plotting the magnetization as function of g

III) Extract the central charge using "entanglement scaling": $S = \frac{c}{6} \log \xi + \text{const}$

Tensor Networks and the Many-Body Problem

I) Entanglement and Matrix-Product States

II) Time Evolving Block Decimation

III) Density-Matrix Renormalization Group

IV) Extracting topological invariants 👉

V) (isometric Tensor-Network States)

Topological Phases of Matter

"Non invertible"

-FQHE
-Toric Code
-Spin Liquids

Anyons

"SET"

"Symmetry protected top."

-Topological Insulators
-Haldane Phase

Symmetry Fractionalization

"Symmetry broken"

"Chiral top."

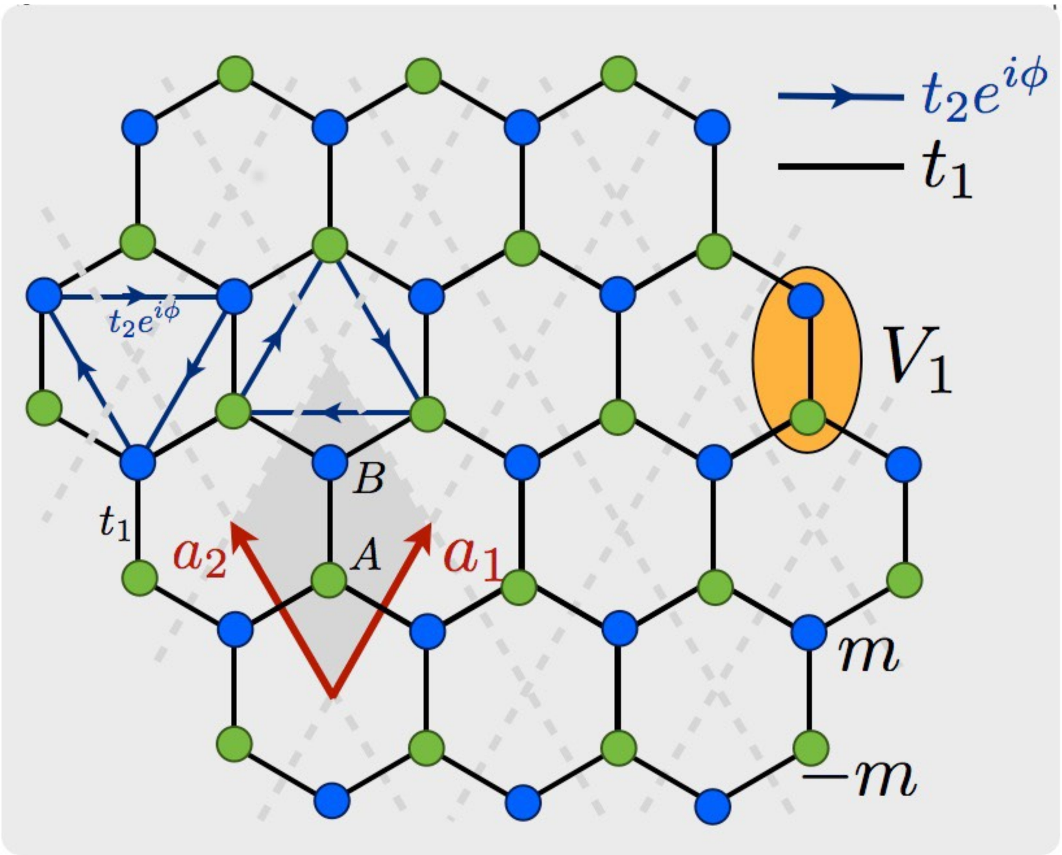
-IQHE
-Chern Insulator

Chiral Edge Modes

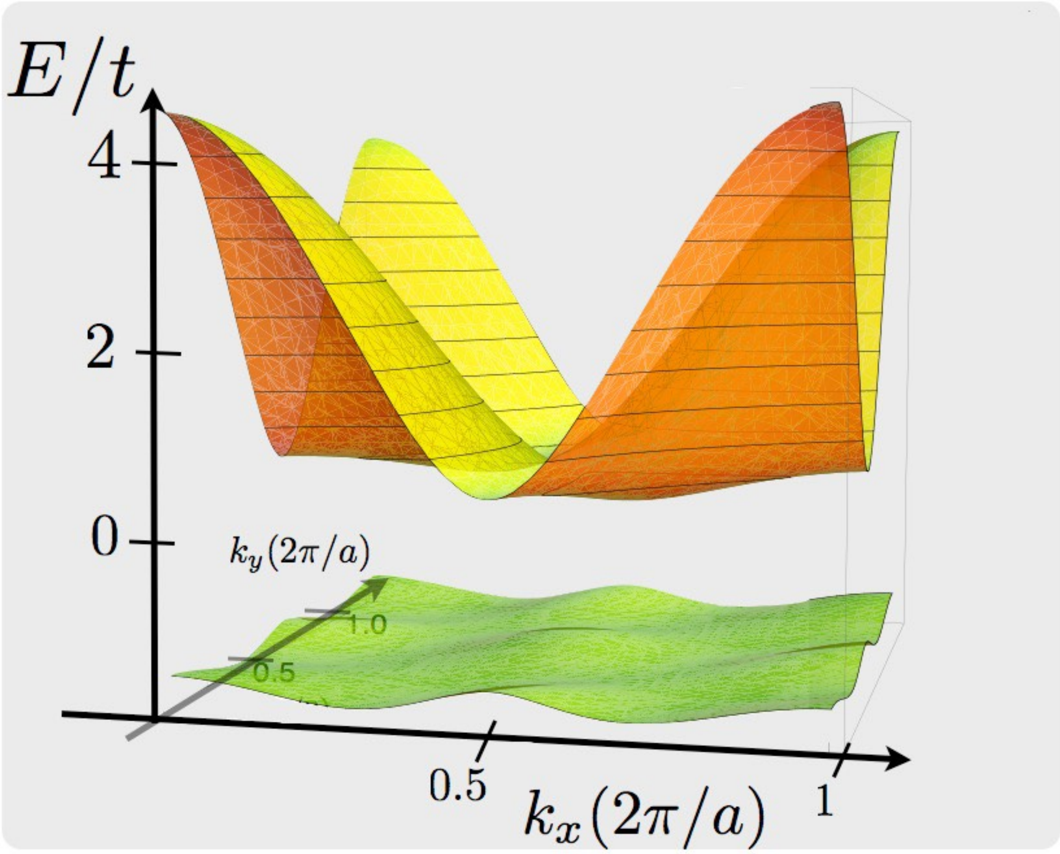
"Trivially disordered"

All phases can be identified using DMRG!

(Fractional) Chern Insulators



[Haldane '88]



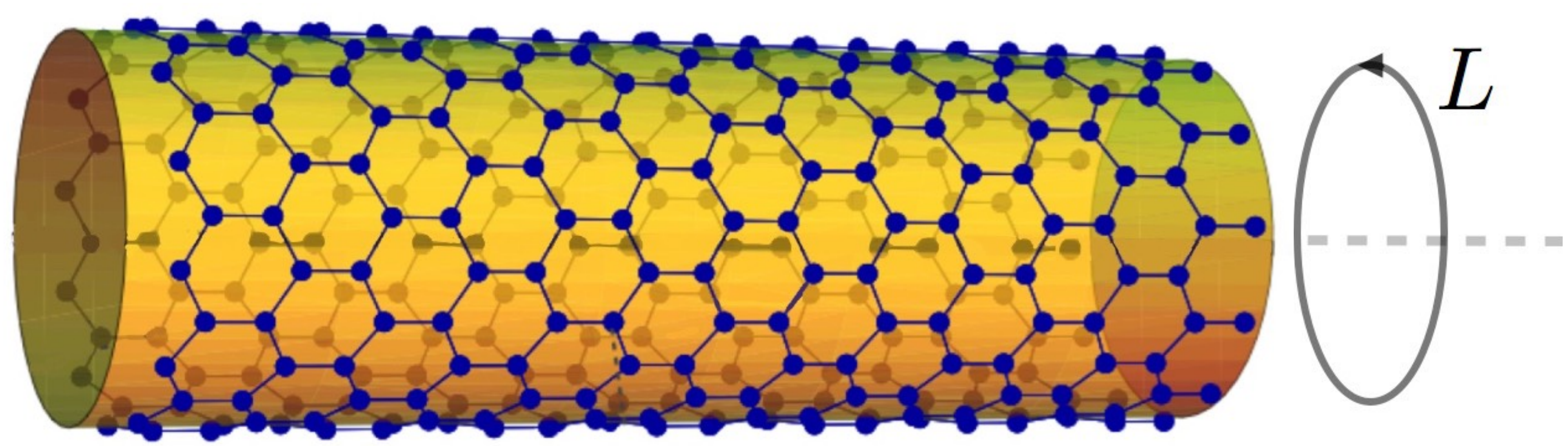
D. Sheng, Z.-C. Gu, K. Sun, and L. Shen '11
N. Regnault and B. A. Bernevig '11
S. Kourtis, T. Neupert, C. Chamon, and C. Mudry, '12
S. Kourtis, J. W. F. Venderbos, and M. Daghofer '13
....

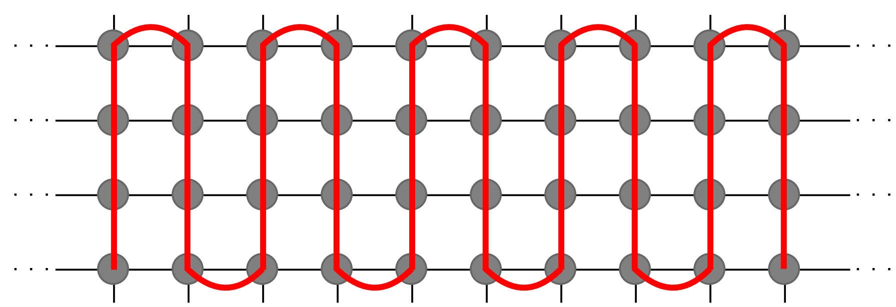
Chern Insulator ($\nu = 1$): Lattice version of the Integer Quantum Hall Effect

Fractional Chern Insulator (e.g., $\nu = 1/3$): Interactions are important!

DMRG for 2D Systems

DMRG on cylinders with circumference up to $L = 12$





2D physics at cost of long range interaction in 1D representation!

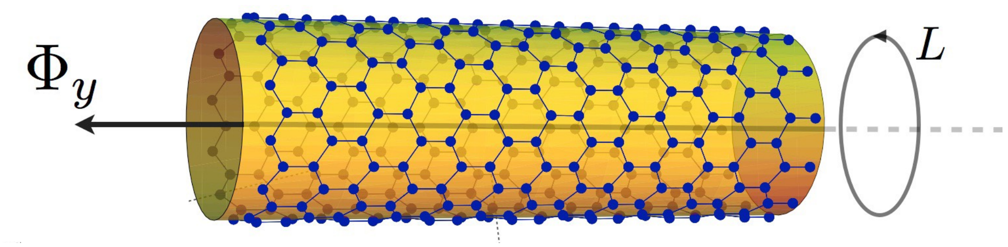
- MPO representation of the Hamiltonian (bond dimension scales **polynomially** with L)
- Area law: MPS dimensions of the ground state grows **exponentially** with L!

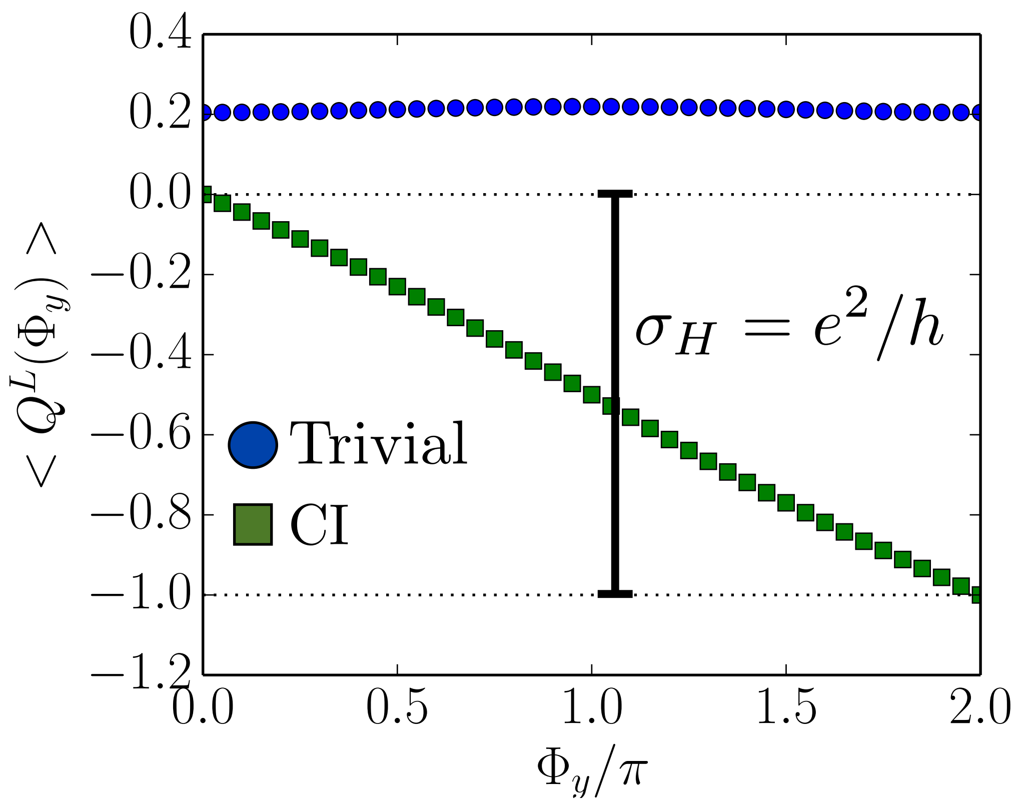
Grushin, Motruk, Zaletel, FP, PRB 91, 035136 (2015).

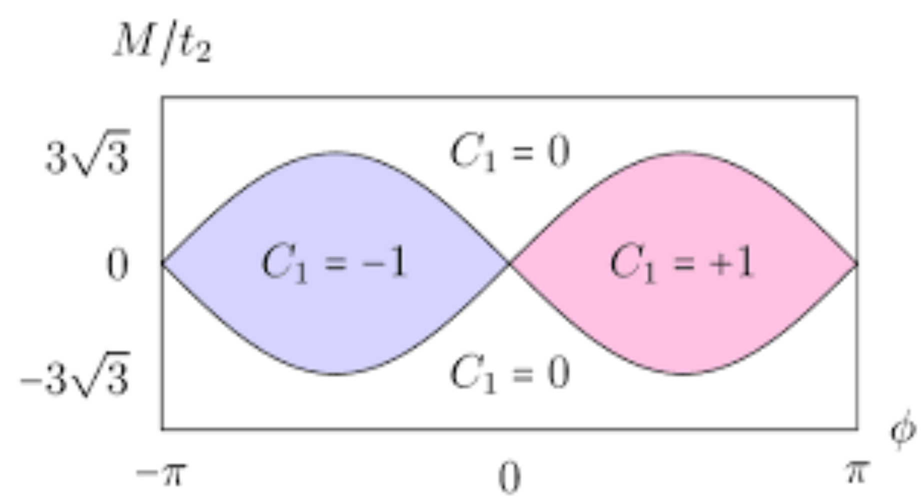
Chern Insulators

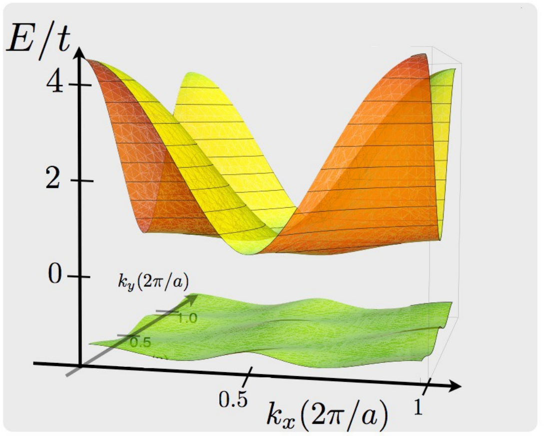
Laughlin charge pump

- Generate flux ϕ_y using complex hopping at the boundary
- Start from $\phi_y = 0$ and adiabatically insert a flux 2π
- Measure the charge pumped from left to right





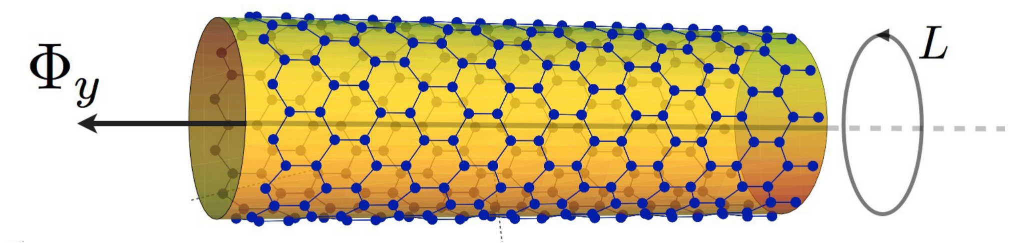


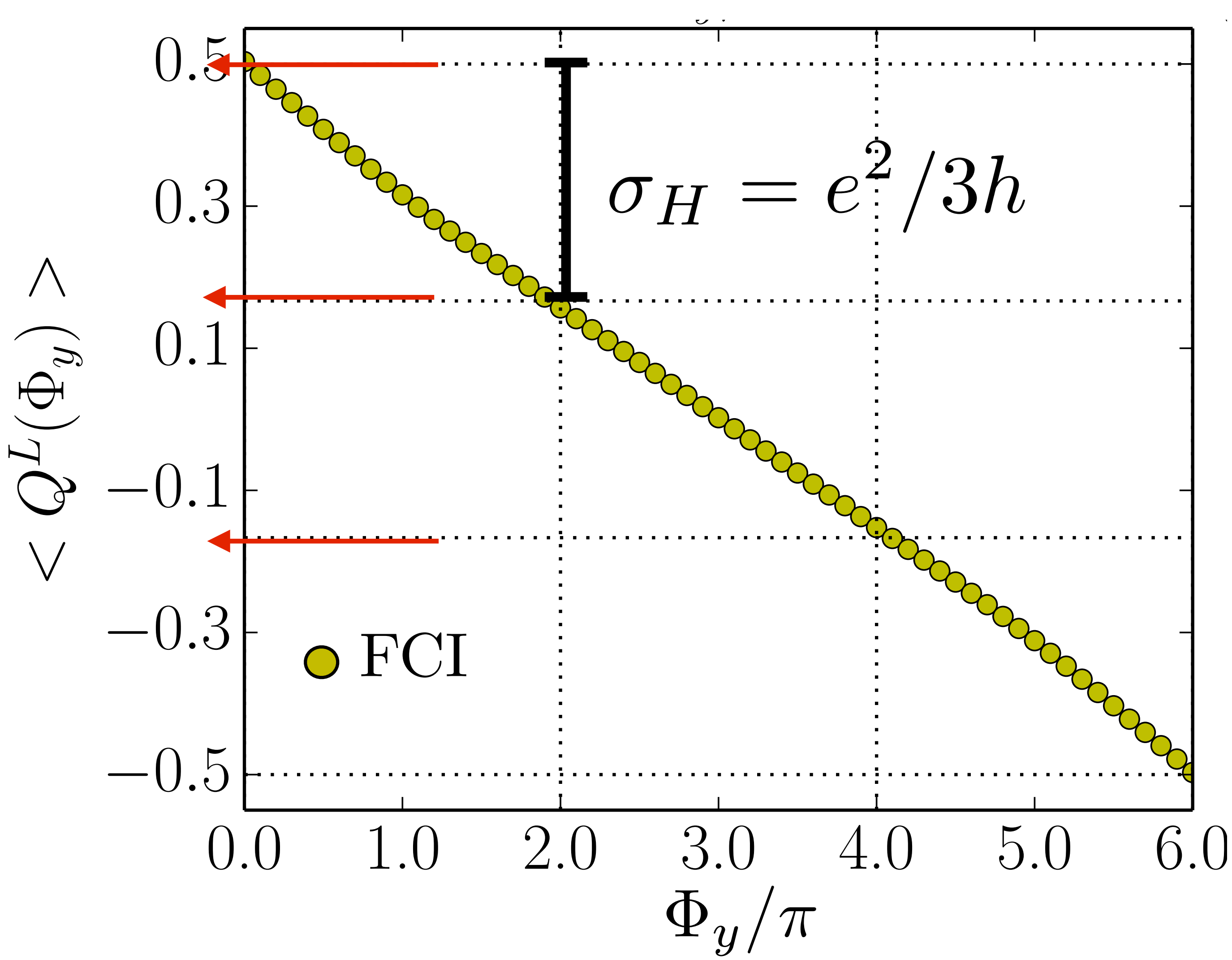


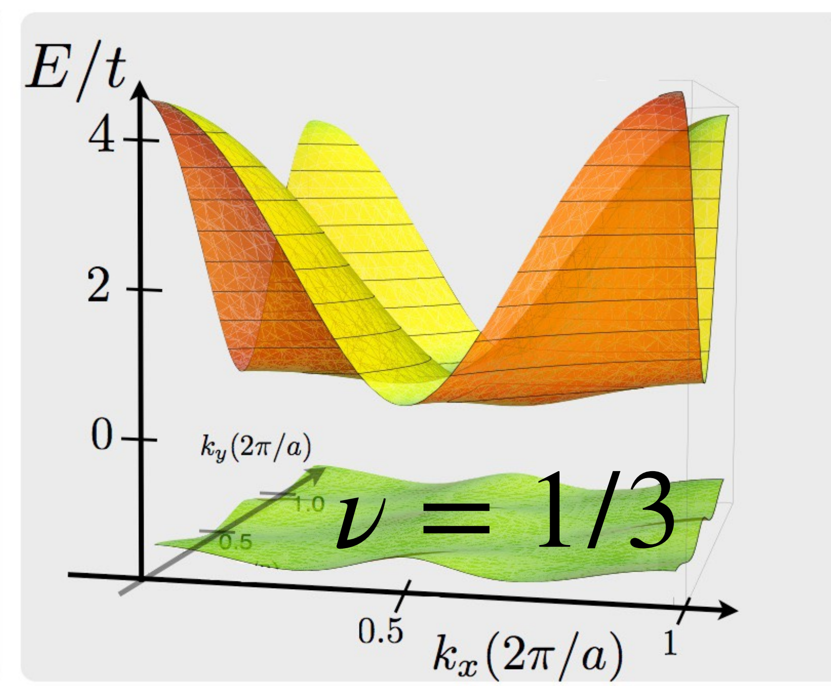
Grushin, Motruk, Zaletel, FP, PRB 91, 035136 (2015).

Fractional Chern Insulators

Laughlin charge pump





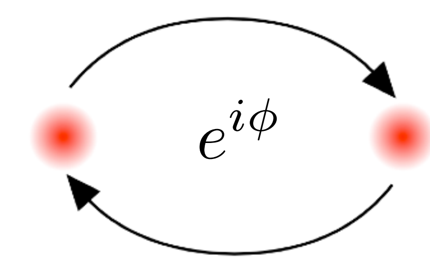


Grushin, Motruk, Zaletel, FP, PRB 91, 035136 (2015).

Fractional Chern Insulators: Top. Entanglement

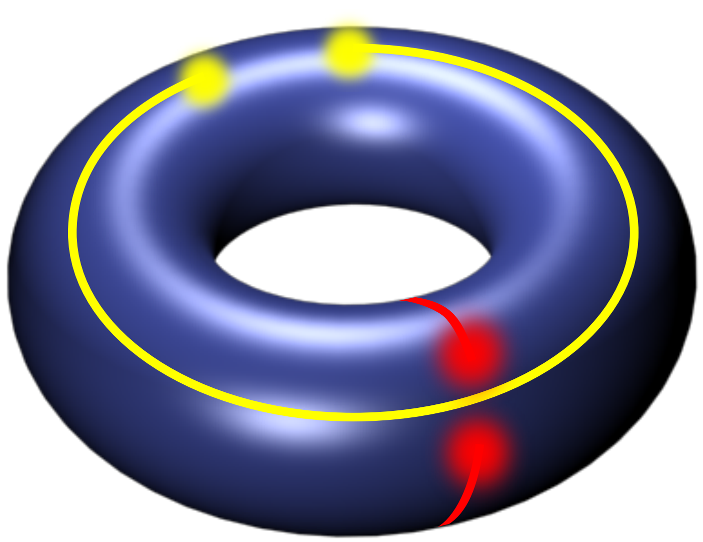
Intrinsic topological order: Gapped quantum phases that are robust to any small (local) perturbation

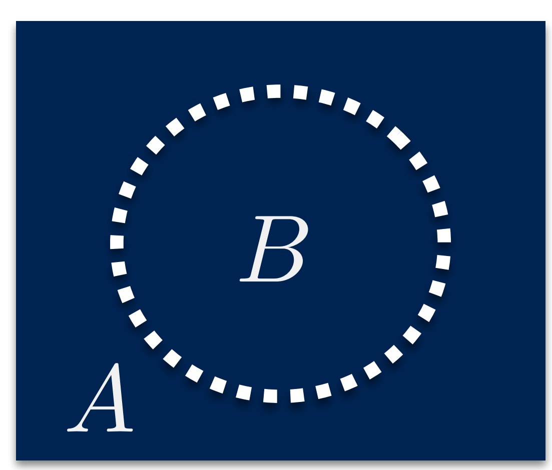
Characterized by quasiparticle excitations that obey **fractional statistics "anyons"** [Wen '90]



- Topological degeneracy on torus/cylinder (= number of anyons)
- Topological entanglement entropy $\gamma : S = \alpha L - \gamma$

[Kitaev and Preskill '06, Levin and Wen '06]





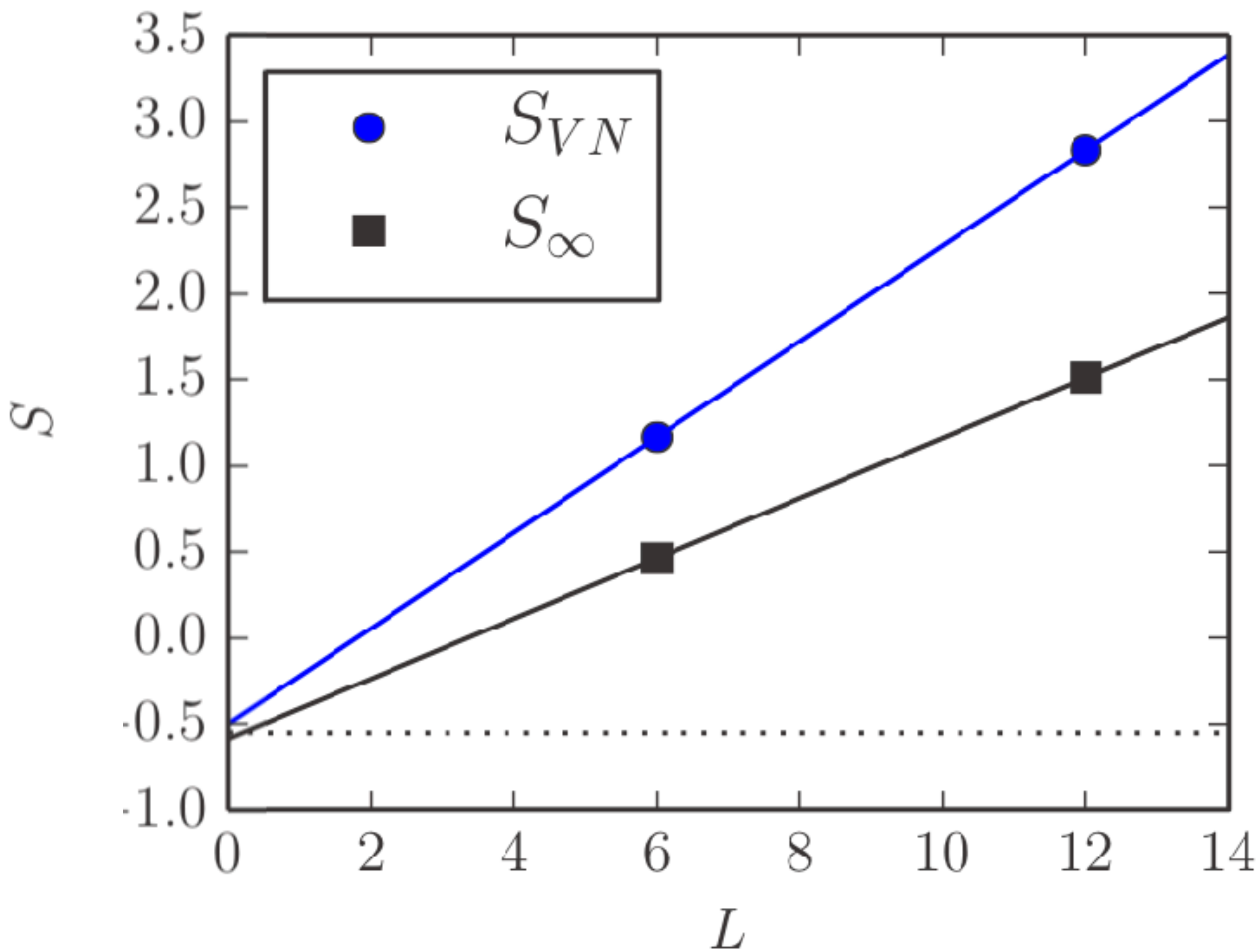
$$|\psi\rangle = \sum_\alpha \sqrt{p_\alpha} \; |\phi_\alpha^A\rangle \; |\phi_\alpha^B\rangle$$

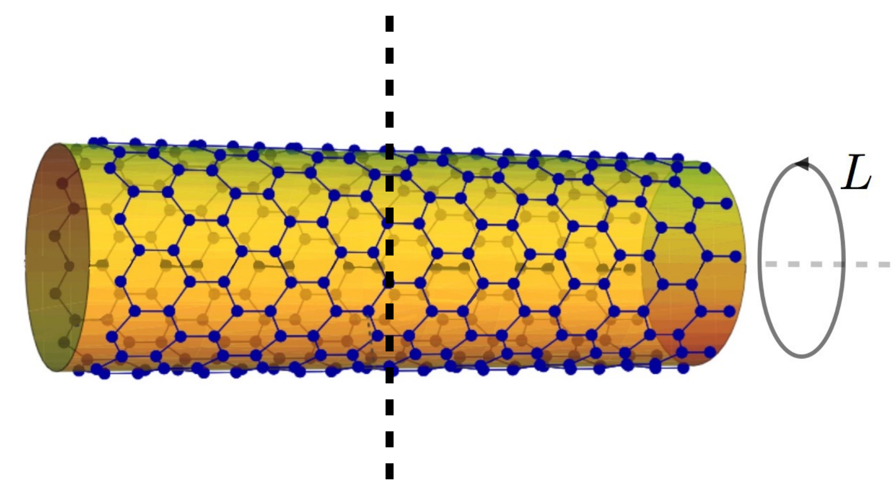
$$S = -\sum_\alpha p_\alpha \log p_\alpha$$

Abelian: $\gamma = \log(\sqrt{\#\text{anyons}})$

Fractional Chern Insulators: Top. Entanglement

Topological entanglement in the FCI phase





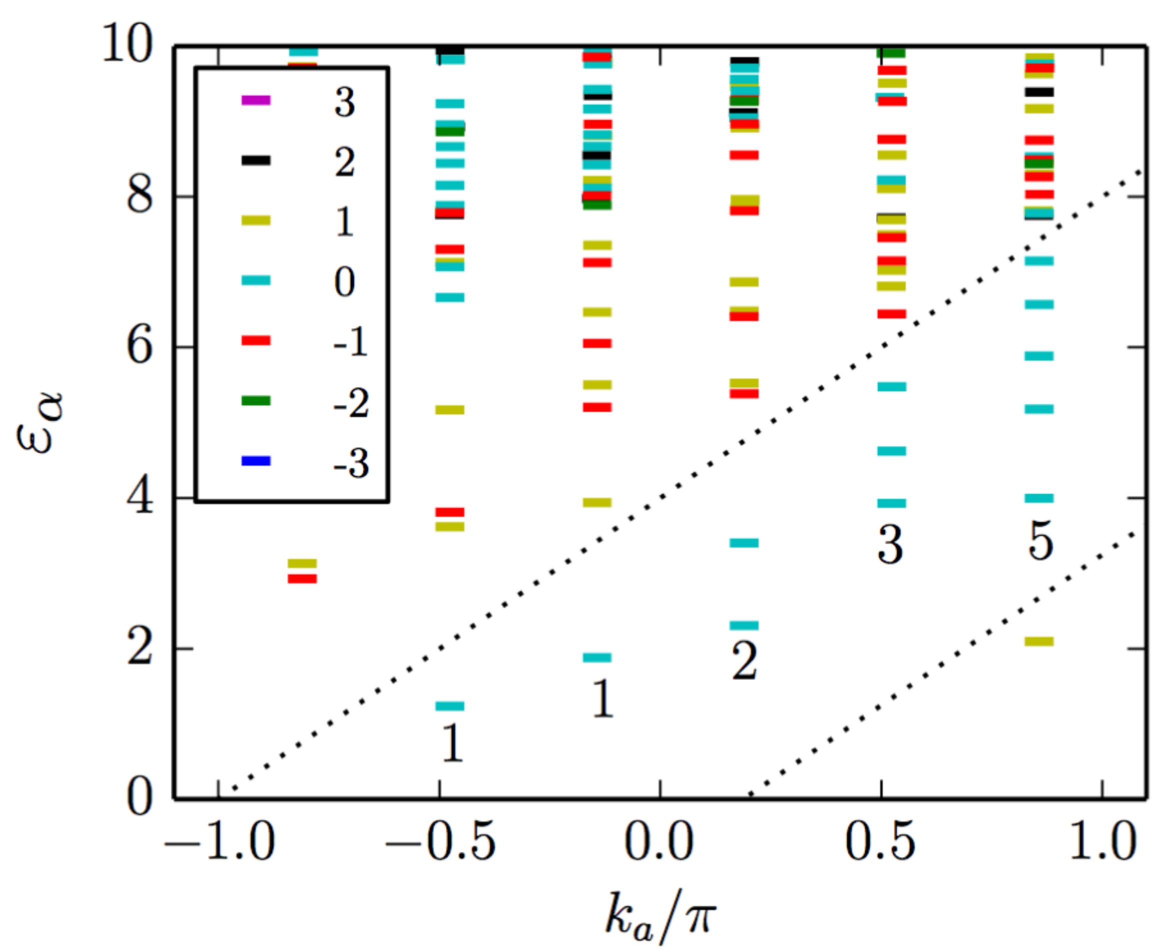
$$S = \alpha L - \gamma$$

$$\log \sqrt{3}$$

Grushin, Motruk, Zaletel, FP, PRB 91, 035136 (2015).

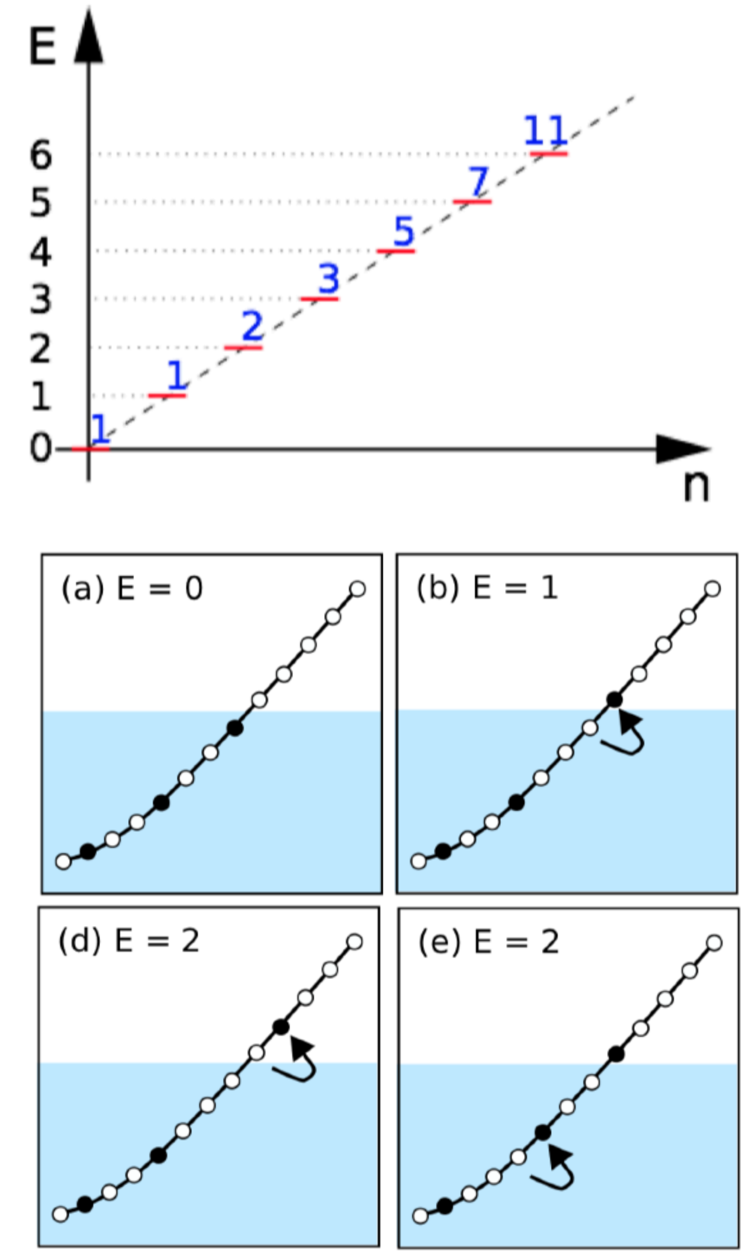
Fractional Chern Insulators: Chiral Edge States

Chiral edges states



[Li & Haldane '08]

$$\rho^{\mathrm{red}} = \sum_{\alpha} \exp(-\epsilon_\alpha)|\phi_\alpha\rangle\langle\phi_\alpha|$$



Grushin, Motruk, Zaletel, FP, PRB 91, 035136 (2015).

Tutorial: (D) TeNPy Entanglement Spectrum

http://go.tum.de/475840

I) Calculate the entanglement spectrum of the Haldane model. Can you get the right counting?

II) Calculate the expectation value of the charge density operator and explore the regime of large interactions.

Tutorial: (E) TeNPy Laughlin Pump

http://go.tum.de/475840

I) Explore a few points in the phase diagram of the Haldane using the Laughlin pump.

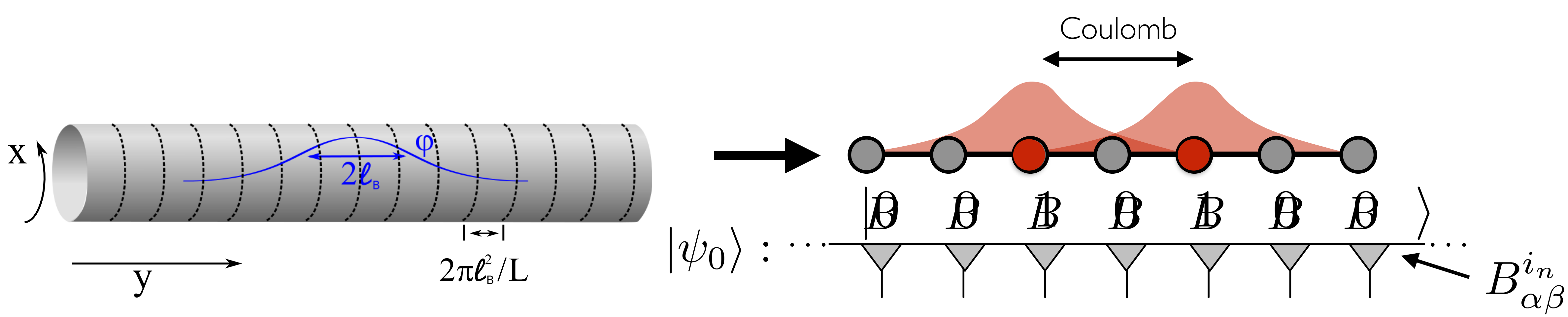
II) Open ended: Can you find parameters to stabilize an FCI?

Fractional Quantum Hall

Density Matrix Renormalization Group
to simulate FQHE on infinite cylinders

Orbitals in first Landau level are localized along the cylinder: **Quasi 1D model**

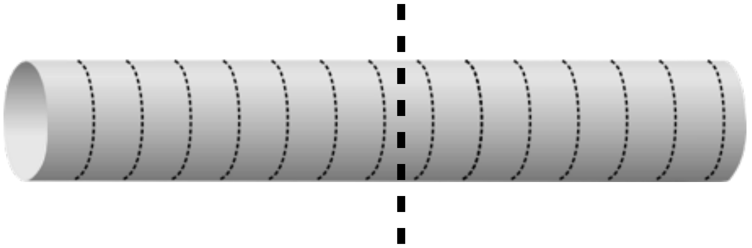
[Tao & Taoless 88, Haldane & Rezayi '94; Bergholtz et al. '05, Seidel et al. '05]



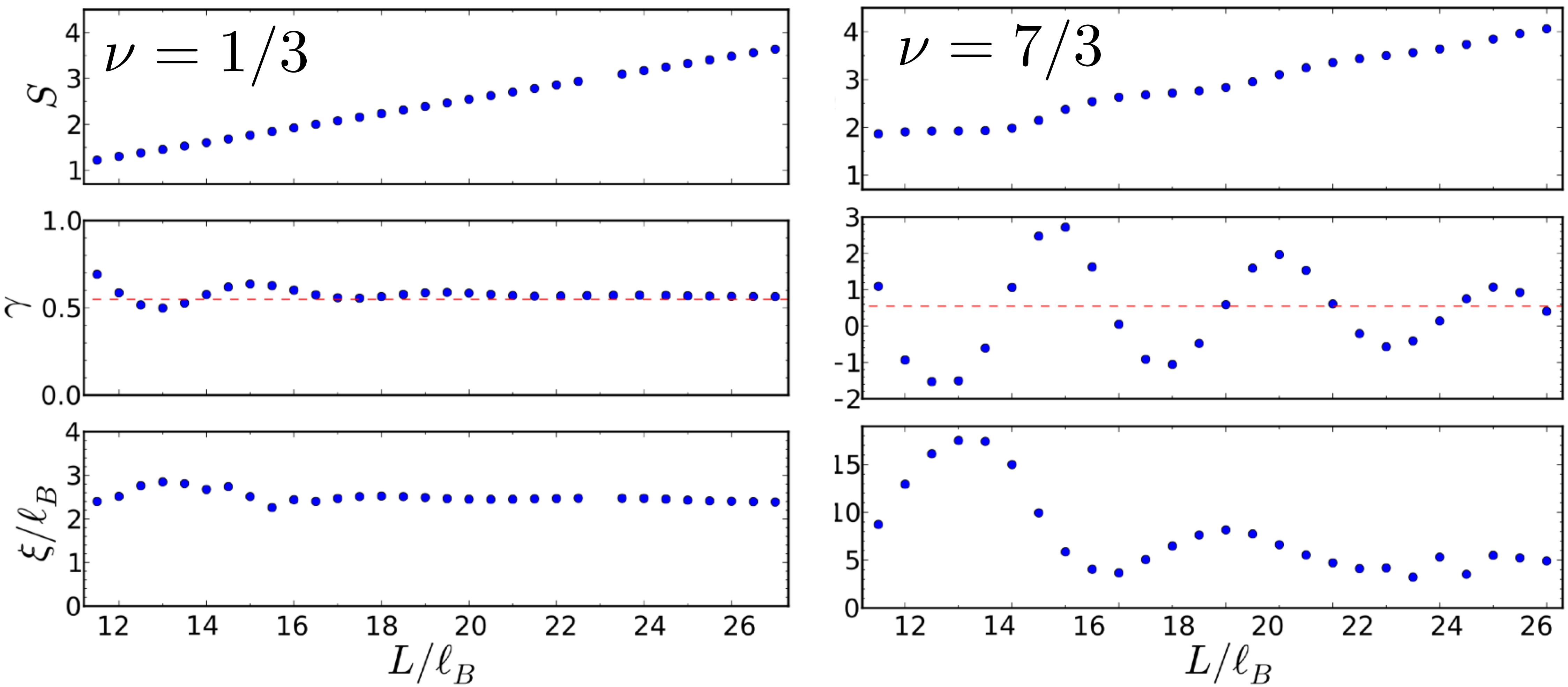
M. P. Zaletel, R. S. K. Mong, FP, PRL **110**, 236801 (2013).

Fractional Quantum Hall

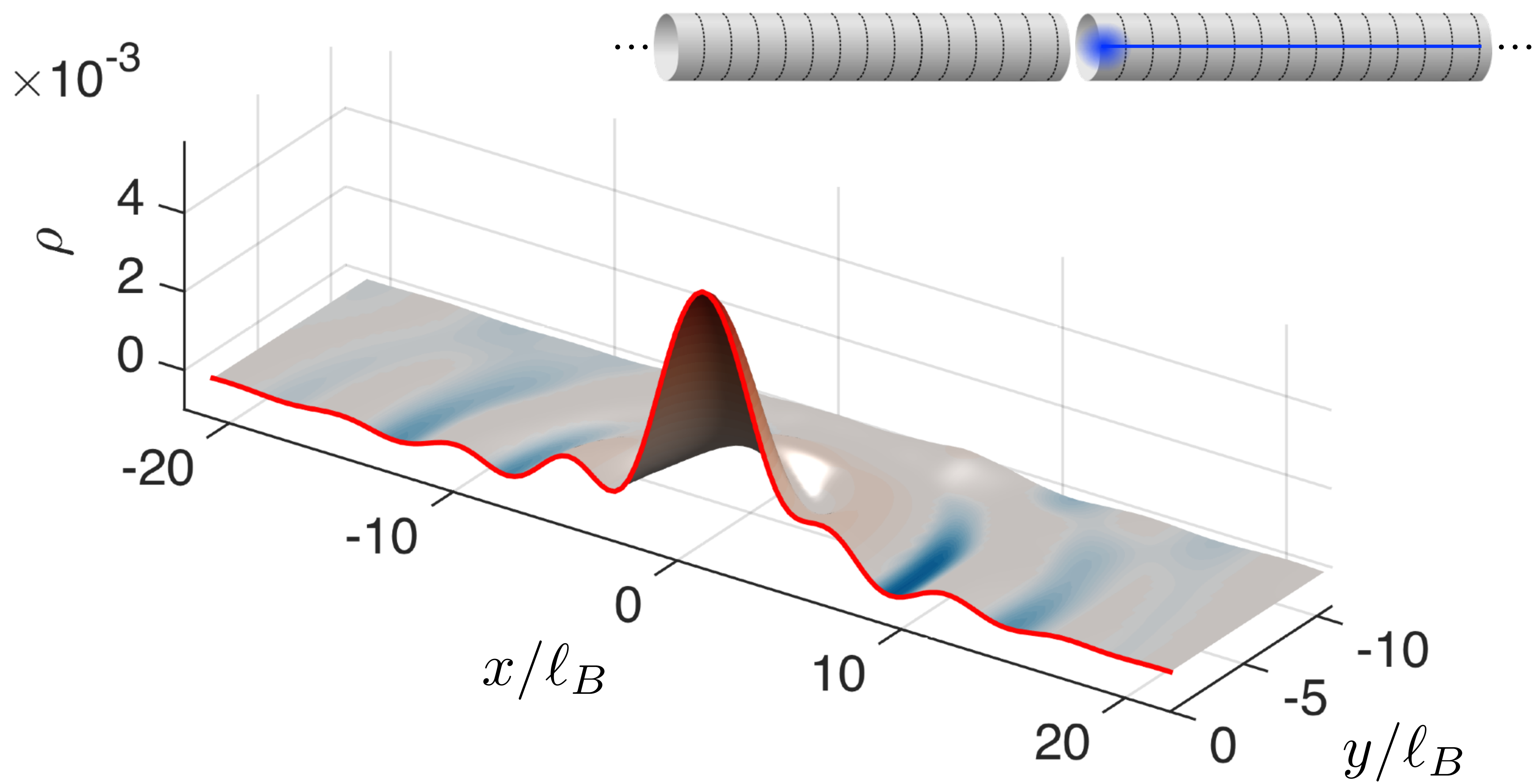
Topological entanglement entropy of the FQHE with Coulomb interactions ("minimally entangled states")



$$S = \alpha L - \gamma$$



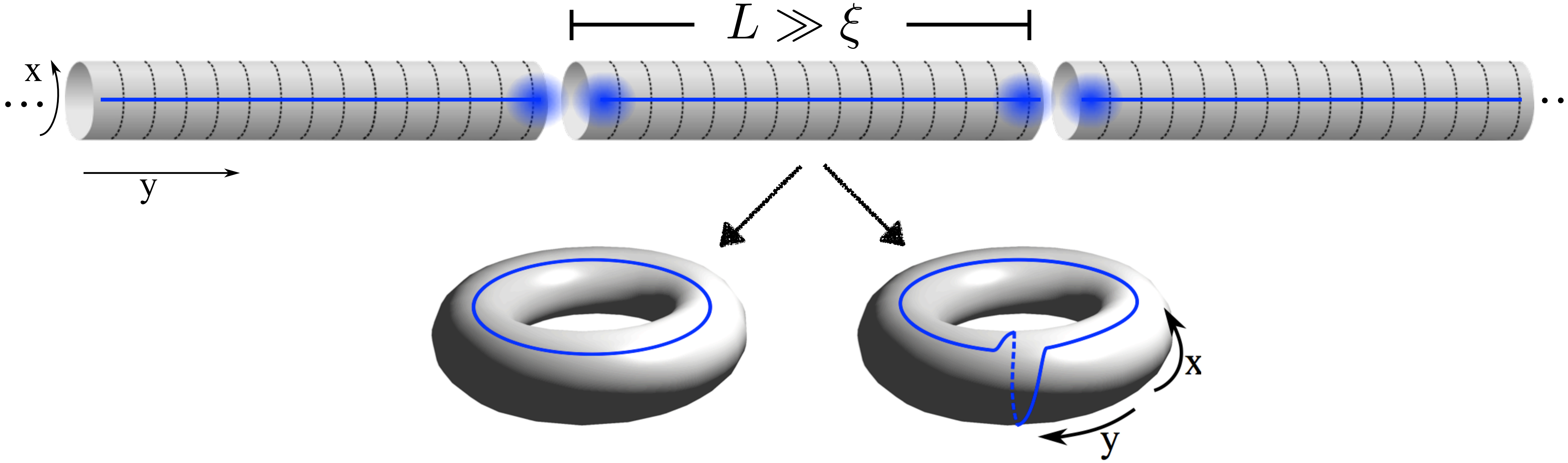
Fractional Quantum Hall



Density profile for Fibonacci anyons with charge **e/5**

Fractional Quantum Hall

Extracting topological content by adding a “twist”



Momentum polarization: **topological spin, central charge, Hall viscosity**

$$U_{T;ab} = \delta_{ab} \exp\left[2\pi i \left(h_a - \frac{c}{24} - \frac{\eta_H}{2\pi\hbar} L_x^2\right)\right]$$

Tensor Networks and the Many-Body Problem

I) Entanglement and Matrix-Product States

II) Time Evolving Block Decimation

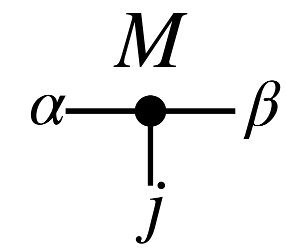
III) Density-Matrix Renormalization Group

IV) Extracting topological invariants

V) (isometric Tensor-Network States) 👉

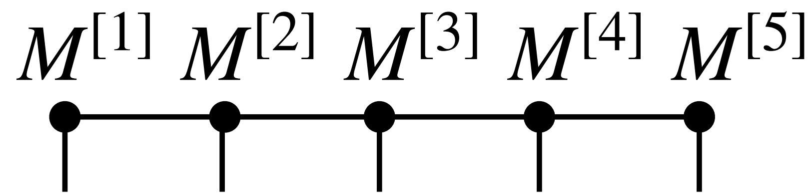
Matrix-Product States

Matrix-product states (MPS): Reduction of the number of variables: $d^L \rightarrow L d \chi^2$ [M. Fannes et al. 92]

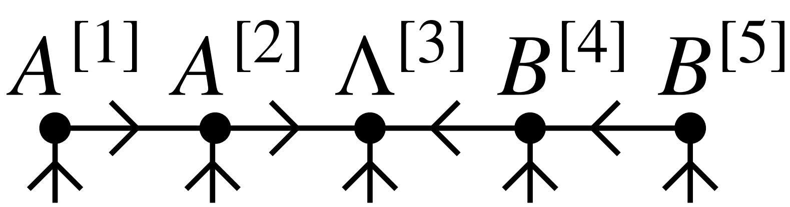
$M^j_{\alpha,\beta} =$ 

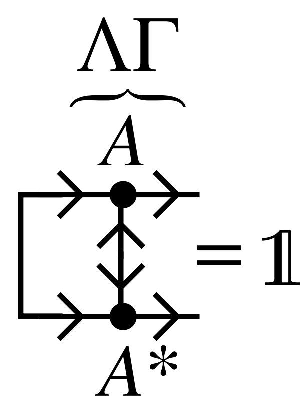
$\alpha, \beta = 1 \ldots \chi$

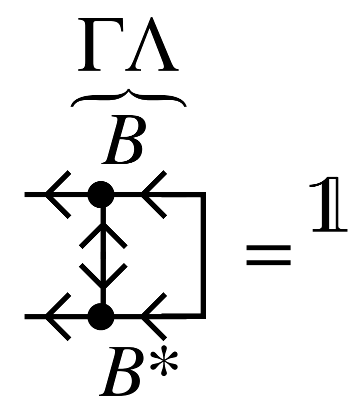
$j = 1 \ldots d$

$\psi_{j_1,j_2,j_3,j_4,j_5} =$ 

Canonical form: Use the gauge degree of freedom ($A^j = X M^j X^{-1}$) to find a convenient representation







(Isometries)

Center matrix Λ represents wave function

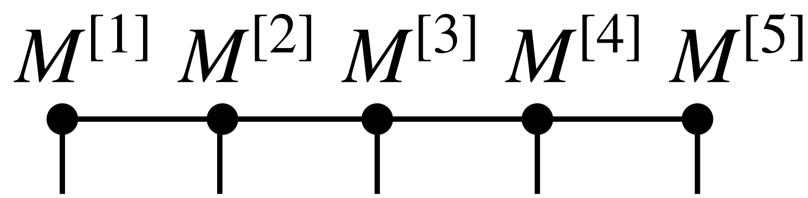
$$|\psi\rangle = \sum_{\alpha,\beta,j} \Lambda^j_{\alpha,\beta} |\alpha\rangle |j\rangle |\beta\rangle$$

(orthogonal states $|j\rangle$, $|\alpha\rangle$, $|\beta\rangle$)

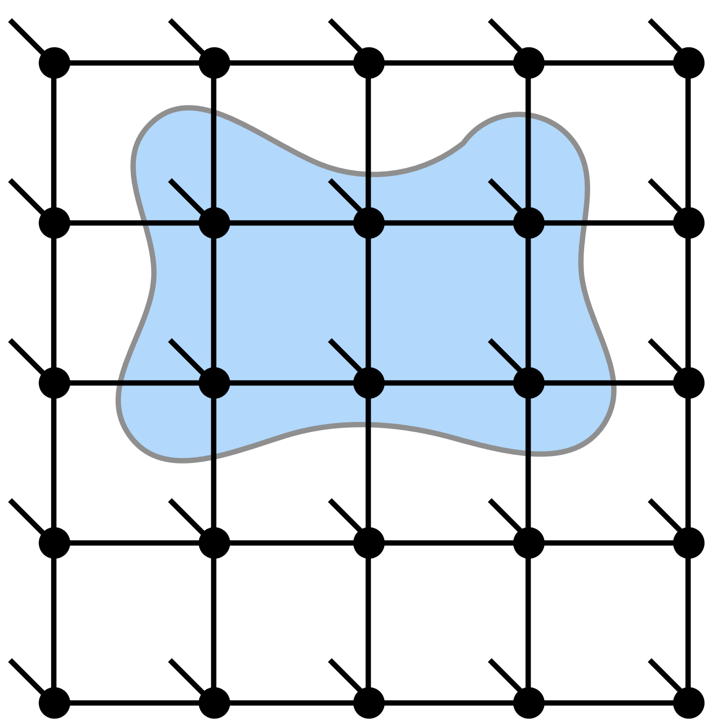
Tensor Network States in 2D

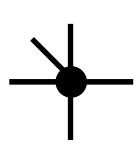
MPS capture 1D area law → Exponential scaling in 2D

$$\psi_{j_1,j_2,j_3,j_4,j_5} \approx$$



How to generalize the MPS approach to 2D?

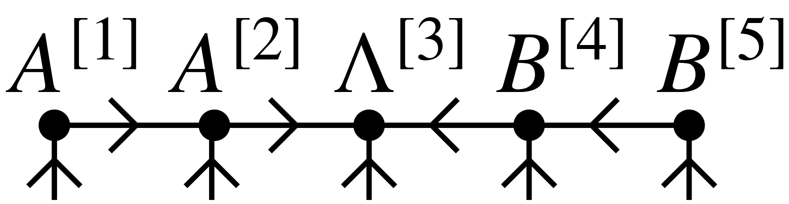


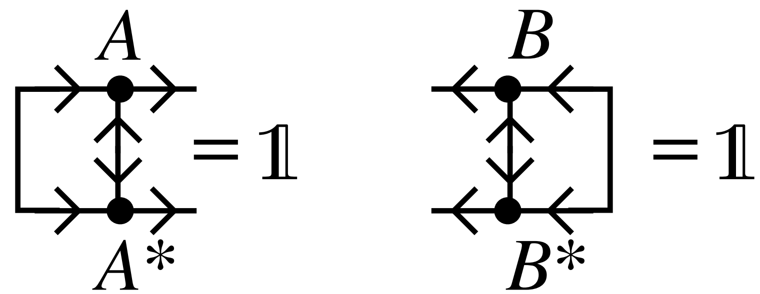
$T^j_{\alpha,\beta,\gamma,\delta} =$ 

- Tensor Network States (TNS)
  [Maeshima et al. '01, Verstraete and Cirac '04]
- Capture 2D area law* 😊
- Difficult to handle numerically:
  Exact contraction of the 2D network is still **exponentially hard** 🙁

Isometric Tensor Network States in 2D

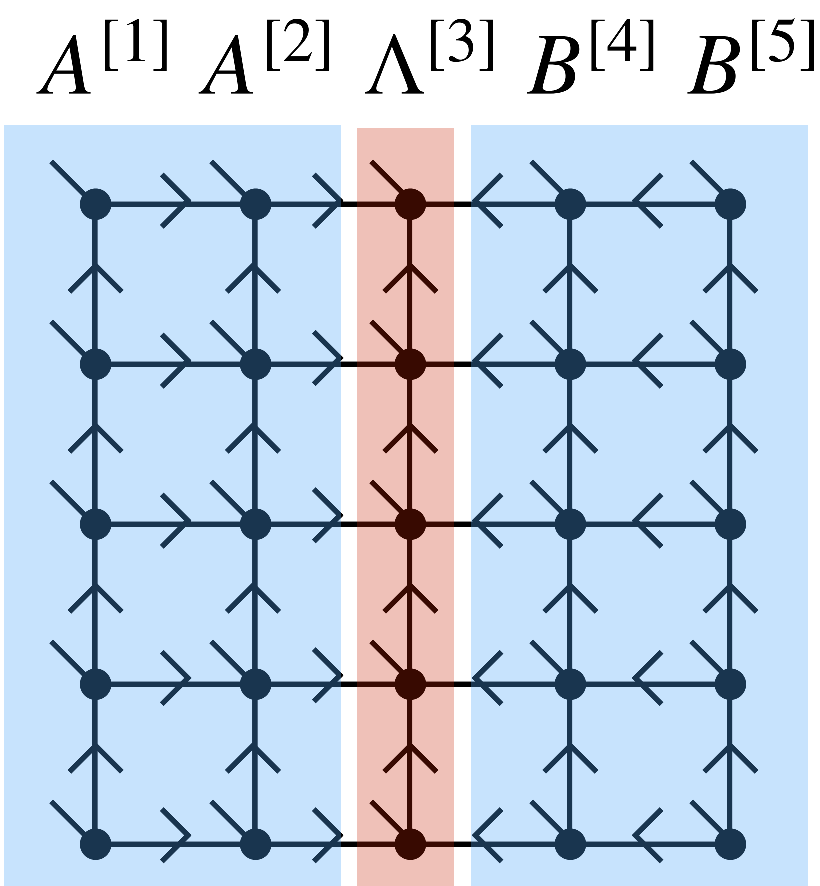
Recall: Canonical form of 1D MPS





(Isometries)

Isometric TNS

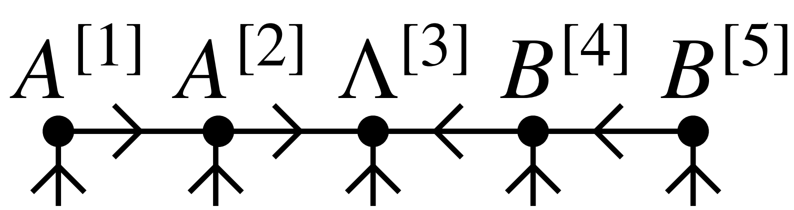


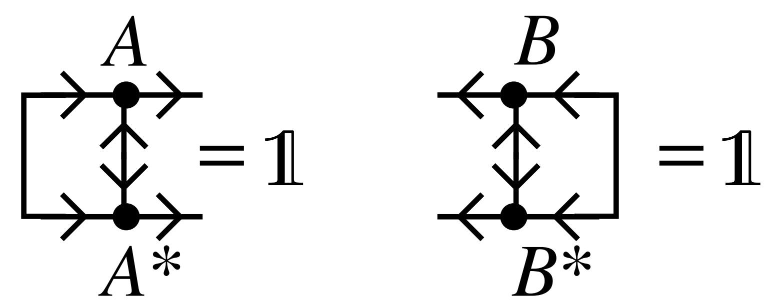
- Isometric tensors are **efficiently contractable**
- Orthogonality center column is a **1D MPS**: Standard DMRG techniques
- Subset of TNS: Unclear what its variational power is!

[Zaletel and FP, PRL **124**, 037201 (2020)]

Isometric Tensor Network States in 2D

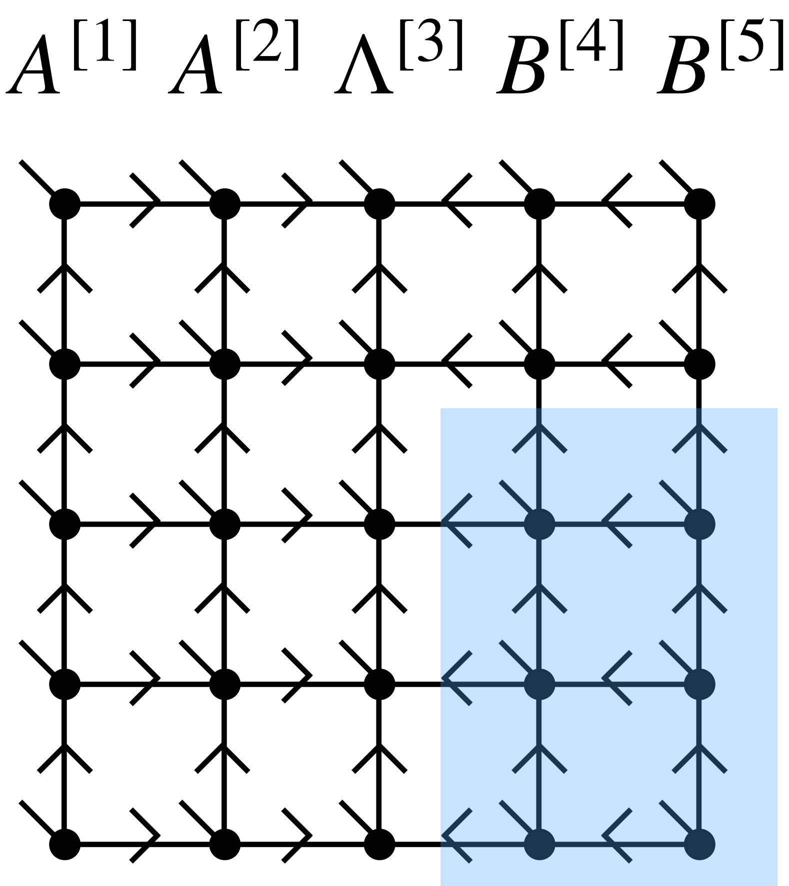
Recall: **Canonical form of 1D MPS**





(Isometries)

Isometric TNS



- **Subregions** with only outgoing arrows have isometric boundary maps
- **Causal structure**: time flows opposite to the direction of the arrows

[Zaletel and FP, PRL **124**, 037201 (2020)]

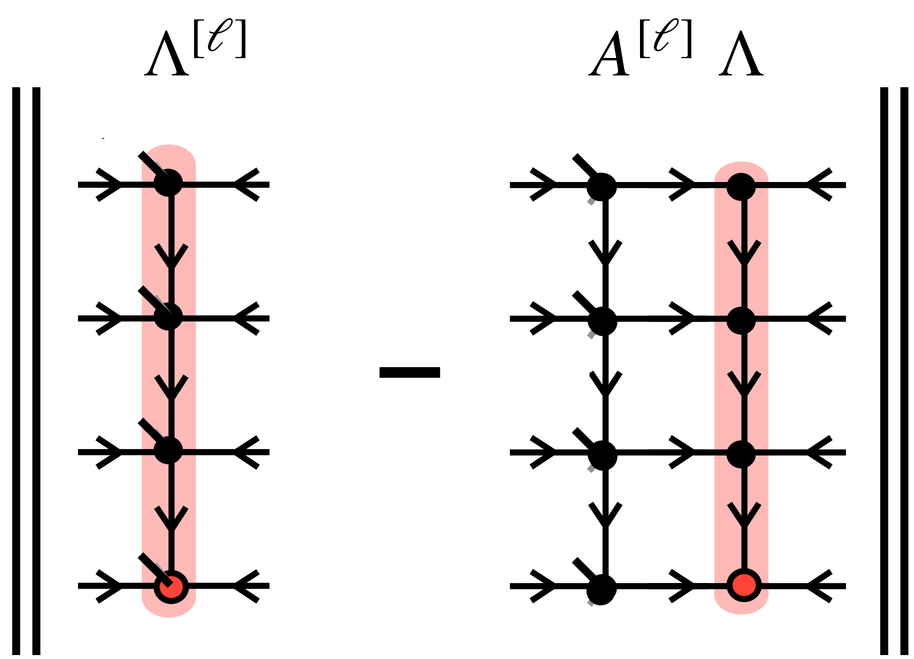
Isometric Tensor Network States in 2D

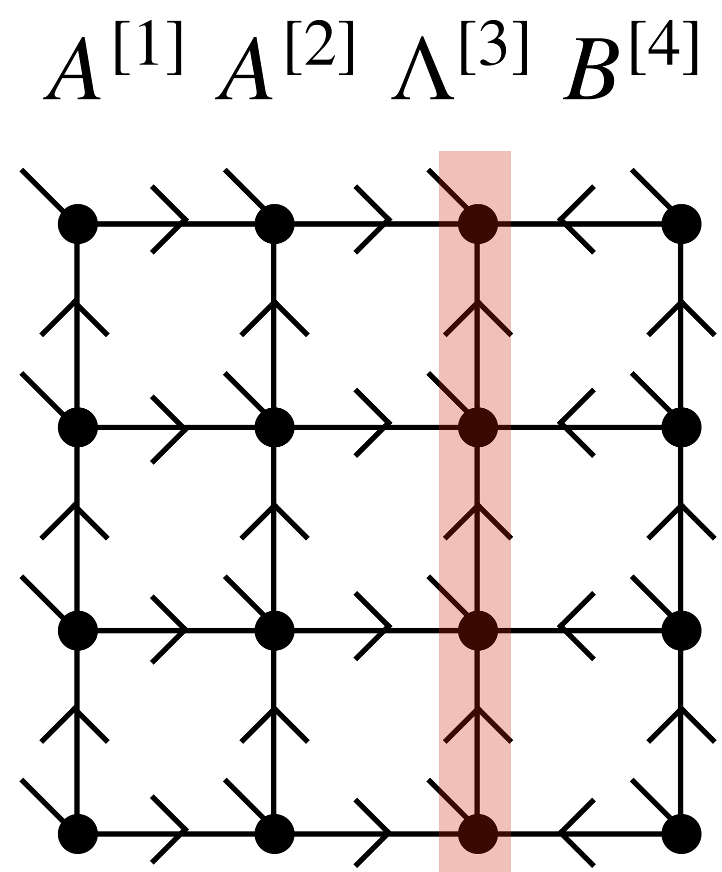
How to shift the orthogonality center?

Recall: 1D MPS $\Lambda^{\ell} B^{[\ell+1]} = A^{[\ell]} \Lambda^{[\ell+1]}$ solved by QR or SVD

Not possible for 2D TNS as it would destroy the locality of Λ

Solve the variational problem:

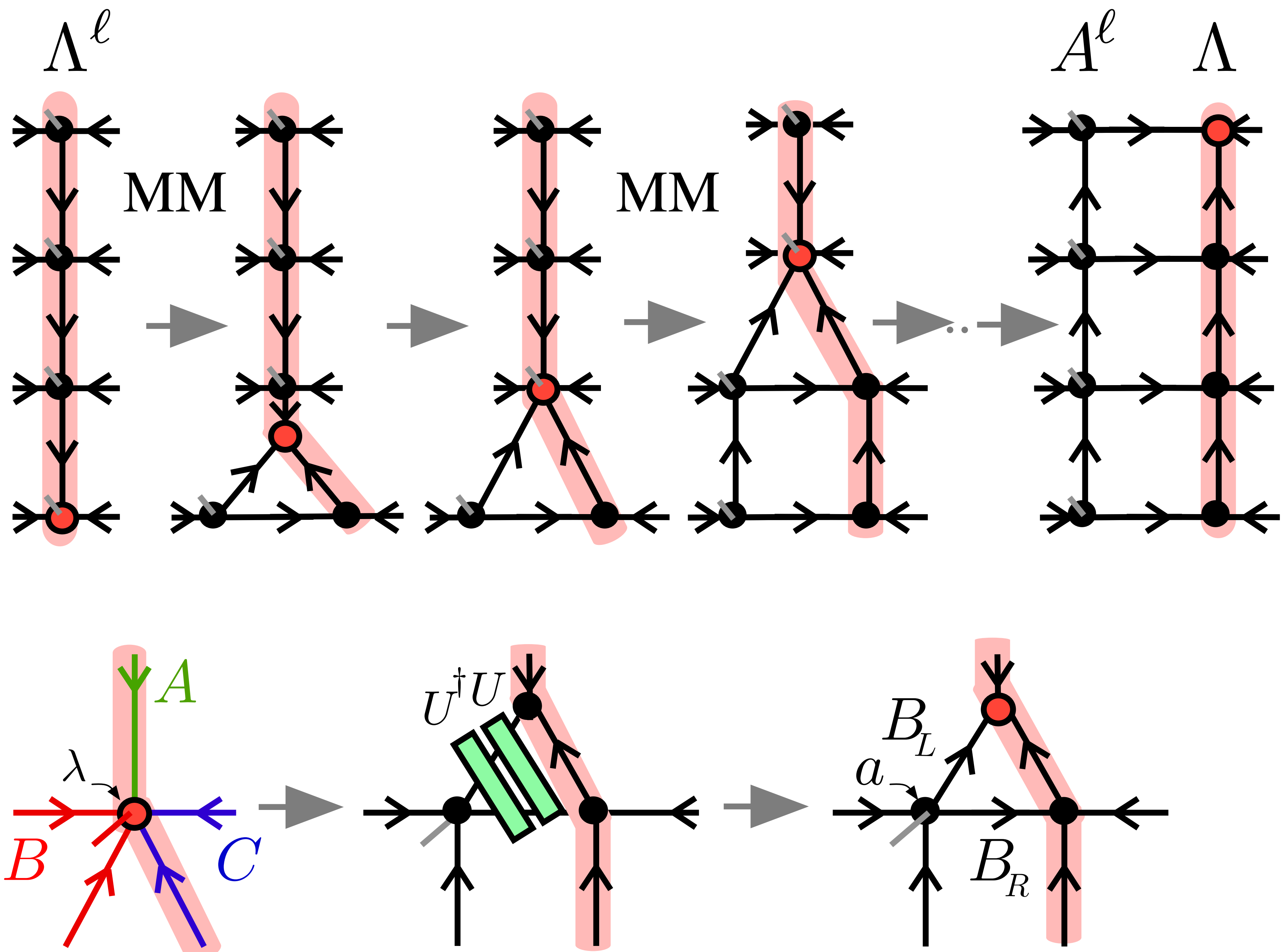
min 



[Zaletel and FP, PRL **124**, 037201 (2020)]

Isometric Tensor Network States in 2D

Sequential splitting based on disentangling: **"Moses Move" (MM)**

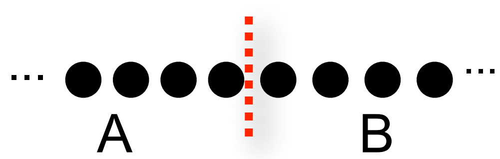


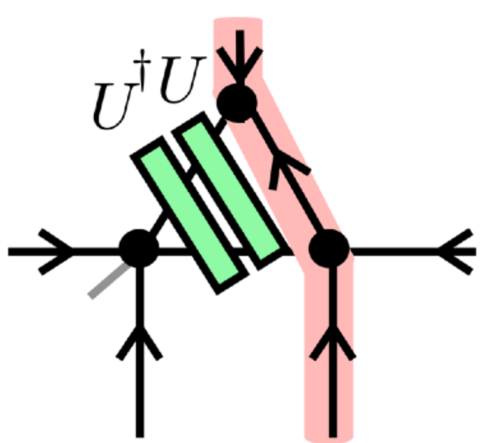
[Zaletel and FP, PRL **124**, 037201 (2020)]

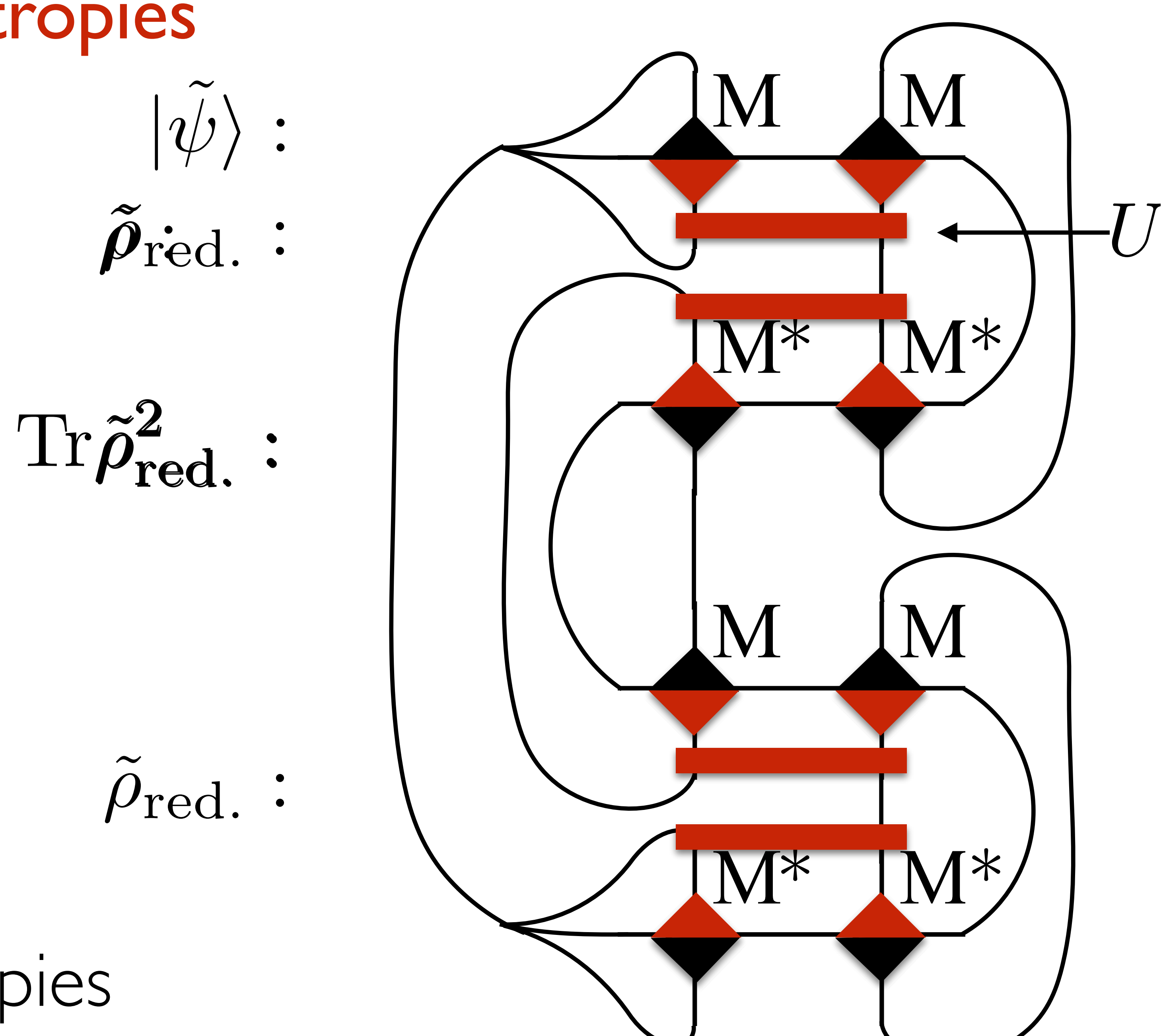
Finding the disentangler

Variationally disentangle the state: $S_2 = -\ln \mathrm{Tr}\rho^2_{\mathrm{red.}}$

minimize Renyi entropies





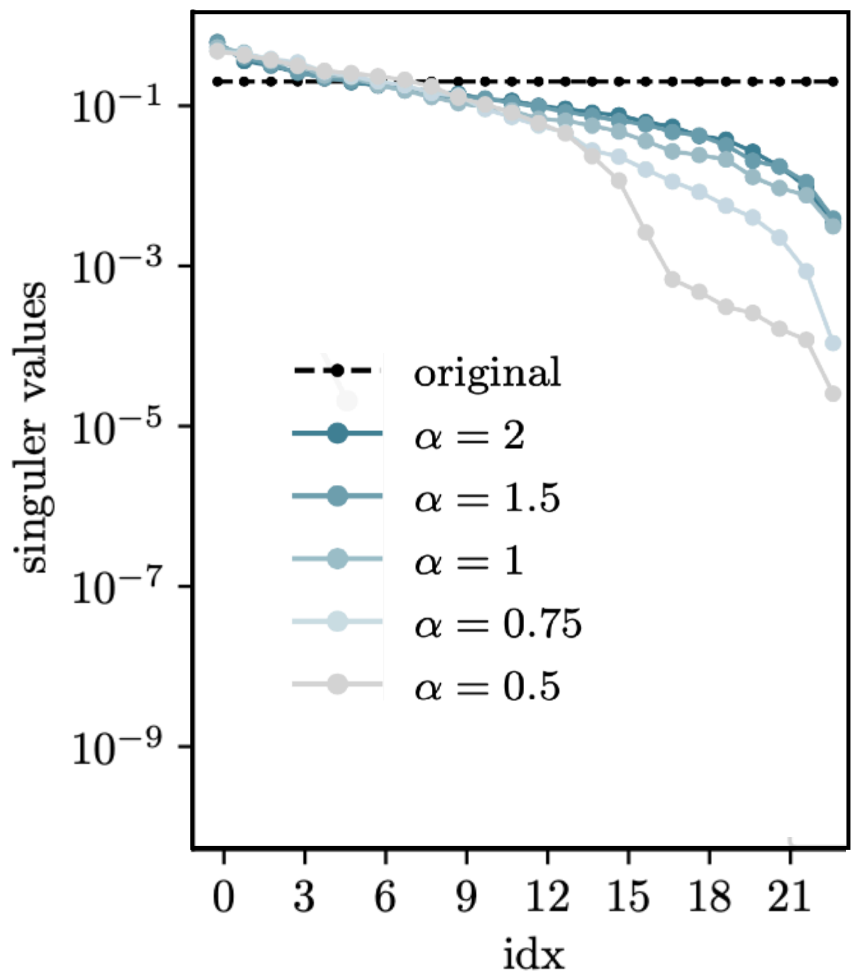


- Other Rényi entropies can also be minimized

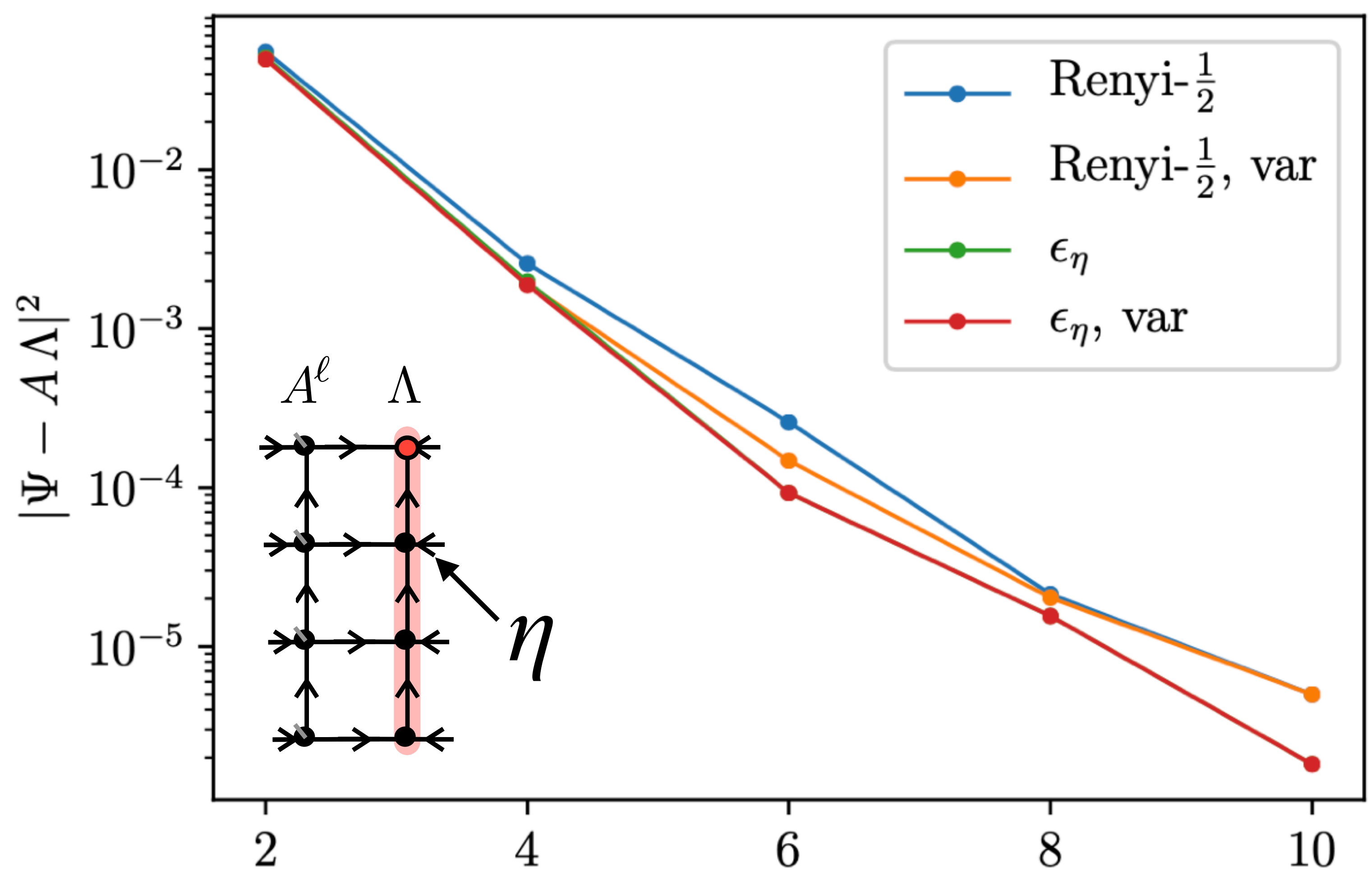
[Evenbly & Vidal '09]

Finding the disentangler

Role of the disentangler:



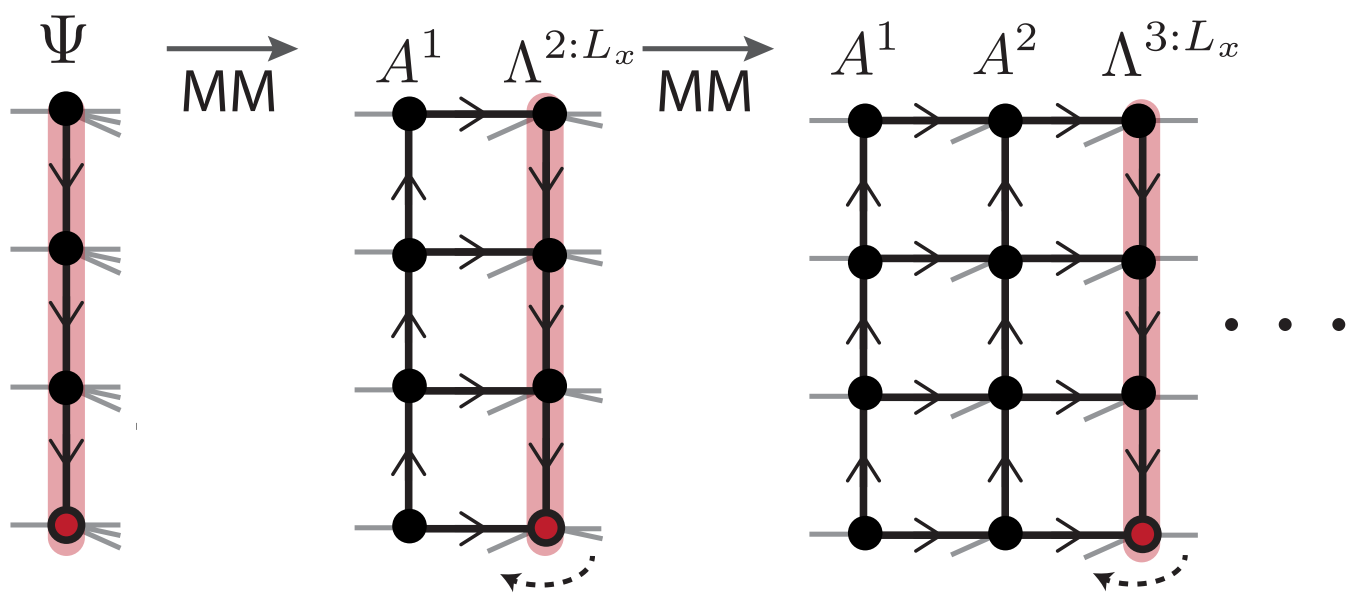
Variational vs. Moses Move:



[Zaletel and FP, PRL **124**, 037201 (2020)]

Convert quasi 1D MPS to isometric TNS

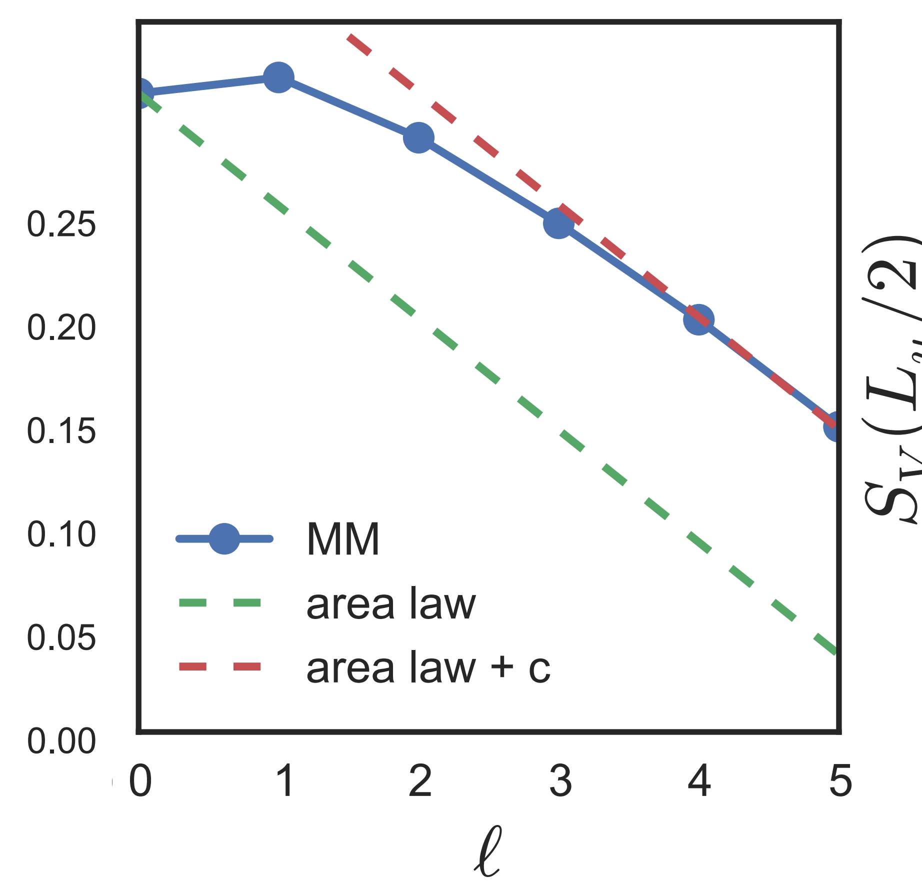
"Peel off" layers from MPS representation of 2D state



- Sequentially disentangle the state
- Efficient compression

2D transverse field Ising Model ($g = 3.5$)

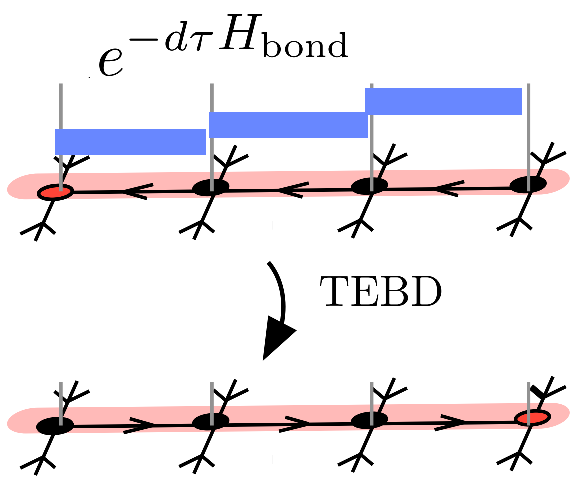
$$H = -\sum_{\langle i,j\rangle} \sigma_i^z \sigma_j^z - g \sum_i \sigma^x$$

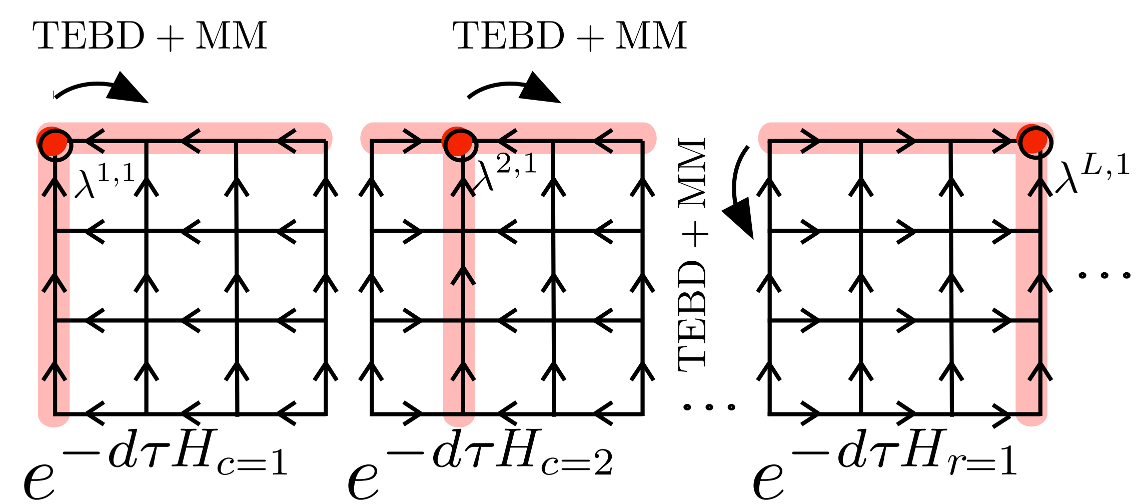


[Zaletel and FP, PRL **124**, 037201 (2020)]

(I) Time evolution of 2D Hamiltonians (TEBD²)

Sequentially apply **1D Time-Evolving Block Decimation (TEBD)** algorithm on the center columns/rows: 2nd order [Vidal '03]

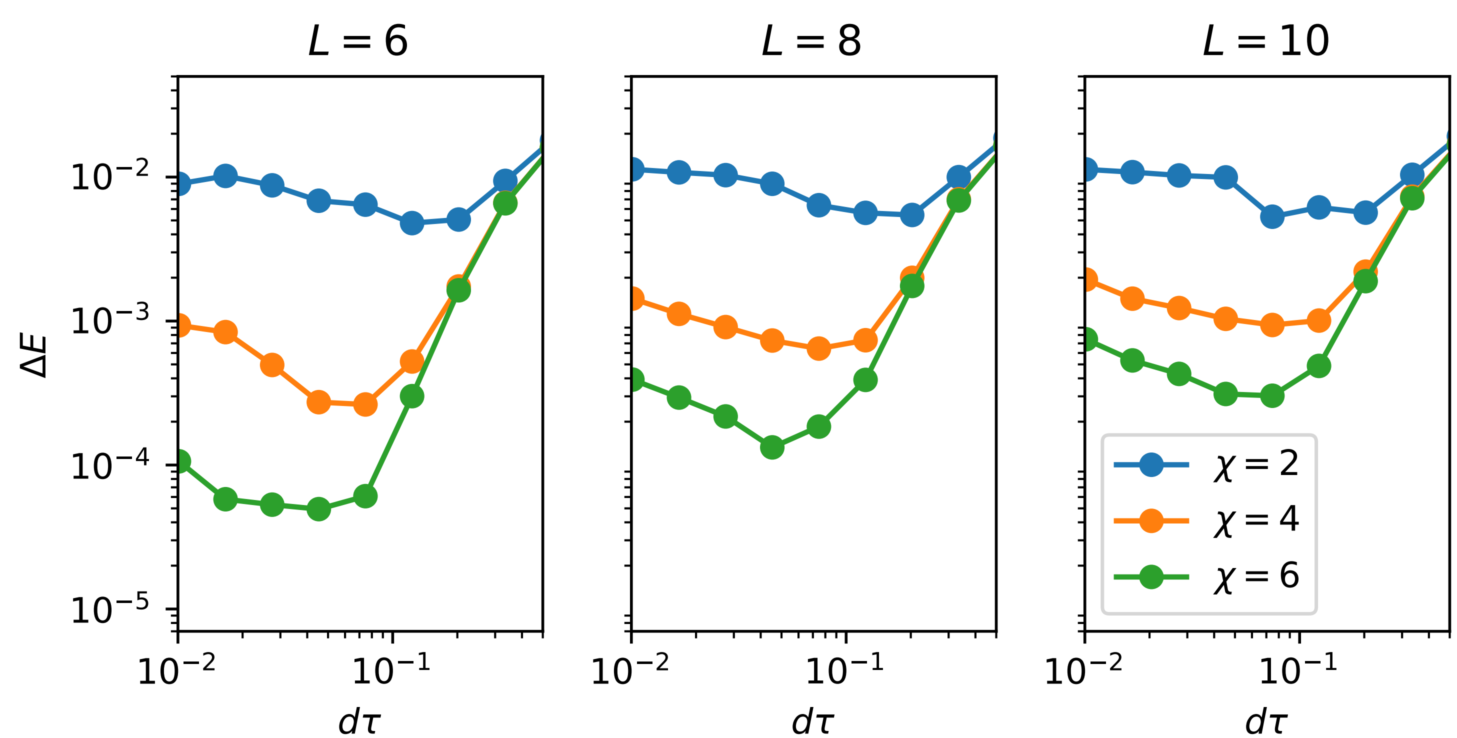




2D transverse field Ising Model ($g = 3.5$)

$$H = -\sum_{\langle i,j\rangle} \sigma_i^z \sigma_j^z - g \sum_i \sigma^x$$

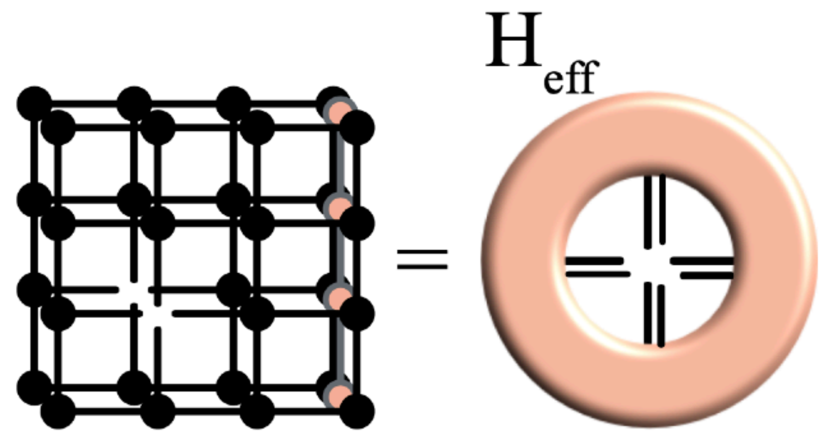
Imaginary time evolution: $|\psi_0\rangle$



[Zaletel and FP, PRL **124**, 037201 (2020)]

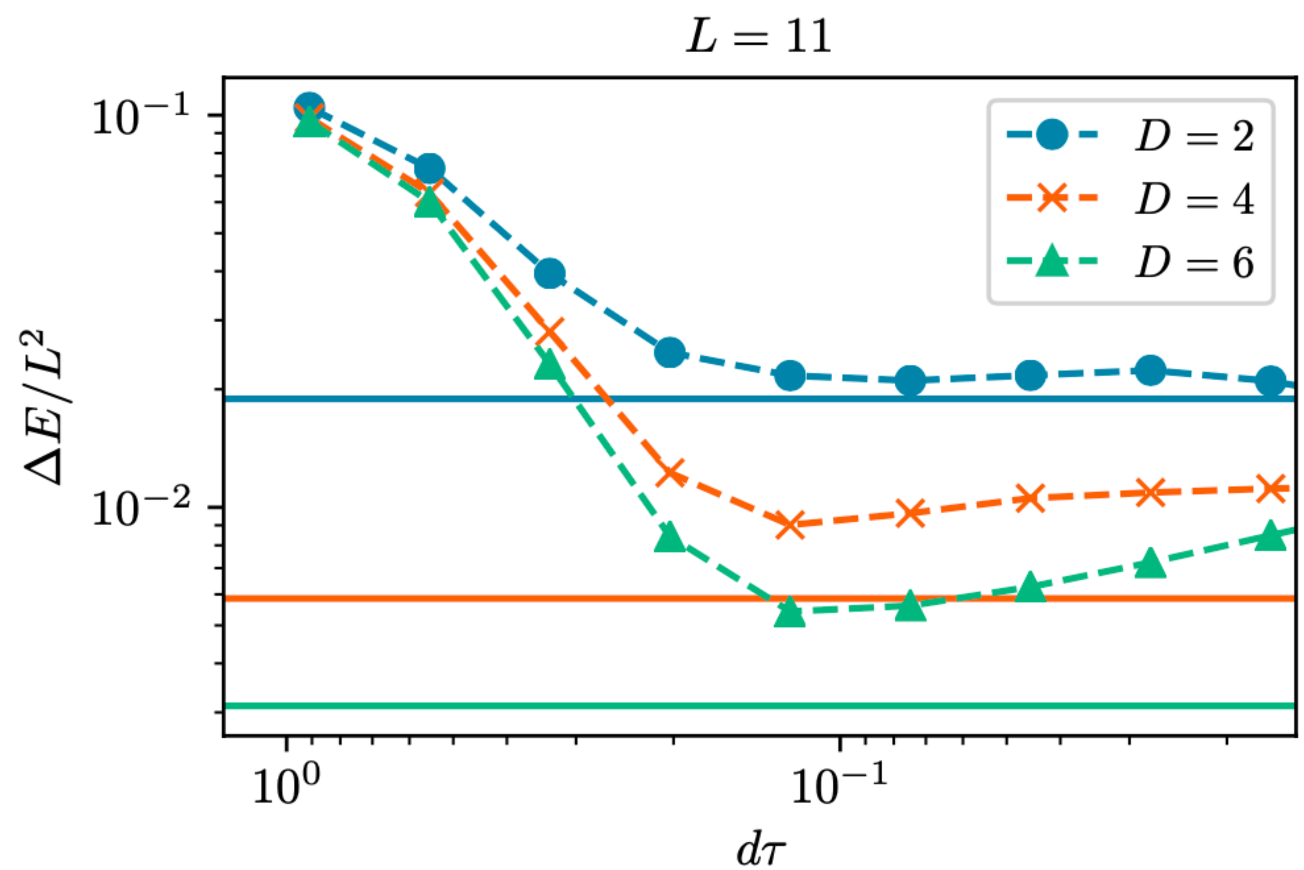
(II) Variational optimization (DMRG²)

Iteratively minimize the energy by sequentially optimizing the isometries



2D transverse field Ising Model ($g = 3.0$)

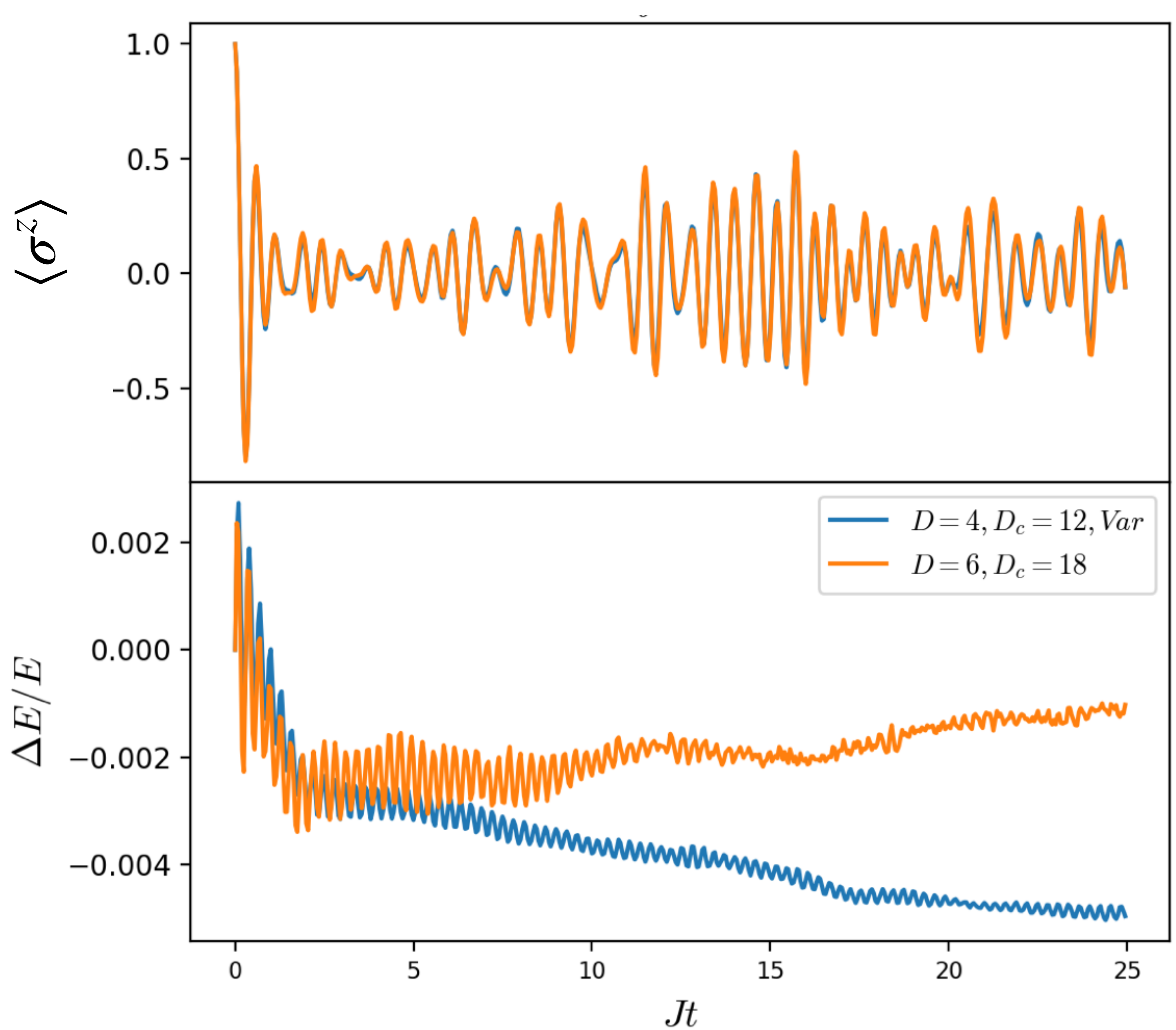
$$H = -\sum_{\langle i,j \rangle} \sigma_i^z \sigma_J^z - g \sum_i \sigma^x$$

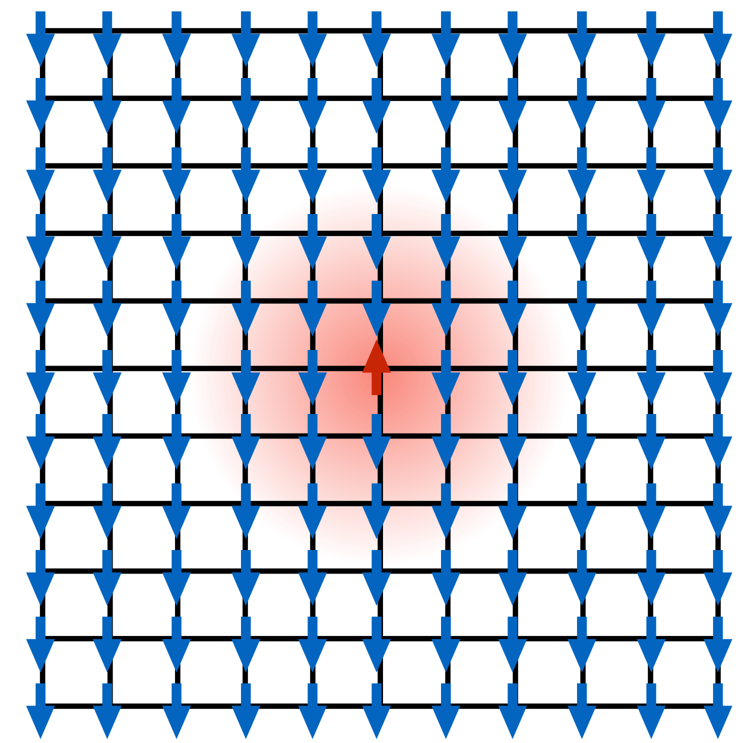


[**Lin**, Zaletel and FP; work in progress]

(III) Dynamical spin structure factors from isoTNS

Real time evolution of $|\psi_0(t)\rangle = e^{-iHt}\,\sigma^y|\psi_0\rangle$ for the transverse field Ising model (paramagnetic phase)





- Good convergence at small bond dimension χ

[**Lin**, Zaletel and FP; work in progress]

(III) Dynamical spin structure factors from isoTNS

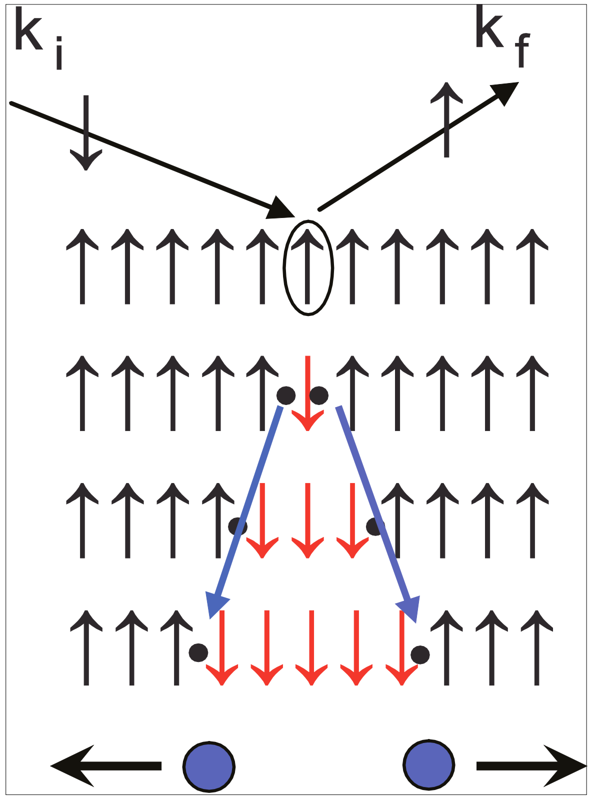
Numerical calculation of the **dynamical structure factor**

$$S(k,\omega) = \sum_x \int_{-\infty}^{\infty} dt \; e^{-i(kx+\omega t)} C(x,t)$$

with $C(x,t) = \langle \psi_0 | \sigma_x^y(t) \sigma_0^y(0) | \psi_0 \rangle$

(1) Find the ground state $|\psi_0\rangle$: DMRG[2]

(2) Time evolve $\sigma_0^y |\psi_0\rangle$ to obtain $C(x,t)$



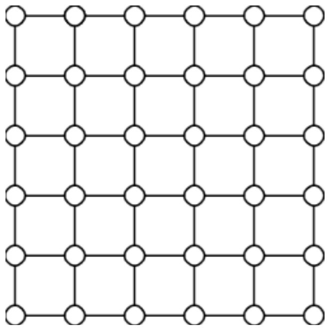
Slow growth of entanglement: Long times!

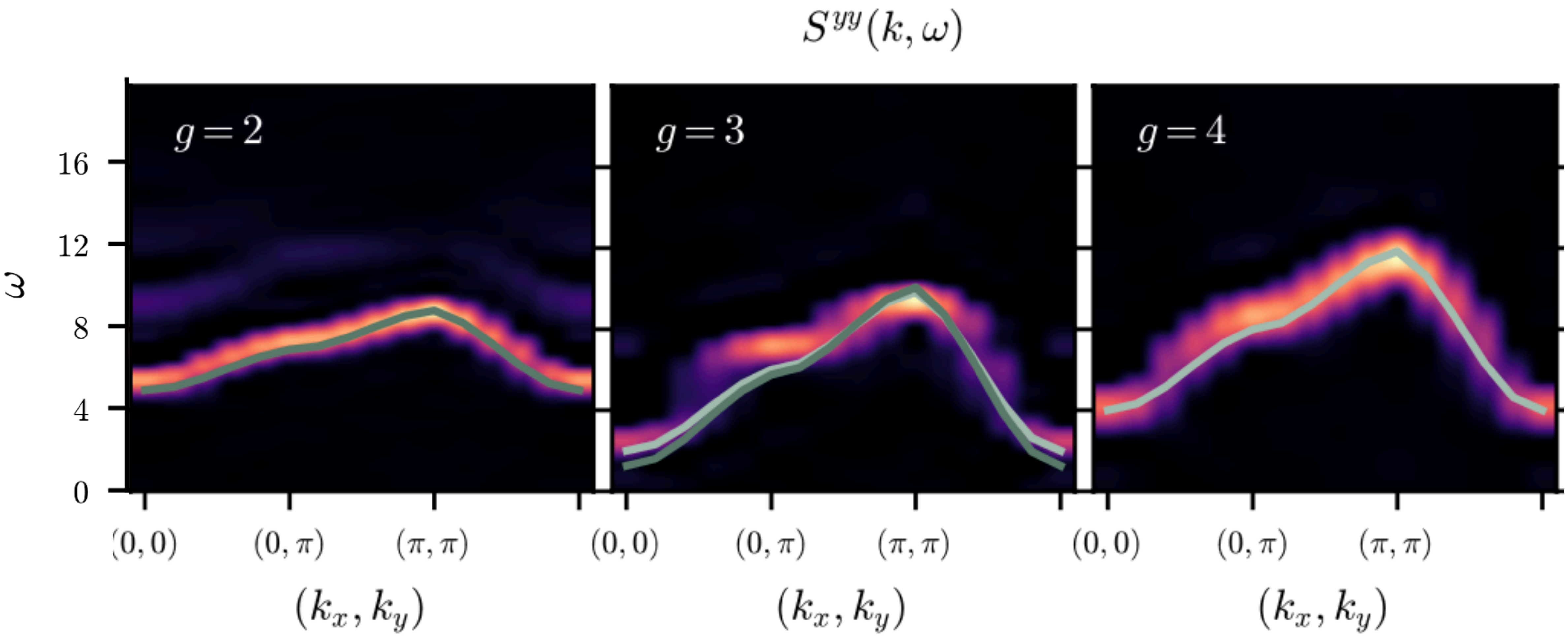
[**Lin**, Zaletel and FP; work in progress]

(III) Dynamical spin structure factors from isoTNS

Dynamical structure factor: Transverse field Ising

$$H = -\sum_{\langle i,j \rangle} \sigma_i^z \sigma_j^z - g \sum_i \sigma^x$$



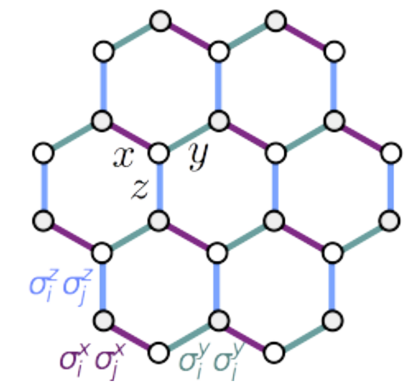


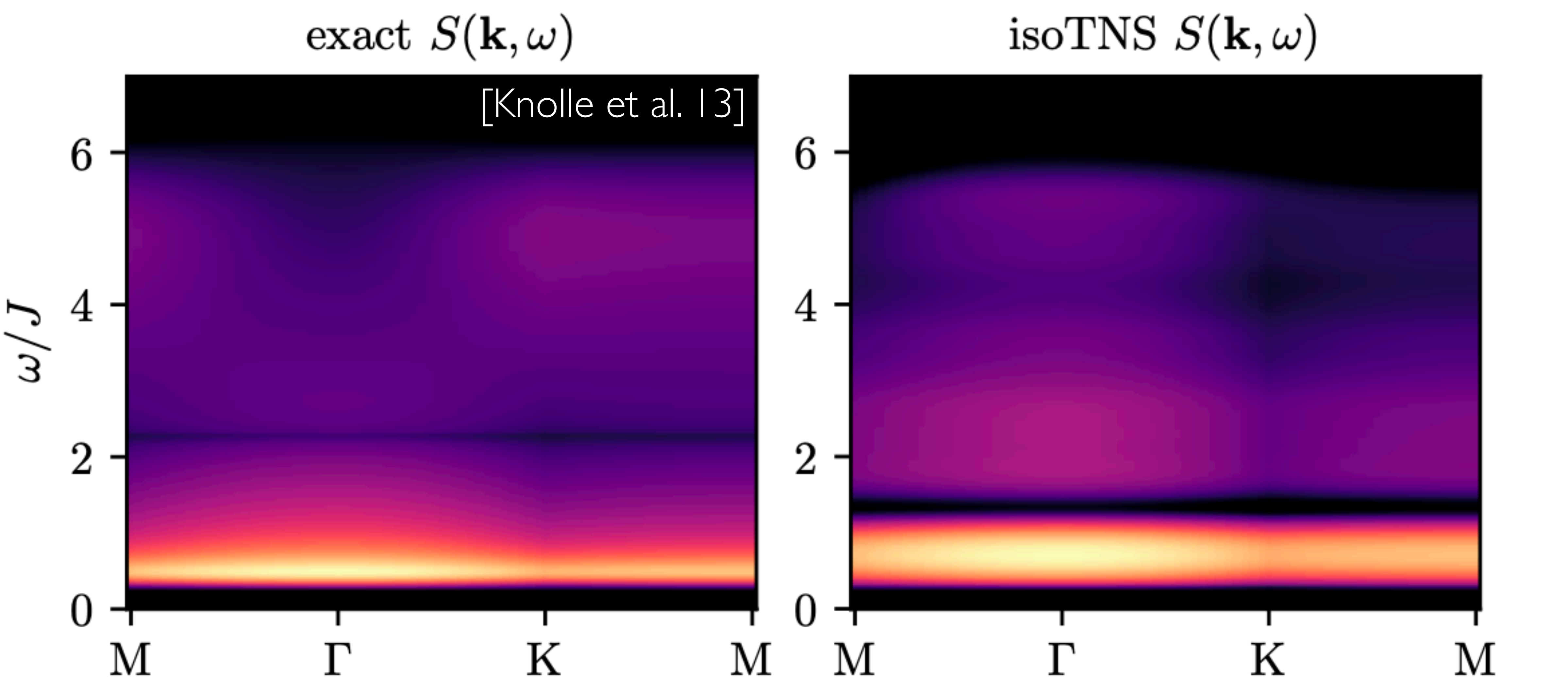
[**Lin**, Zaletel and FP; work in progress]

(III) Dynamical spin structure factors from isoTNS

Dynamical structure factor: Kitaev model

$$H = J \sum_{\langle i,j \rangle_{\alpha=x,y,z}} \sigma_i^\alpha \sigma_j^\alpha$$





[**Lin**, Zaletel and FP; work in progress]

Tensor networks and the many-body problem

I) Entanglement and Matrix-Product States

II) Time Evolving Block Decimation

III) Density-Matrix Renormalization Group

IV) Extracting topological invariants

V) (isometric Tensor-Network States)

Python toolbox for MPS/TPS simulations:

https://github.com/tenpy/tenpy

Lecture notes on MPS:

Hauschild and FP, arxiv:1805.00055

Thank You!