TNS APPLICATIONS: LATTICE GAUGE THEORIES

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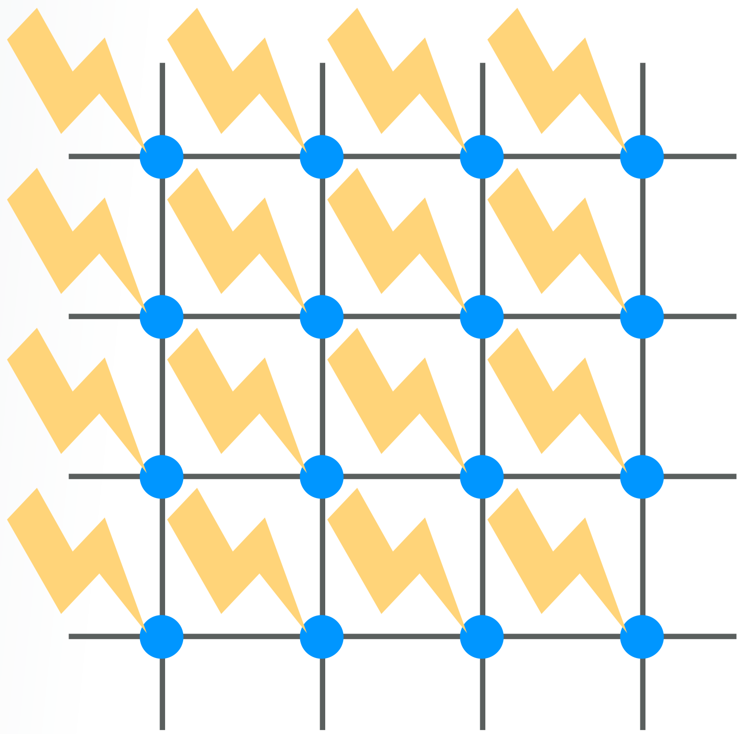
TENSOR25

what are LGT?

theory with a local (gauge) symmetry

Ising model



$$H = \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z + g \sum_i \sigma_i^x$$

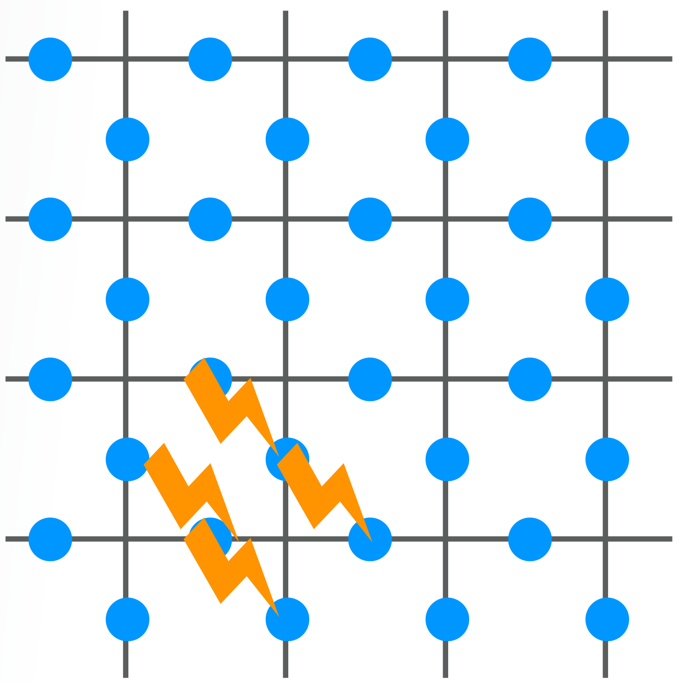
global $\mathbb{Z}_2$ symmetry

$$U = \otimes_i \sigma_i^x$$

$U \sigma_k^z U^\dagger = -\sigma_k^z$ flips all spins at once

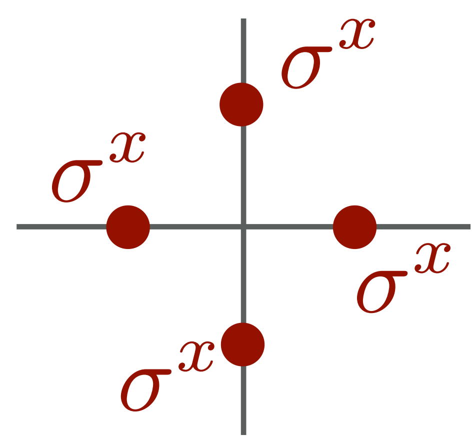
Ising lattice gauge theory

Wenger 1971



$$H = \sum_{\Box} \sigma^z_{\ell_1}\sigma^z_{\ell_2}\sigma^z_{\ell_3}\sigma^z_{\ell_4} + g\sum_{\ell}\sigma^x_{\ell}$$

local Z_2 symmetry

$G_i \sigma^z_\ell G_i = -\sigma^z_\ell$ $G =$ 

$[G_i, H] = 0 \qquad \forall i$

$G_i|\Psi\rangle = |\Psi\rangle$ Gauss' law

phase transition: deconfined / confined

promoting a global symmetry to local

gauging the symmetry

fundamental role in HEP

QED

$$\mathcal{L} = i\bar{\psi}\gamma_\mu\partial^\mu\psi - m\bar{\psi}\psi$$ Dirac fermion

global $\mathrm{U}(1)$ symmetry $\psi(x) \to e^{i\theta}\psi(x)$

global phase

local $\mathrm{U}(1)$ $\psi(x) \to e^{i\theta(x)}\psi(x)$

$$\bar{\psi}\partial^\mu\psi \to \bar{\psi}\left(\partial^\mu + i\partial^\mu\theta\right)\psi$$

solution $\partial^\mu \to D^\mu = \partial^\mu + iA^\mu$

$A^\mu \to A^\mu + \partial^\mu\theta$ gauge field

gauge invariant dynamical term $\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu\psi - m\bar{\psi}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

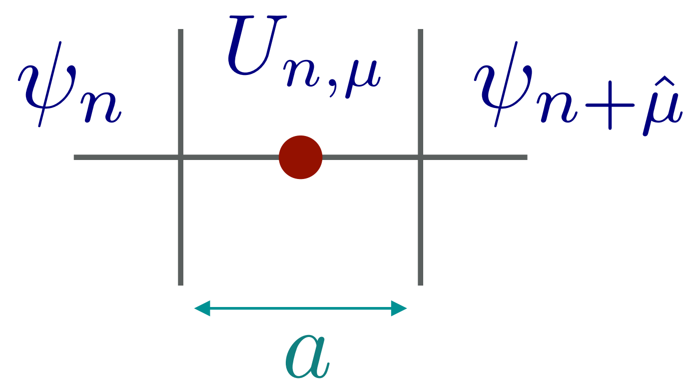
Wilson's LGT

Wilson 1974

discretized action $\rightarrow$ loses gauge invariance

discrete derivatives

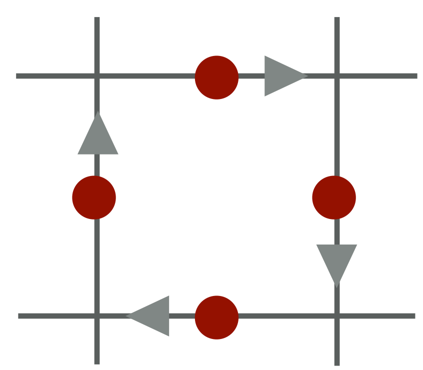
$$\frac{\psi_{n+\hat{\mu}} - \psi_{n-\hat{\mu}}}{2a}$$



$$\bar{\psi}_n \gamma_\mu U_{n,\mu} \psi_{n+\hat{\mu}}$$

$$U_{n,\mu} \rightarrow e^{ig\alpha_n} U_{n,\mu} e^{-ig\alpha_{n+\hat{\mu}}}$$

dynamics of gauge dof

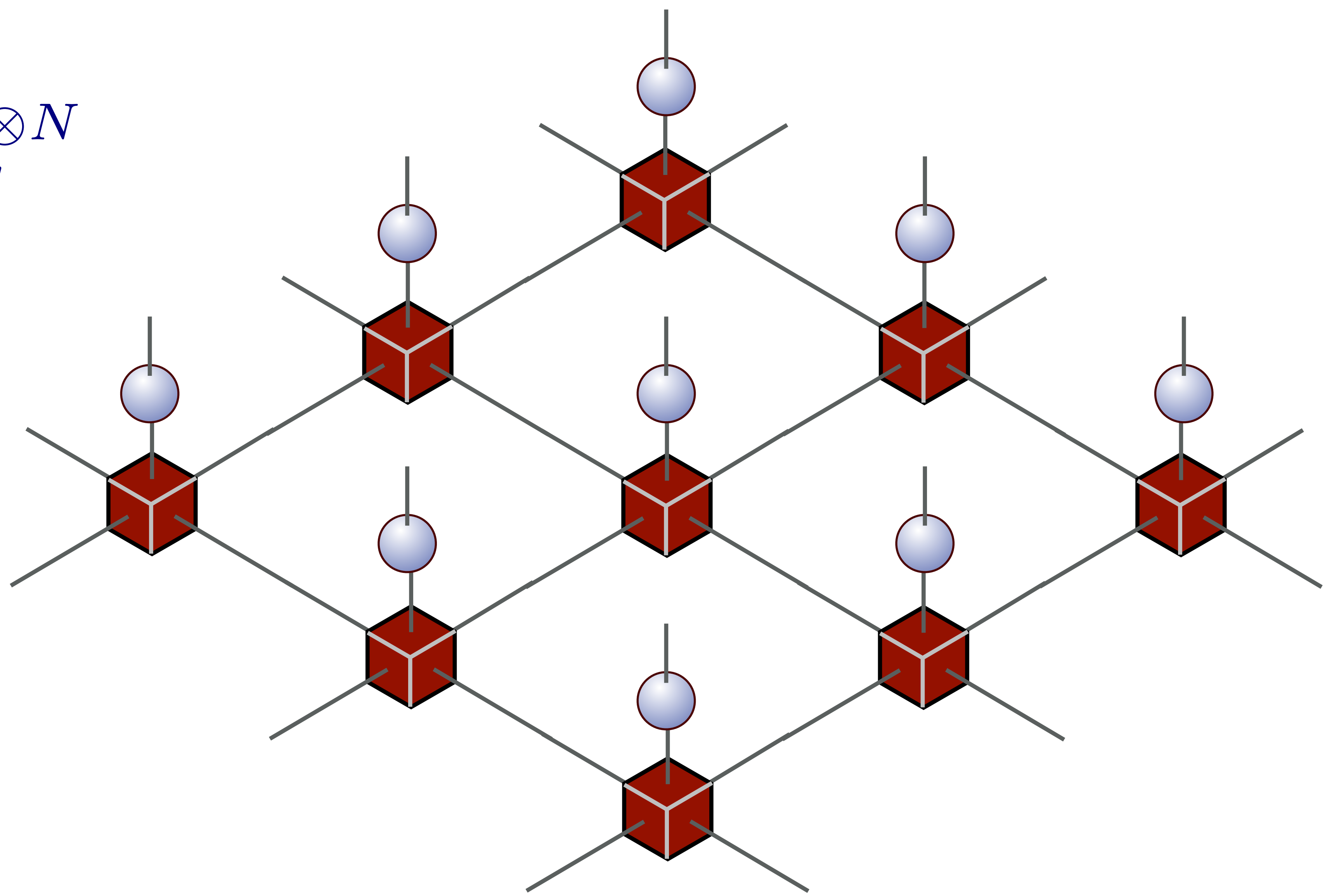


$$\mathrm{Tr}\left(U_\Box\right) + \mathrm{h.c.}$$

$a \rightarrow 0$ continuum QFT

gauging PEPS

$U_g^{\otimes N}$



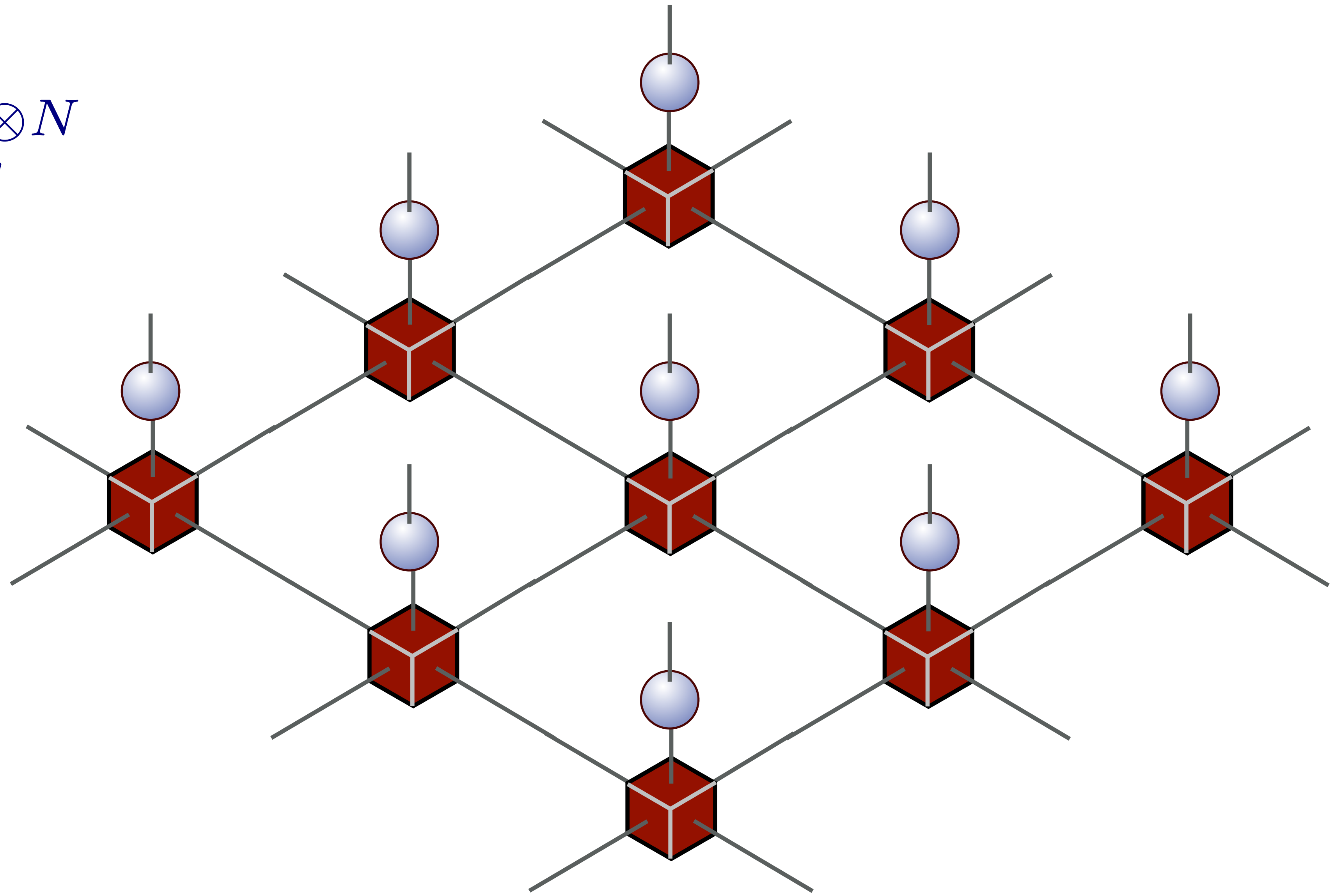
gauging PEPS

state (globally) invariant ⇔

Pérez-García et al., PRL 2008
Sanz et al., PRA 2009
Schuch et al., Ann. Phys. 2010
Singh et al., NJP 2007, PRA 2010

gauging PEPS

$U_g^{\otimes N}$



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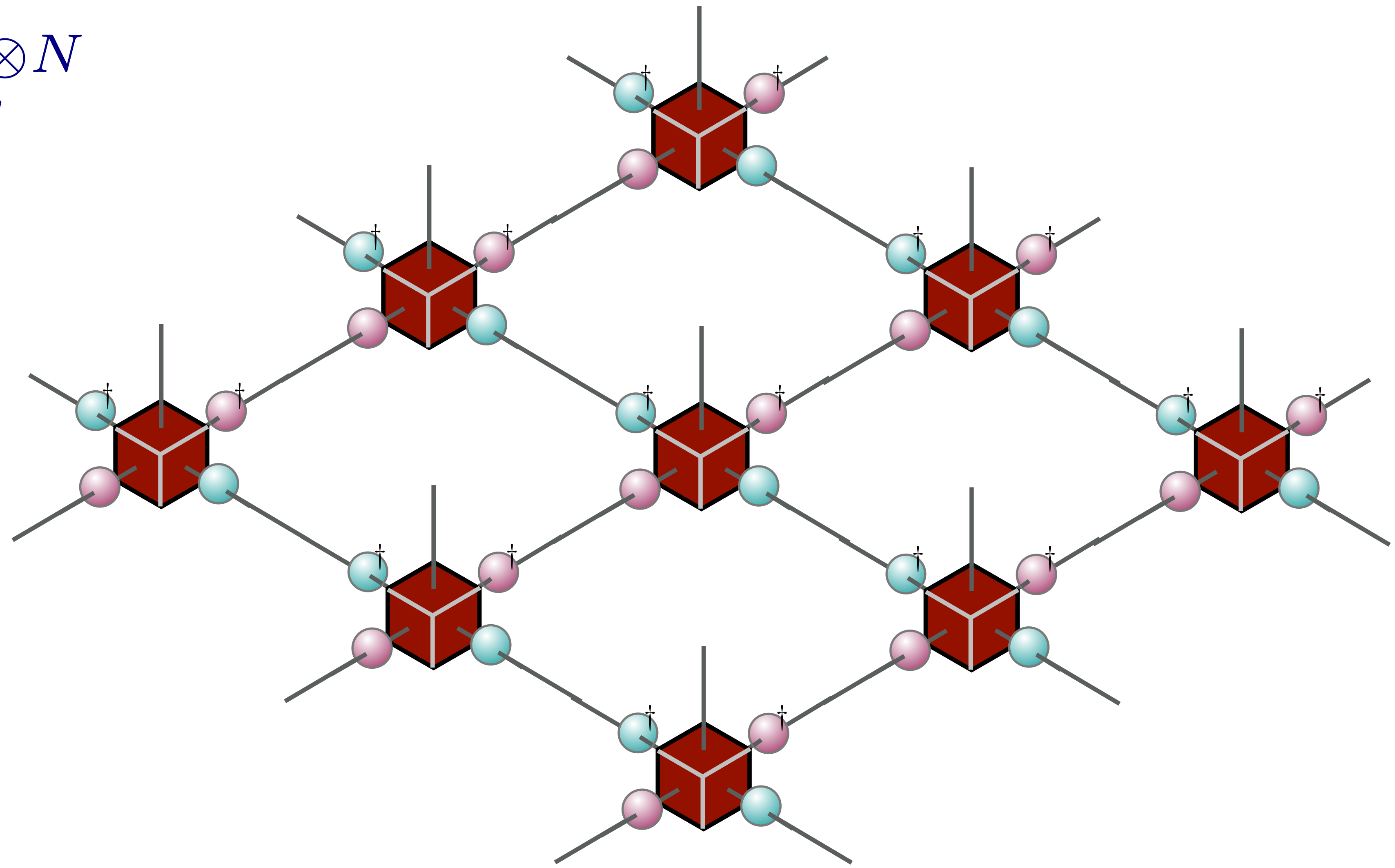

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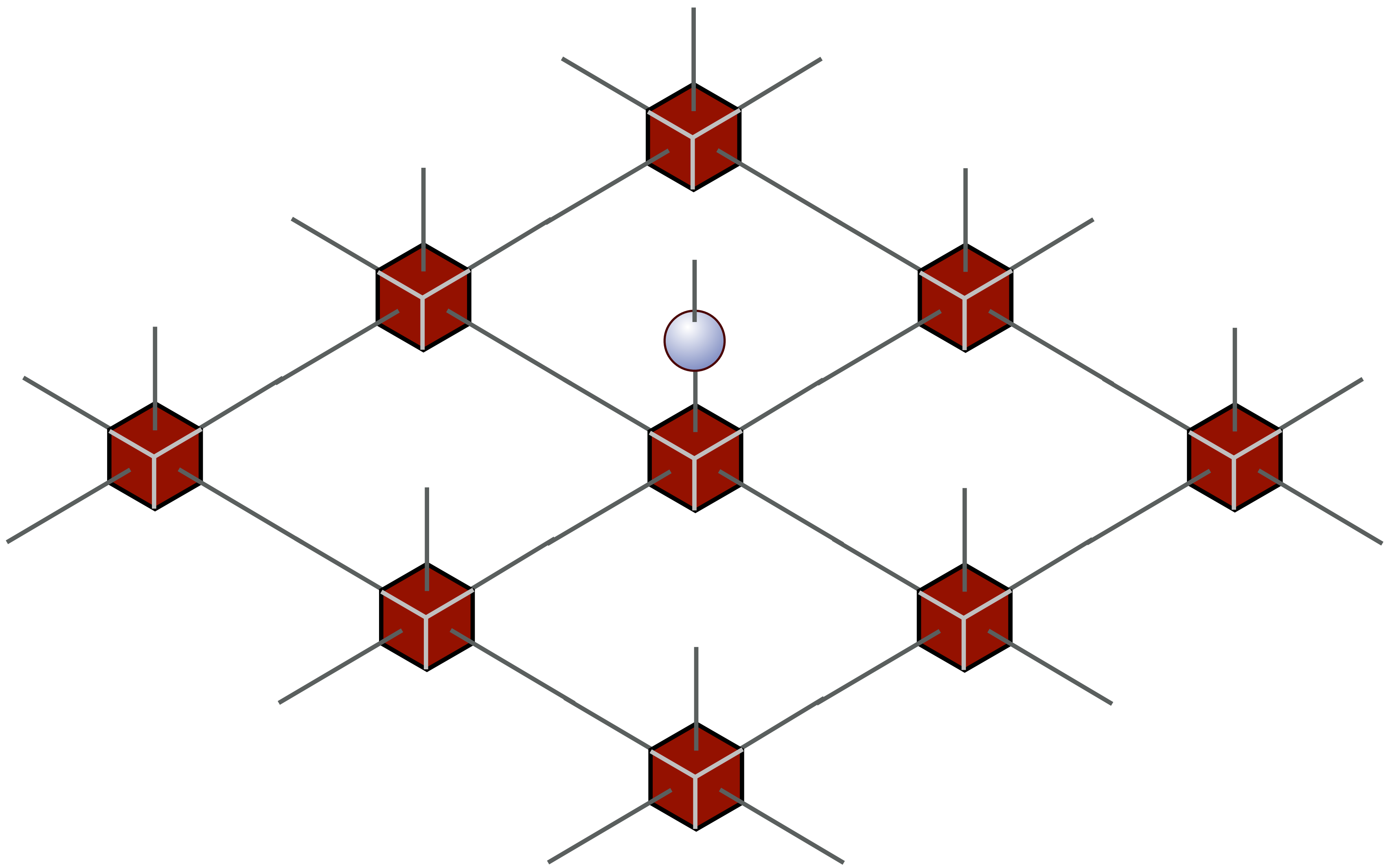
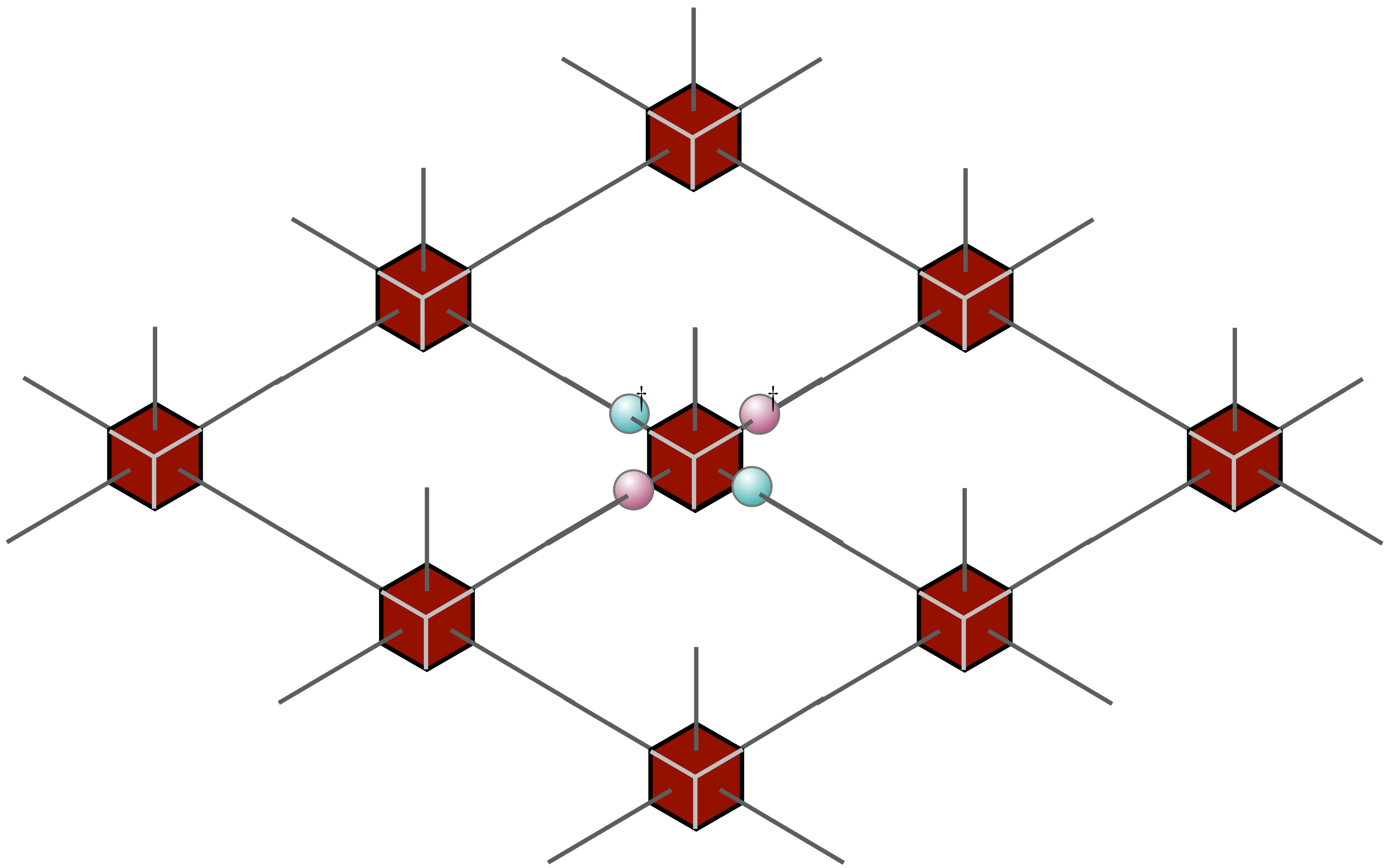


gauging PEPS



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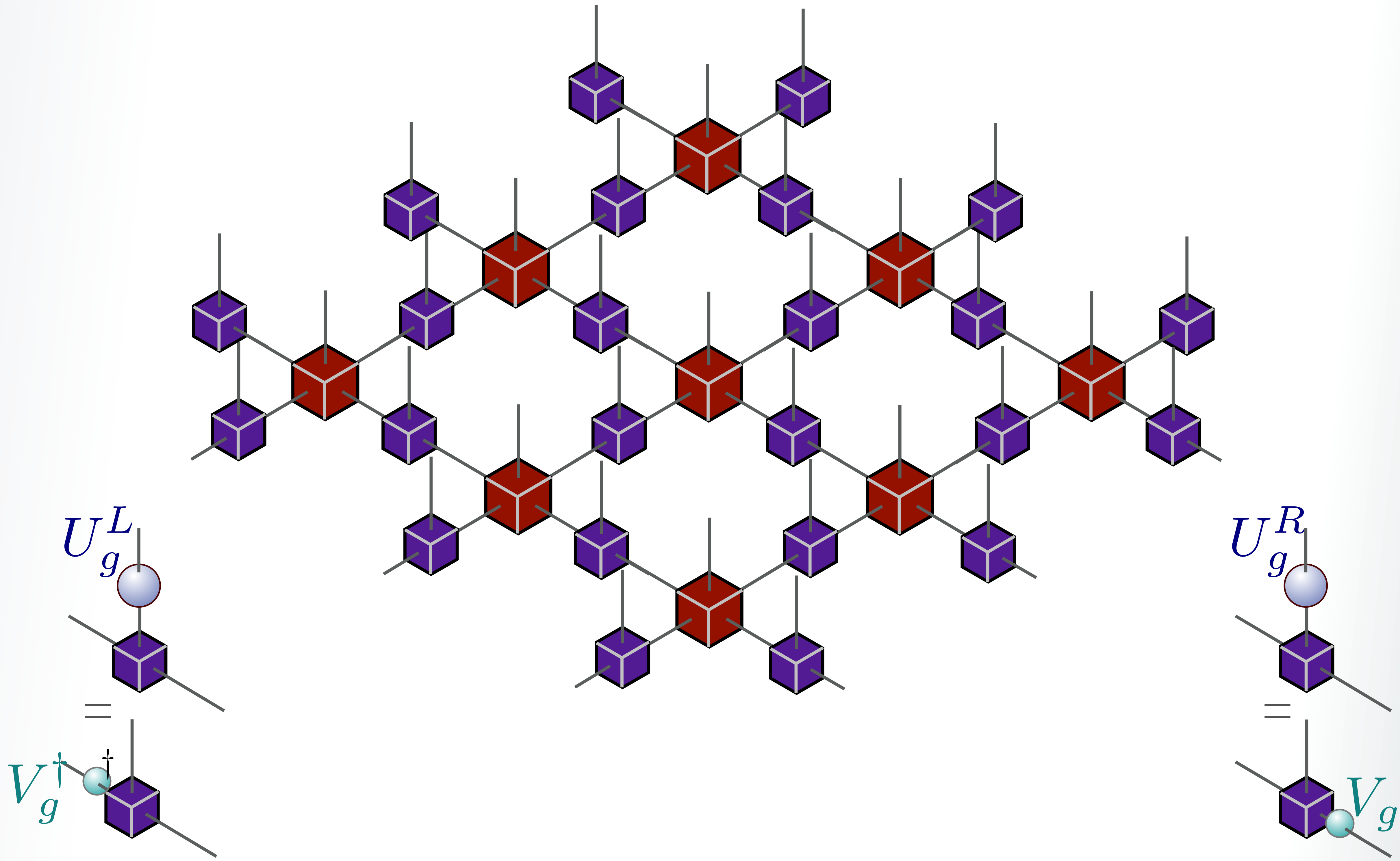
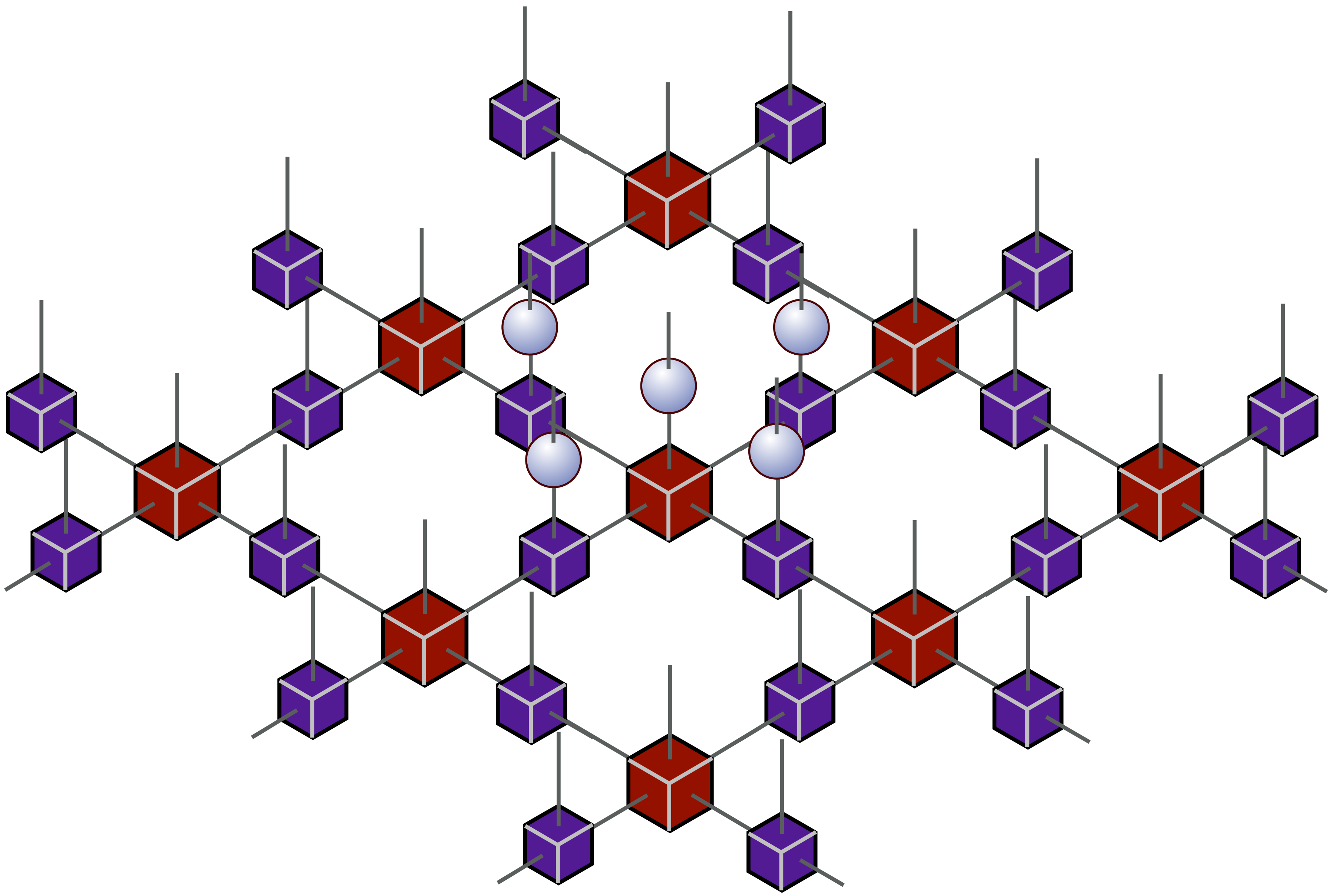
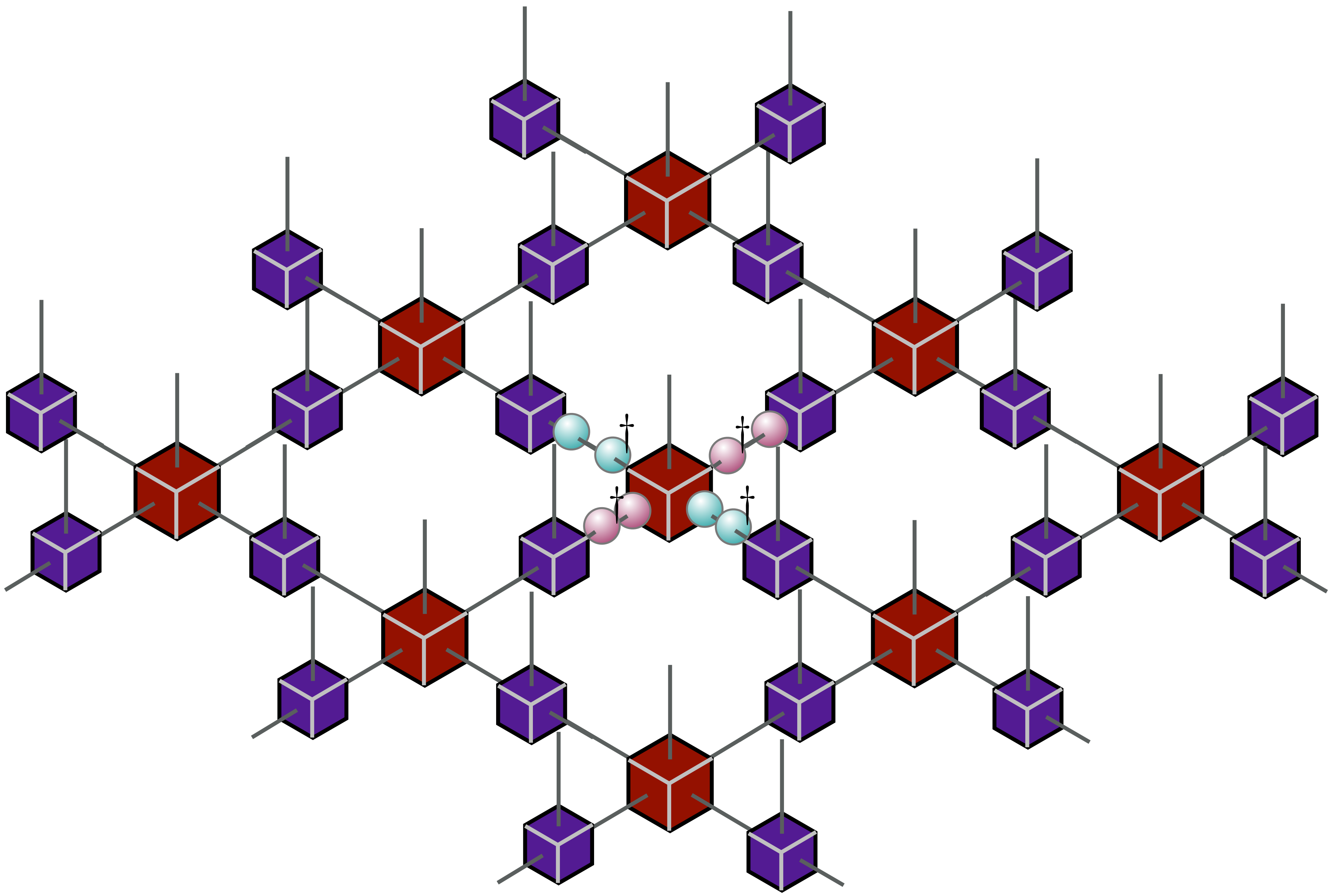


gauging PEPS



gauging PEPS

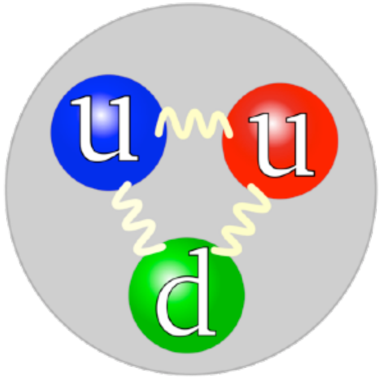


why LGT???

Motivation for LGT: QCD

Wilson, 1974

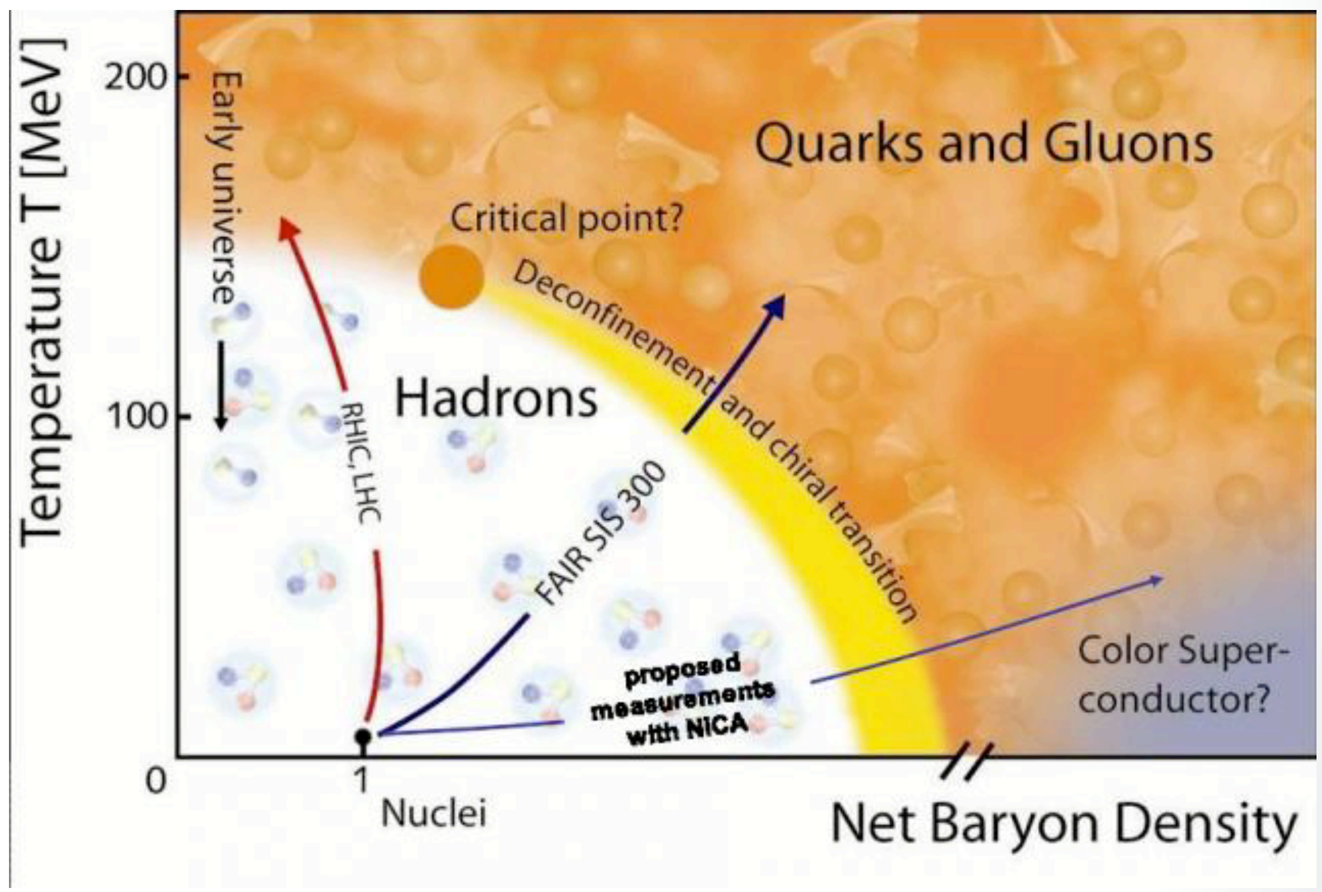
non perturbative at low energy



LQCD

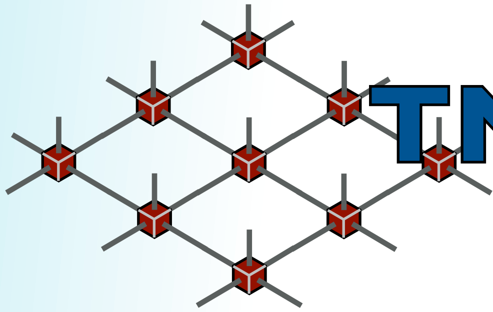
successful spectral calculations

limitations: time, finite density



why with TNS?

a natural connection



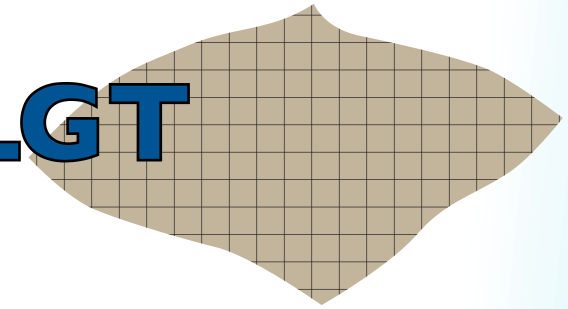
TNS

non-perturbative for Hamiltonian systems

extremely successful for 1D systems (MPS)

consistent developments for higher dimensions

ground states
low-lying excitations
thermal states
time evolution



LGT

non-perturbative way of solving QFT (QCD)

mostly path-integral formalism & MC

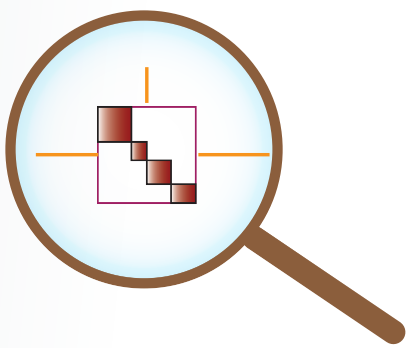
4D lattice

spectrum
finite T
big 3+1 dimensional
chemical potential
time evolution

how TNS FOR LGT???

using TNS for QMB

a formal approach



classifying tensors

constructing states

great descriptive power: phases,
topological chiral states, anyons...

Chen et al PRB 2011
Schuch et al PRB 2011
Wahl et al PRL 2013; Yang et al PRL 2015
Haegeman et al, Nat. Comm. 2015

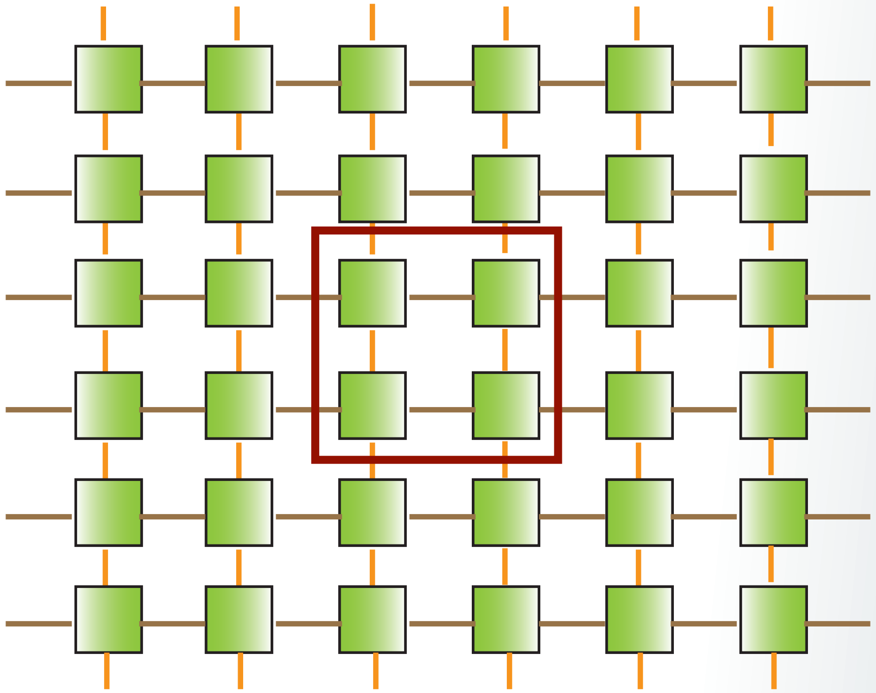
no sign problem

numerical algorithms

tensor networks describe
partition functions (observables)

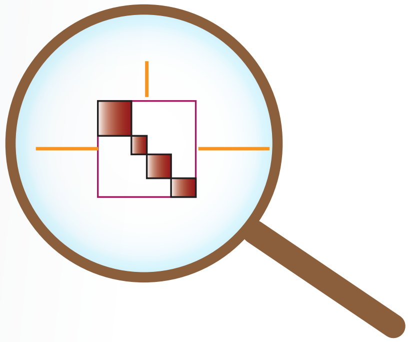
need to contract a TN
TRG approaches

Nishino, JPSJ 1995
Levin & Wen PRL 2008
Xie et al PRL2009; Zhao et al PRB 2010



using TNS for QMB

a formal approach



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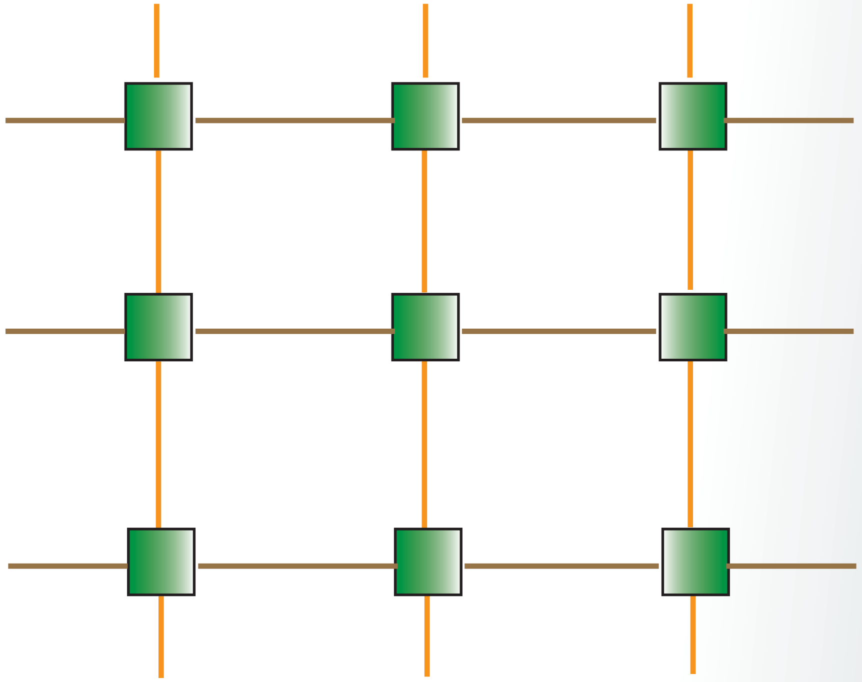

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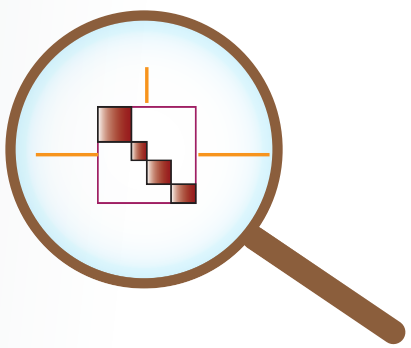

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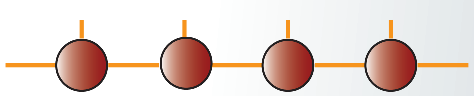
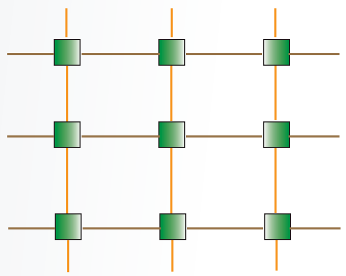


TNS as ansatz for the state



need to contract a TN
TRG approaches

Nishino, JPSJ 1995
Levin & Wen PRL 2008
Xie et al PRL2009; Zhao et al PRB 2010

efficient algorithms for GS, low
excited states, thermal, dynamics

White PRL 1992; Schollwöck RMP 2011
Vidal PRL 2003; Verstraete et al PRL 2004
Verstraete et al Adv Phys 2008; Orús Ann Phys 2014

using TNS for LGT

formal approach

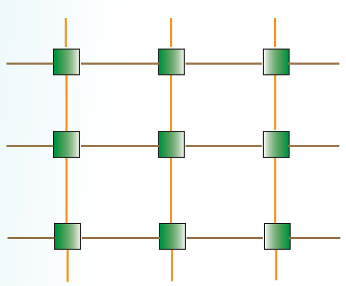
gauging the symmetry

explicitly invariant states

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

general prescriptions, U(1), SU(2)

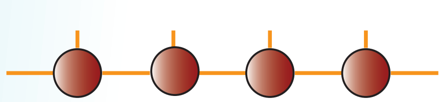
numerical simulations $\boxed{\text{no sign problem}}$



TN describe partition functions (observables)

TRG approaches to classical and quantum models

Liu et al PRD 2013; Shimizu, Kuramashi, PRD 2014; Kawauchi, Takeda 2015; review Meurice et al. RMP 2022



TN describe states

Photo by Maria Teneva on Unsplash

early works with DMRG/TNS

Byrnes PRD2002; Sugihara NPB2004
Tagliacozzo PRB2011; Sugihara JHEP2005
Meurice PRB2013

Schwinger model
U(1) in 1D

precise equilibrium
simulations,
feasibility of QSim

MCB et al JHEP11(2013)158;
Rico et al PRL 2014; Buyens et al. PRL 2014;
Kühn et al., PRA 90, 042305 (2014);
MCB et al PRD 2015, Buyens et al. PRD 2016;
Pichler et al. PRX 2016;
review Dalmonte, Montangero, Cont. Phys. 2016
MCB, Cichy, Cirac, Jansen, Kühn, arXiv:1810.12838

MCB, K. Cichy 1910.00257
QTFLAG Collab.1911.00003

3+1 dimensions

Magnifico et al. Nat. Comm. 2021

2+1 dimensions

Felser et al. PRX 2020
Robaina et al. PRL 2021
Emonts et al. PRD 2020

Non-Abelian in 1D

string breaking dynamics

S. Kühn et al., JHEP 07 (2015) 130;
Silvi et al., Quantum 2017
S. Kühn et al. PRX 2017

SU(3)QLM

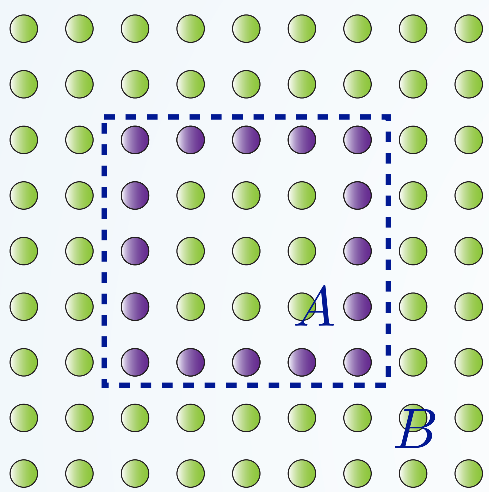
Silvi et al, PRD 2019

finite density

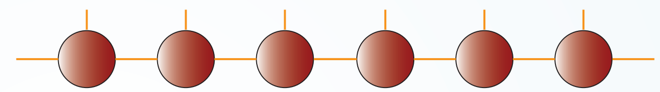
S. Kuehn et al, PRL118 (2017) 071601

TNS = entanglement based ansatz

area law

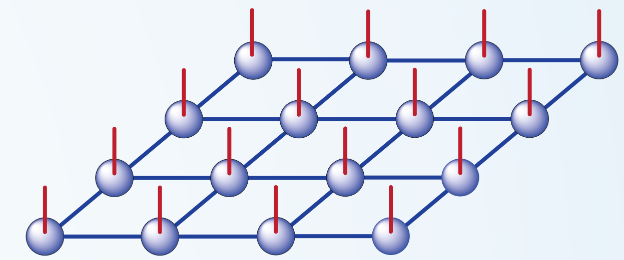


MPS



Schollwöck Ann.Phys.2011

PEPS



Verstraete et al. Adv. Phys. 2008

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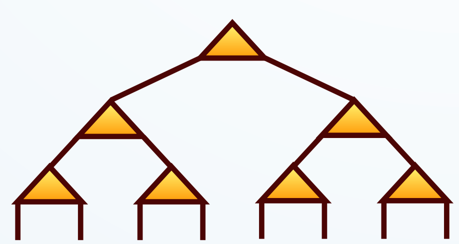
other TNS

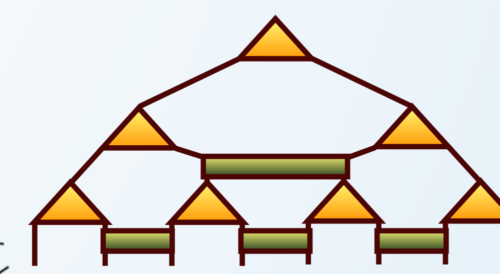
TTN

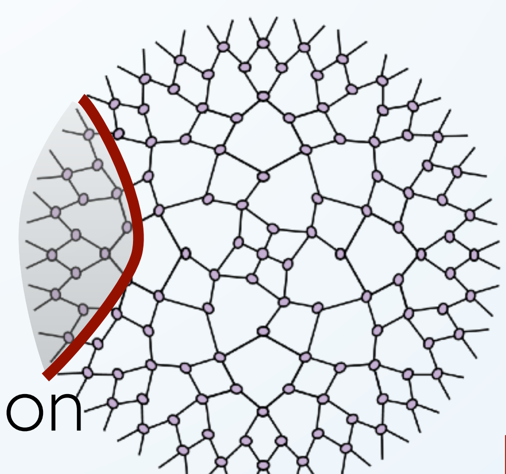


Shi et al PRA 2006

Vidal PRL 2007

MERA





suggested connection to AdS/CFT

Swingle PRD 2012
Molina JHEP 2013
Nozaki et al JHEP 2012
Bao et al PRD 2015

in principle, they can all be used for LGT simulations

general strategy

Hamiltonian formulation

acting on a Hilbert space

⟶ choose proper basis

Finite dimensional degrees of freedom

fermions

⟶ ✓ no sign problem

gauge bosons require attention

⟶ truncating, integrating out (also QLinks)

★ Common ingredients for quantum simulation

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

early works with DMRG/TNS

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Schwinger model U(1) in 1D

precise equilibrium simulations, feasibility of QSim

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Falser et al. arXiv:1911.09693
Robaina et al. arXiv:2007.11630
Emonts et al. PRD102, 074501 (2020)

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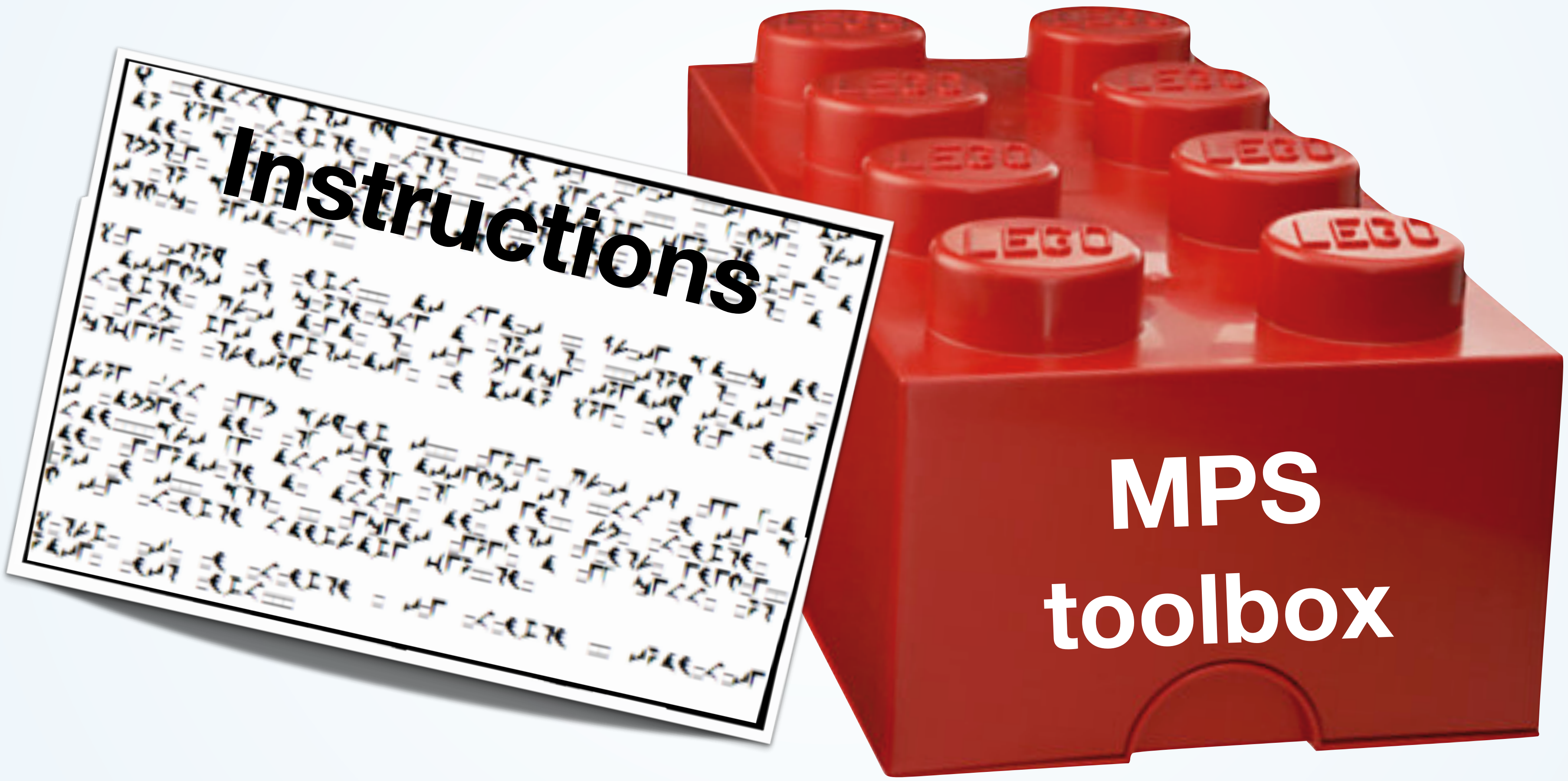


start with the simplest case: 1D
(1+1) LGT

recall...

MPS properties

MPS

good approximation of ground states

Verstraete, Cirac, PRB 2006

Hastings, J. Stat. Phys 2007

gapped finite range Hamiltonian ⇒
area law (ground state)

Cramer, Eisert, Plenio, RMP 2009

efficient calculation of expectation values

exponentially decaying correlations

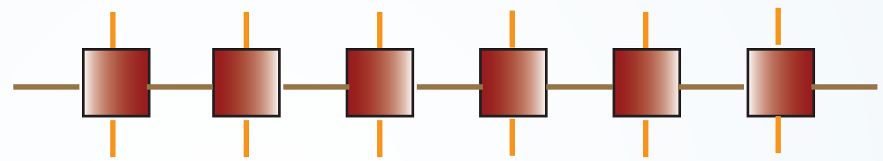
can be prepared efficiently

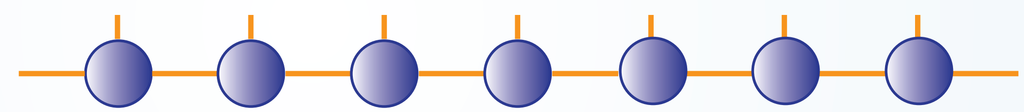
recall...

basic algorithms

variational minimization of energy

local
Hamiltonian

$H =$ 

$|E_0\rangle \simeq$ 

ground state
excitations

apply local operators → simulate time evolution

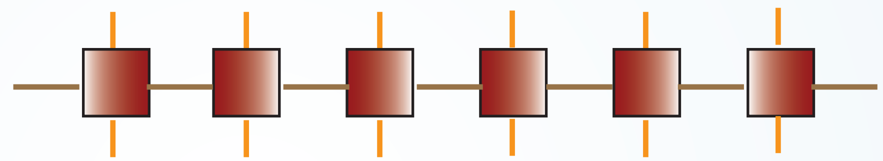
imaginary time → ground state
thermal state

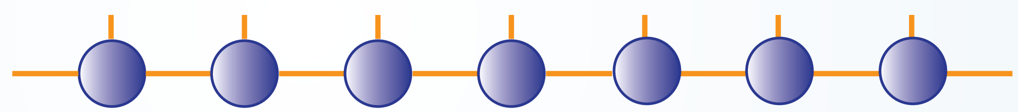
recall...

basic algorithms

variational minimization of energy

local Hamiltonian

$H =$ 

$|E_0\rangle \simeq$ 

ground state
excitations

apply local operators → simulate time evolution

imaginary time → ground state
thermal state

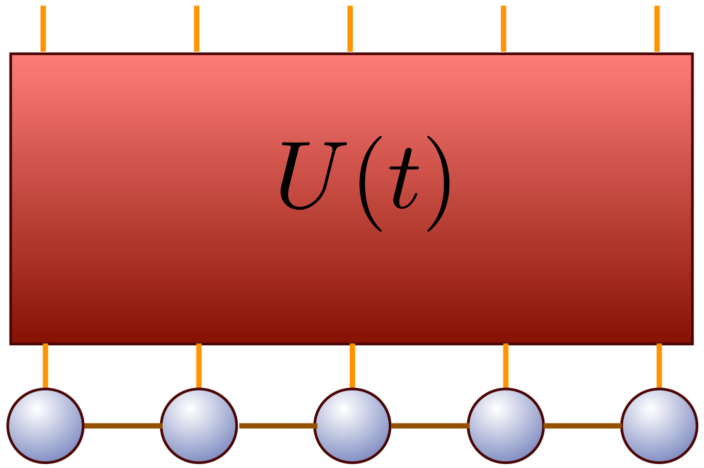
alternatively: TDVP

regarding dynamics

recall...

basic time evolution algorithms

initial MPS



TEBD, t-DMRG

Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

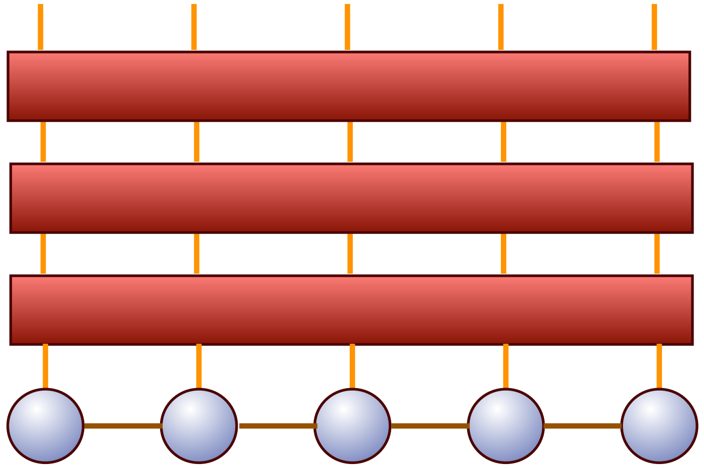
recall...

basic time evolution algorithms

initial MPS

discrete time

$$U(t) \to [U(\delta)]^M$$



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recall...

basic time evolution algorithms

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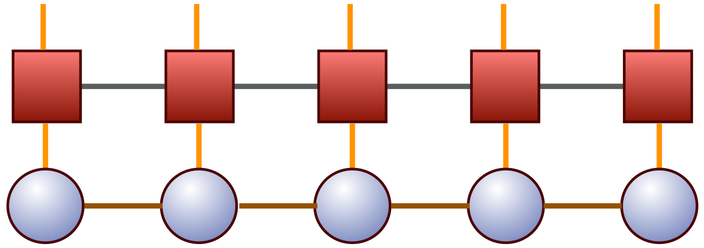
$$U(t) \to [U(\delta)]^M$$

Suzuki-Trotter expansion

$$U(\delta) \approx e^{-iH_e\delta}e^{-iH_o\delta}$$



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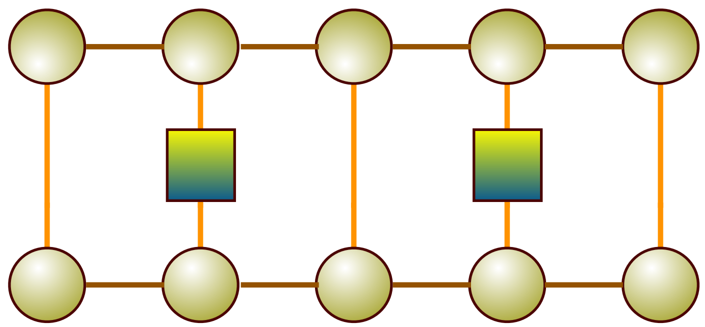
Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

recall...

basic time evolution algorithms

time evolved state approximated by MPS



TEBD, t-DMRG

Vidal, PRL 2003, 2004

Verstraete, García-Ripoll, Cirac, PRL 2004

alternative: TDVP Haegeman et al, PRL 2011

initial MPS

discrete time

$$U(t) \rightarrow [U(\delta)]^M$$

Suzuki-Trotter expansion

$$U(\delta) \approx e^{-iH_e\delta} e^{-iH_o\delta}$$

truncate bond dimension

iterate

compute observables

recall...

mixed states

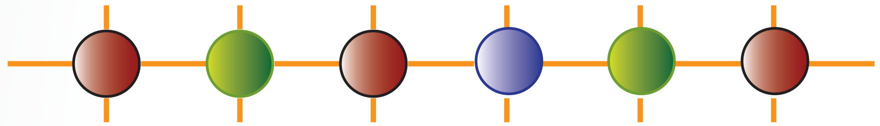
MPO = Matrix Product Operator

Similar problems can be attacked

equilibrium ➞ thermal states

imaginary time evolution

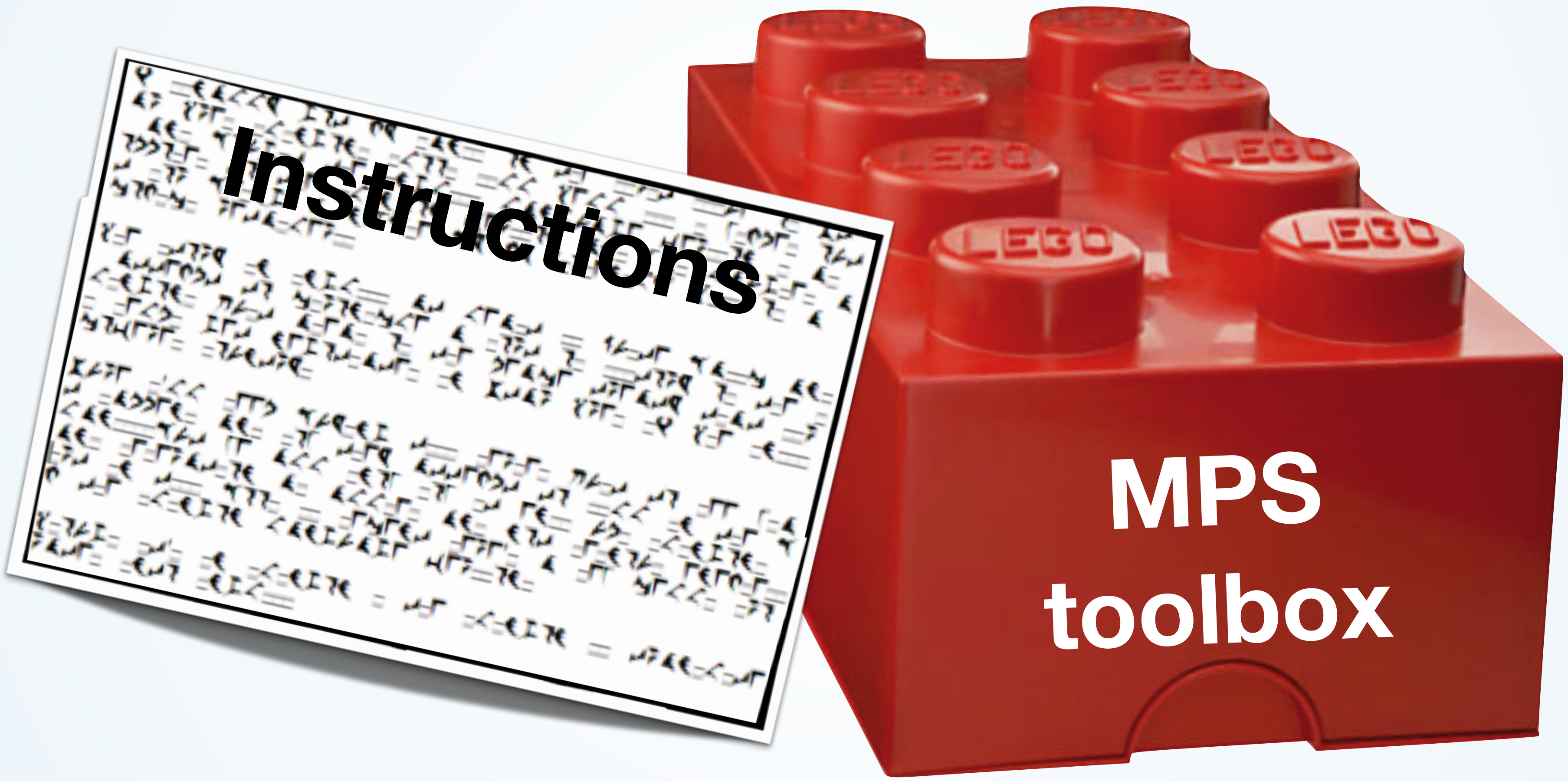
time-dependent ➞ real time evolution



unitary $\rho(t) = U(t)\rho(0)U(t)^{\dagger}$

non-unitary $$\frac{d\rho(t)}{dt} = \mathcal{L}(\rho)$$

Verstraete et al., PRL 2004
Prosen, Znidaric PRL 2008
Cai, Barthel, PRL 2013,...


Instructions
MPS
toolbox

In this talk…

Why using TNS/MPS for LGT? ✓

testbench:
Schwinger model

- spectral calculations
- finite temperature
- real-time
- chemical potential

higher dimensional problems

Schwinger model as laboratory

Schwinger model

Schwinger '62

Simplest gauge theory with matter

QED in 1+1 dimensions

electrons & photons

Shows some of the features of *full* QCD

confinement → bound states (massive bosons)

fermion condensate

A testbench for lattice techniques

Schwinger model

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \tfrac{1}{4} F_{\mu\nu}F^{\mu\nu}$$

in 1+1 D single adimensional parameter ***m/g***

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$$

U(1) gauge invariance

$$\Psi(x) \to e^{-ig\phi(x)}\Psi(x)$$

$$A_\mu(x) \to A_\mu(x) - \partial_\mu \phi(x)$$

equations of motion

$$\partial_\alpha \frac{\partial \mathcal{L}}{\partial \Phi_{,\alpha}} - \frac{\partial \mathcal{L}}{\partial \Phi} = 0 \quad \text{for} \quad \Phi = A_\mu,\ \Psi$$

$$(i\gamma^\mu \partial_\mu - g\gamma^\mu A_\mu - m)\,\Psi = 0$$

$$\partial_\mu F^{\mu\nu} = g\bar{\Psi}\gamma^\nu \Psi$$

Schwinger model

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Hamiltonian formulation

$$A^0 = 0$$

$$E = -\dot{A}^1$$

$$H = \int dx\, \big[\underbrace{-i\bar{\Psi}\gamma^1\partial_1\Psi}_{\text{fermion kinetic term}} + \underbrace{g\bar{\Psi}\gamma^1 A_1\Psi}_{\text{fermion-photon coupling}} + \underbrace{m\bar{\Psi}\Psi}_{\text{fermion mass}} + \underbrace{\tfrac{1}{2}E^2}_{\text{electrostatic energy}}\big]$$

plus a constraint: $\partial_1 E = g\bar{\Psi}\gamma^0\Psi$ Gauss' law

quantization

$$\{\Psi_i(x), \Psi_j^\dagger(y)\} = \delta_{ij}\delta(x-y)$$
$$\{\Psi_i(x), \Psi_j(y)\} = 0$$
$$[A_1(x), E(y)] = i\delta(x-y)$$

Schwinger model

$$\mathcal{L} = \bar{\Psi}(i\gamma_\mu \partial^\mu - g\gamma_\mu A^\mu - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Hamiltonian formulation

$$A^0 = 0$$

$$E = -\dot{A}^1$$

$$H = \int dx \left[-i\bar{\Psi}\gamma^1\partial_1\Psi + g\bar{\Psi}\gamma^1 A_1\Psi + m\bar{\Psi}\Psi + \frac{1}{2}E^2\right]$$

fermion kinetic term

fermion-photon coupling

fermion mass

electrostatic energy

plus a constraint: $\partial_1 E = g\bar{\Psi}\gamma^0\Psi$ Gauss' law

quantization

$$\{\Psi_i(x), \Psi_j^\dagger(y)\} = \delta_{ij}\delta(x-y)$$
$$\{\Psi_i(x), \Psi_j(y)\} = 0$$
$$[A_1(x), E(y)] = i\delta(x-y)$$

discretize

Schwinger model

on the lattice

discrete Hamiltonian (staggered) formulation

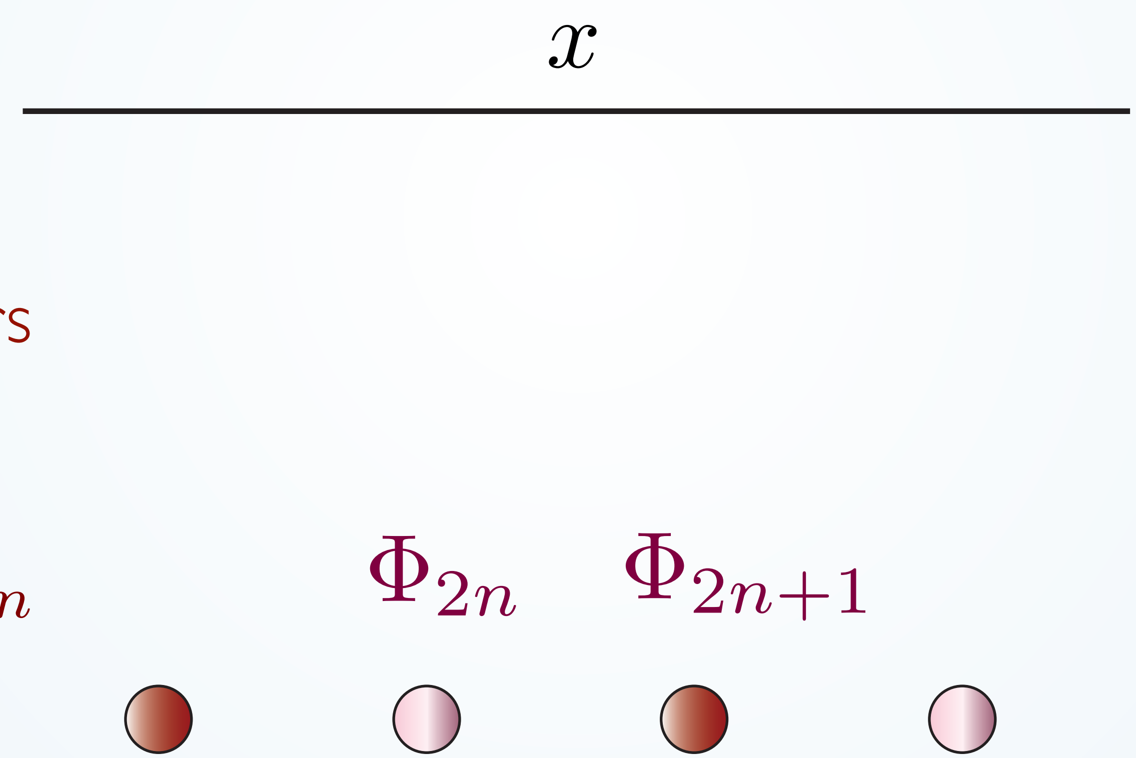
$$x$$

$$\begin{pmatrix} \Psi^{(1)}(x) \\ \Psi^{(2)}(x) \end{pmatrix}$$

Schwinger model

on the lattice

discrete Hamiltonian (staggered) formulation



fermionic operators

$\{\Phi_m, \Phi_n\} = 0$

$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$

Schwinger model

on the lattice

discrete Hamiltonian (staggered) formulation

$$x$$

$$U(x, x+\epsilon) = e^{ig\epsilon A_1(x)}$$

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

$$\{\Phi_m, \Phi_n^\dagger\} = \delta_{mn}$$

Φ_{2n} Φ_{2n+1}

Schwinger model

on the lattice

discrete Hamiltonian (staggered) formulation

fermionic operators

$$\{\Phi_m, \Phi_n\} = 0$$

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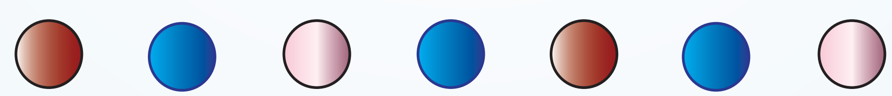
Φ_{2n} Φ_{2n+1}



$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \to -A^1(x)$$

$$gL_n \to E(x)$$

$$[\theta_n, L_m] = ig\delta_{nm}$$

Schwinger model

discretized

on the lattice

$$H = -\frac{i}{2a}\sum_n \left(\phi_n^\dagger e^{i\theta_n}\phi_{n+1} - \mathrm{h.c.}\right) + m\sum_n (-1)^n \phi_n^\dagger \phi_n + \frac{ag^2}{2}\sum_n L_n^2$$

plus constraint: Gauss' Law

spinless fermions

$$L_n - L_{n-1} = \phi_n^\dagger \phi_n - \frac{1}{2}\left[1 - (-1)^n\right]$$

1D spins $\Longleftrightarrow$ fermions: Jordan-Wigner

$$\phi_n = \prod_{k<n} (i\sigma_k^z)\sigma_n^-$$

notice: not strictly necessary for TNS

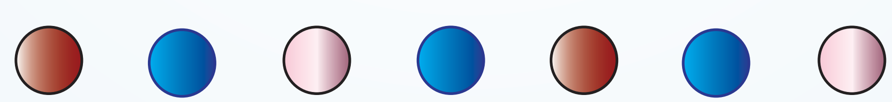
Schwinger model

on the lattice

Jordan-Wigner → spin model

$$H = \frac{1}{2a}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$

$$+\frac{m}{2}\sum_n \left(1 + (-1)^n \sigma_n^3\right) + \frac{ag^2}{2}\sum_n L_n^2$$



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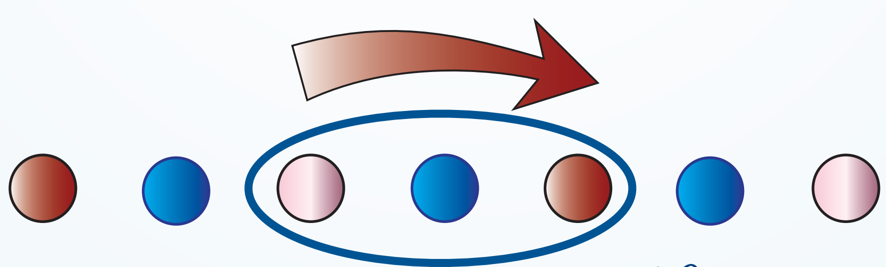
Schwinger model

on the lattice

Jordan-Wigner → spin model

$$H = \frac{1}{2a}\sum_n \left(\sigma_n^+ e^{i\theta_n}\sigma_{n-1}^- + \sigma_{n+1}^+ e^{-i\theta_n}\sigma_n^-\right)$$
$$+\frac{m}{2}\sum_n \left(1+(-1)^n\sigma_n^3\right) + \frac{ag^2}{2}\sum_n L_n^2$$



hopping

$$U_{n,n+1} = e^{i\theta_n}$$

$$\frac{1}{ga}\theta_n \rightarrow -A^1(x)$$

$$gL_n \rightarrow E(x)$$

Schwinger model

continuum QED

1+1 space-time

on the lattice

discrete space-time

continuum limit

fermions

photons

lattice spacing $ag \to 0$

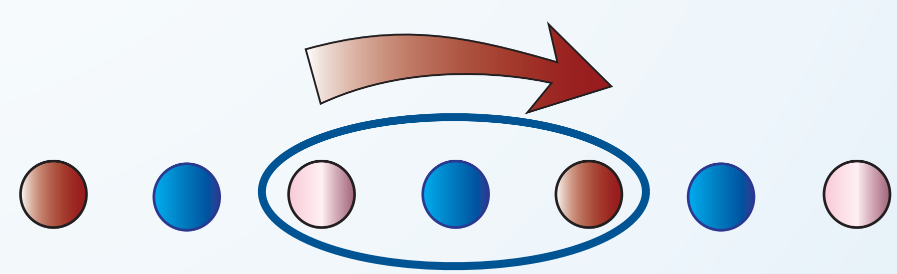
spin Hamiltonian

fermion mass m/g

adimensional

Hamiltonian

+ Gauss law



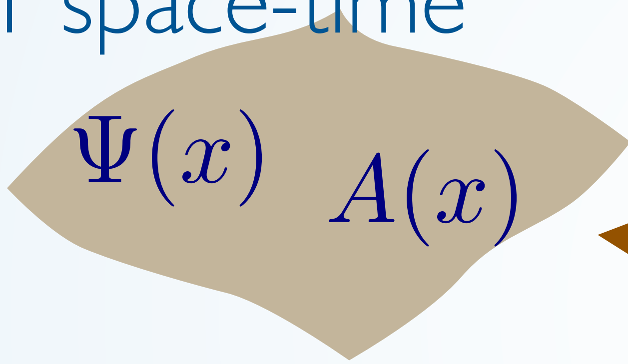
hopping $\sigma_n^+ e^{i\theta_n} \sigma_{n-1}^-$

Schwinger model

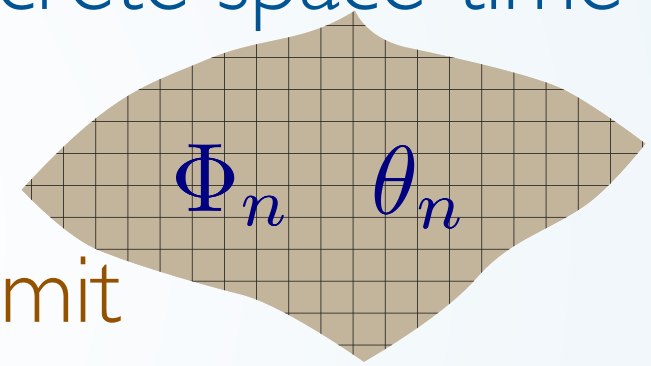
continuum QED

on the lattice

$1+1$ space-time



discrete space-time



continuum limit

fermions

photons

fermion mass m/g

adimensional

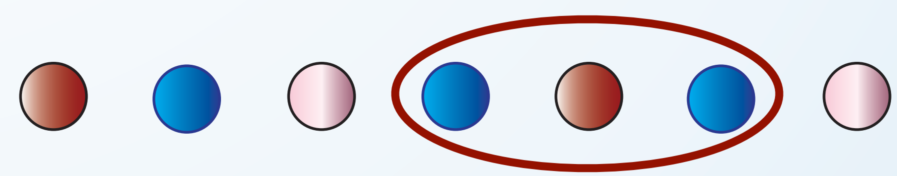
Hamiltonian

+ Gauss law

lattice spacing $ag \to 0$

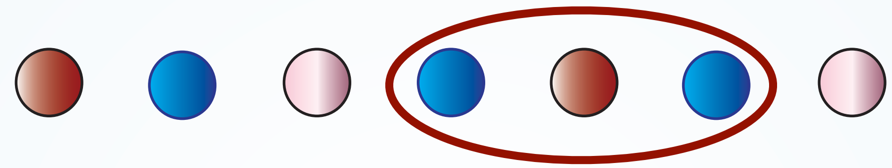
spin Hamiltonian

+ Gauss law



for a TNS we need a basis

Schwinger model



basis $|\ldots s_e \;\ell\; s_o \;\ell\; s_e \;\ell\; s_o \ldots\rangle$

but Gauss' law fixes *photon* content

$$L_n - L_{n-1} = \frac{1}{2}[\sigma_n^3 + (-1)^n]$$

Schwinger model

basis $|\ldots s_e \quad s_o \quad s_e \quad s_o \ldots\rangle$

but Gauss' law fixes *photon* content

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n} \sigma_n^3 + \ldots$$

$\Rightarrow$ eliminate gauge dof

introducing long range interactions

Schwinger model

MPS representation for OPEN BOUNDARIES

$$|\ell_0 \ldots s_e \; s_o \; s_e \; s_o \ldots\rangle$$

non-local terms

$$L_n = \ell_0 + \frac{1}{2}\sum_{k\leq n}\sigma_n^3$$

Long range interactions

$$\sum_n \sum_{k<n} (N-n)\sigma_k^3 \sigma_n^3$$

can be written as a MPO of D=5

both possibilities

Schwinger model

on the lattice

basis $|\ldots s_e\, \ell\, s_o\, \ell\, s_e\, \ell\, s_o \ldots\rangle$ all terms are local

infinite dimensional: truncation

Gauss' law needs to be imposed

works by Buyens et al., PRL 2014; arXiv:1509.00246
Rico et al., PRL 2014; NJP 2014

or integrating out the gauge dof

basis $|\ell_0 \ldots s_e\, s_o\, s_e\, s_o \ldots\rangle$ non-local terms

exact physical subspace

✗ does not generalize to bigger dimensions

finite density

spectrum

entropy

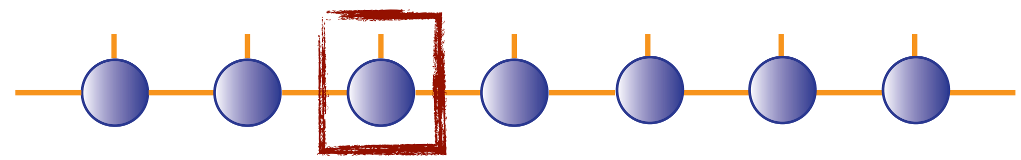
1+1D RESULTS

thermal equilibrium

time evolution

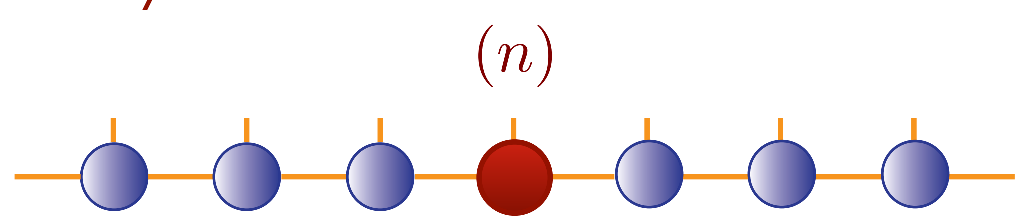
computing the low energy levels

Efficient algorithms to find ground state and excitations

$|E_0\rangle \simeq$ 

variational, imaginary time…

work directly in the TD limit

$\sum_n e^{ikn}$ 

excitation ansatz

Different strategies possible for LGT

truncate the gauge dof, integrate out

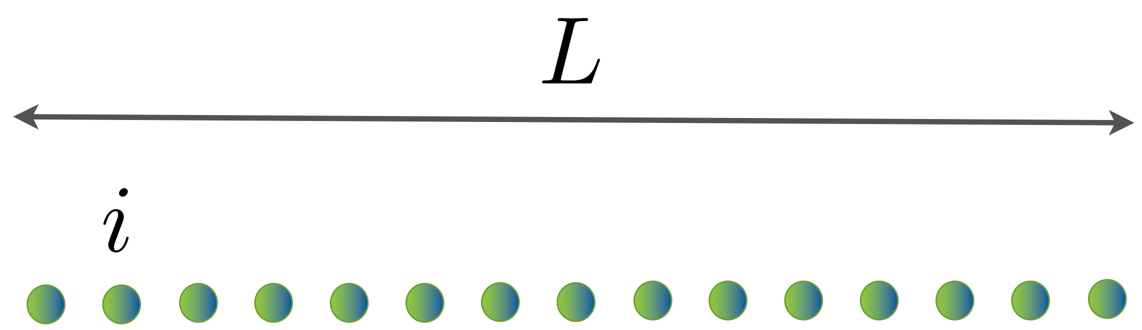
explicit symmetries in tensors

How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size (N)

alternative: infinite size



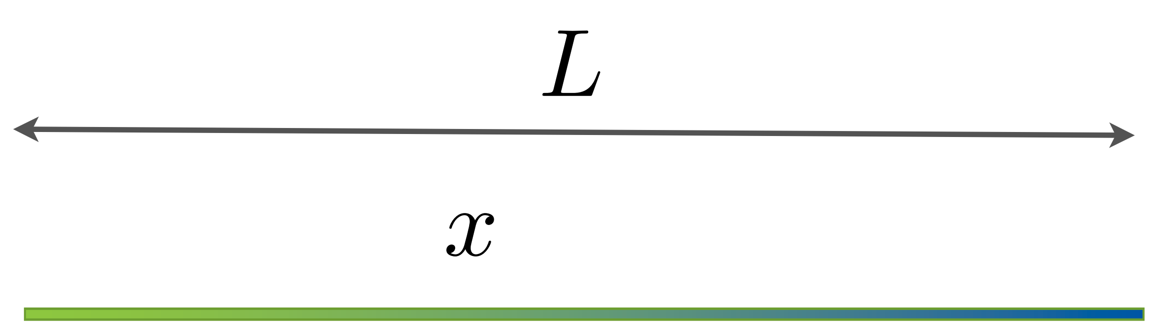
How to do the continuum calculation?

discrete system → finite lattice spacing

reduce lattice spacing → need larger size (N)

alternative: infinite size

extrapolate to vanishing spacing → possible divergences



computing continuum quantities

Scan parameters

m/g

$x \to \infty$

x

finite-size

N

convergence

D

mass gaps and GS energy density in the continuum $x \to \infty$

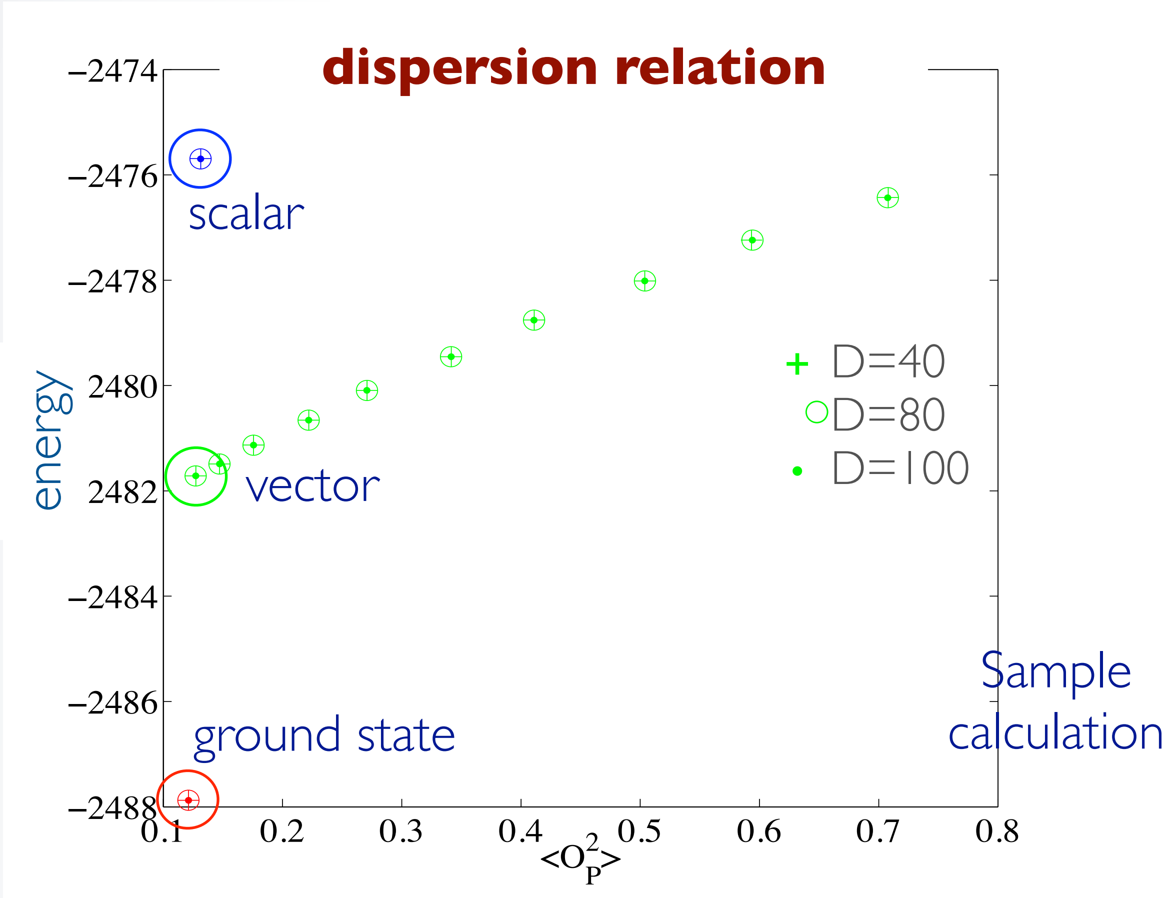
$x \lesssim 600$

$N \propto \sqrt{x}$ (up to ~850)

alternative: infinite size

$D \approx O(100)$

MCB, Cichy, Cirac, Jansen JHEP11(2013)158
Buyens et al. PRL113 (2014) 091601



after the continuum limit

m/g

$x \to \infty$

x

finite-size

infinite chain

N

convergence

D

better precision than any earlier numerics

m/g	DMRG	MPS with OBC [1]	gauge inv. uMPS [2]	SCE	MPS with OBC [1]	gauge inv. uMPS [2]
0	0.5641859	0.56414(26)	0.56418(2)	1,128379	1.1283(10)	-
125	0.53950(7)	0.53946(20)	0.539491(8)	1.22(2)	1.2155(28)	1.222(4)
0.25	0.51918(5)	0.51915(14)	0.51917(2)	1.24(3)	1.2239(22)	1.2282(4)
0.5	0.48747(2)	0.48748(6)	0.487473(7)	1.20(3)	1.1998(17)	1.2004(1)

[1] MCB, Cichy, Cirac, Jansen JHEP11(2013)158
[2] Buyens et al. PRL113 (2014) 091601

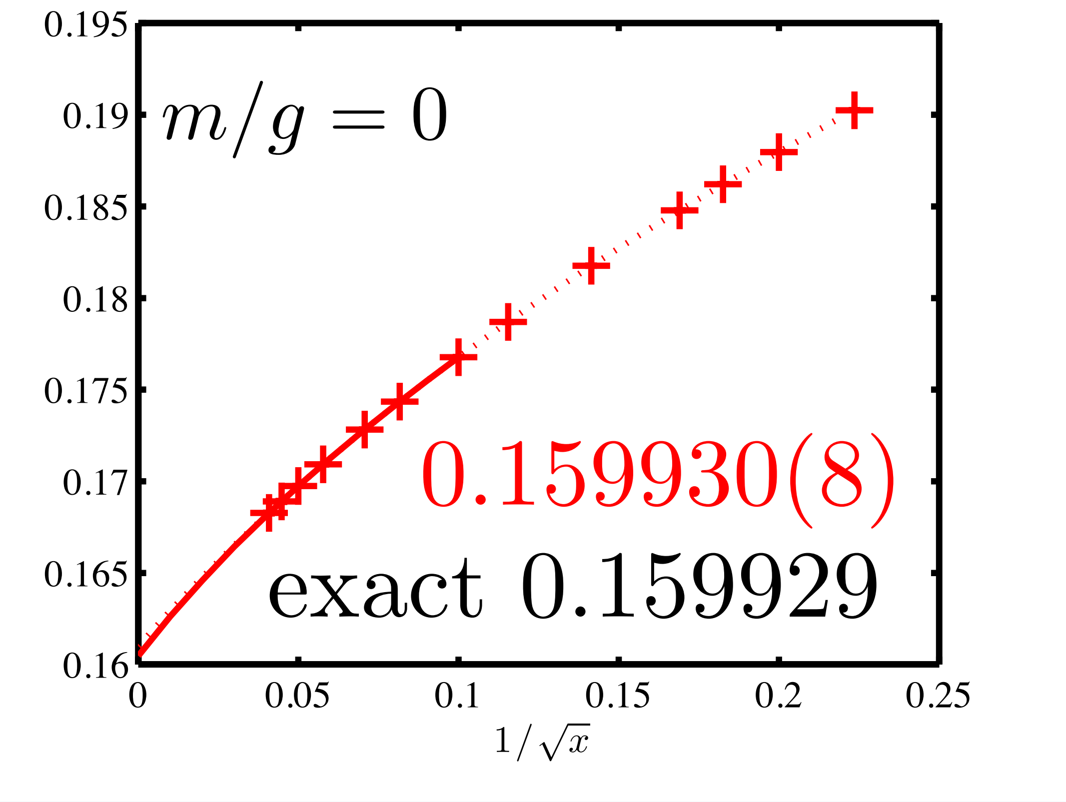
MPS give us access to observables:
expectation values

MPS states → observables

chiral condensate in the GS: order parameter for chiral symmetry breaking (m/g=0)

$$\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$$

in the spin language $$\frac{\sqrt{x}}{L}\sum_n (-1)^n \frac{1+\sigma_n^3}{2}$$



MPS states → observables

chiral condensate in the GS: order parameter for chiral symmetry breaking (m/g=0)

$$\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$$

in the spin language $\frac{\sqrt{x}}{L}\sum_n (-1)^n \frac{1+\sigma_n^3}{2}$

no exact value known for $m/g \neq 0$

only estimations

de Forcrand et al. 97
Hosotani 97

logarithmic divergence → same as in non-interacting case

MPS states → observables

chiral conde… …er $\frac{\Sigma}{g} = \frac{\langle\bar{\Psi}\Psi\rangle}{g}$

for chiral sy…

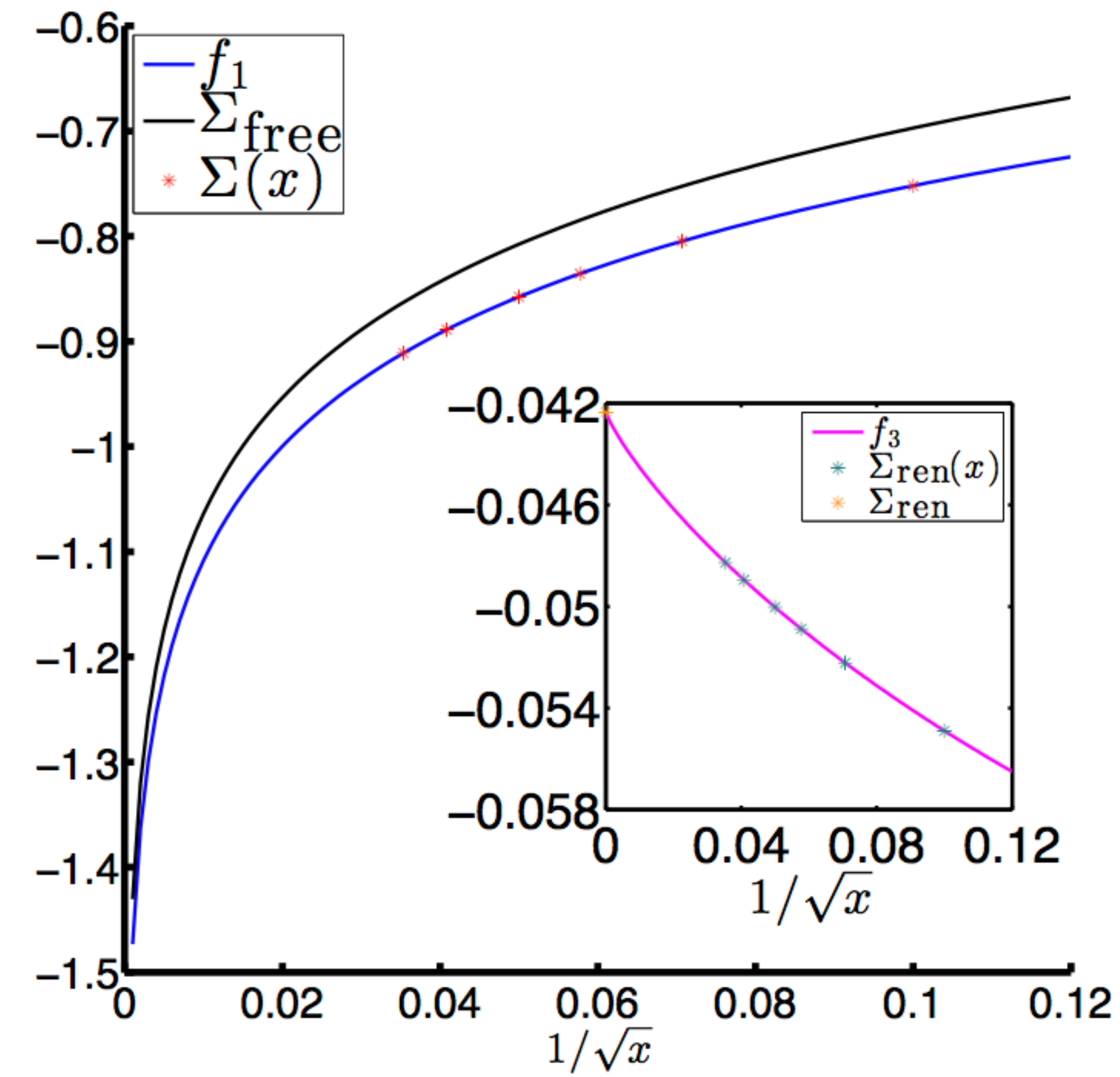
in the spin l…

no exact va…

…Forcrand et al. 97

…sotani 97

logarithmic … …racting case



from Buyens et al. arXiv:1411.0020

MPS states → observables

chiral conde … er $\frac{\Sigma}{g} = \frac{\langle \bar{\Psi}\Psi \rangle}{g}$

for chiral sy

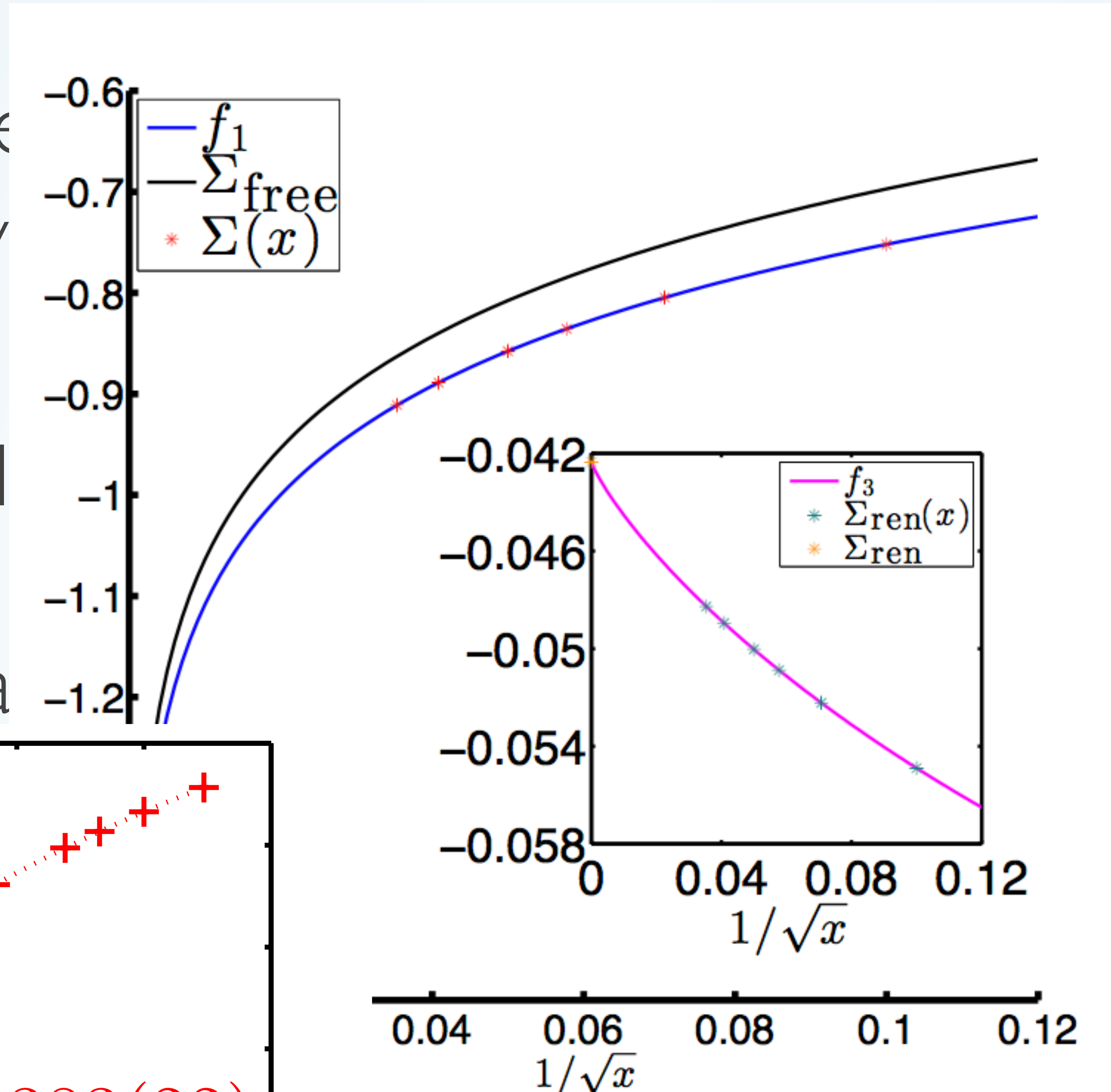
in the spin l

no exact va

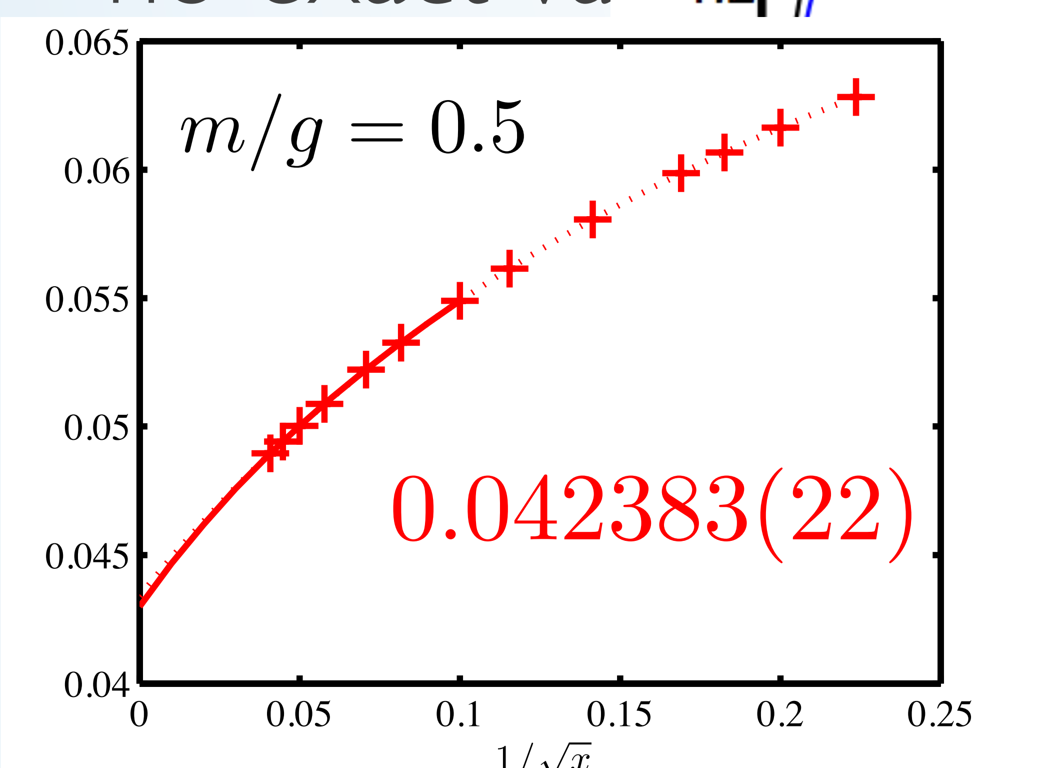
…orcrand et al. 97

…sotani 97

…racting case



from Buyens et al. arXiv:1411.0020



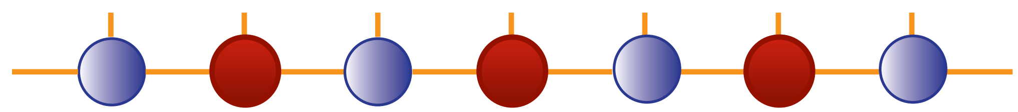
arXiv:1310.4118

alternative: substract exact non-interacting condensate

uMPS: how important is the truncation of gauge dof?

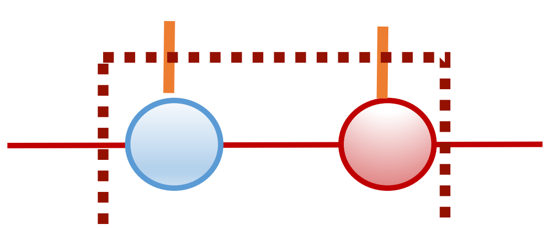
uniform MPS (uMPS)

cannot integrate out gauge $|\ldots s_e \, \ell \, s_o \, \ell \, s_e \, \ell \, s_o \ldots\rangle$



but physical states have to satisfy Gauss' law

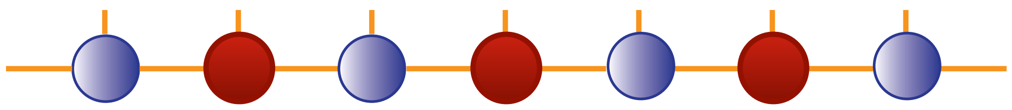
$\Rightarrow$ symmetries



$A^{i\,\ell}_{\alpha\beta}$

uniform MPS (uMPS)

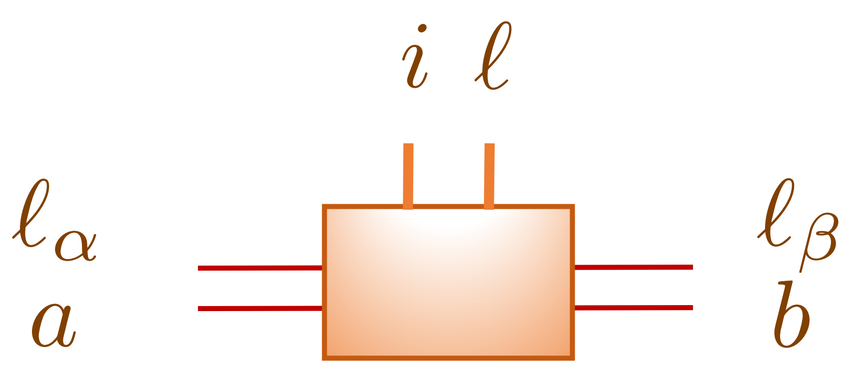
cannot integrate out gauge $|\ldots s_e\,\ell\,s_o\,\ell\,s_e\,\ell\,s_o\ldots\rangle$



truncate

but physical states have to satisfy Gauss' law

$\Rightarrow$ symmetries

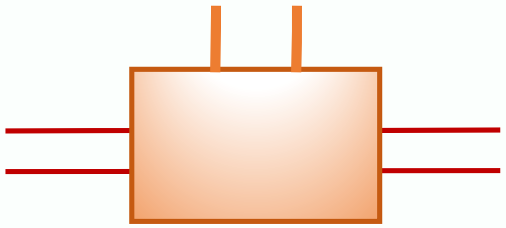


$A^{i\,\ell}_{\alpha\beta}$

$$\alpha = (\ell_\alpha\, a)$$
$$\beta = (\ell_\beta\, b)$$

$$\boxed{\ell = \ell_\beta \qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}}$$

symmetric MPS has block structure



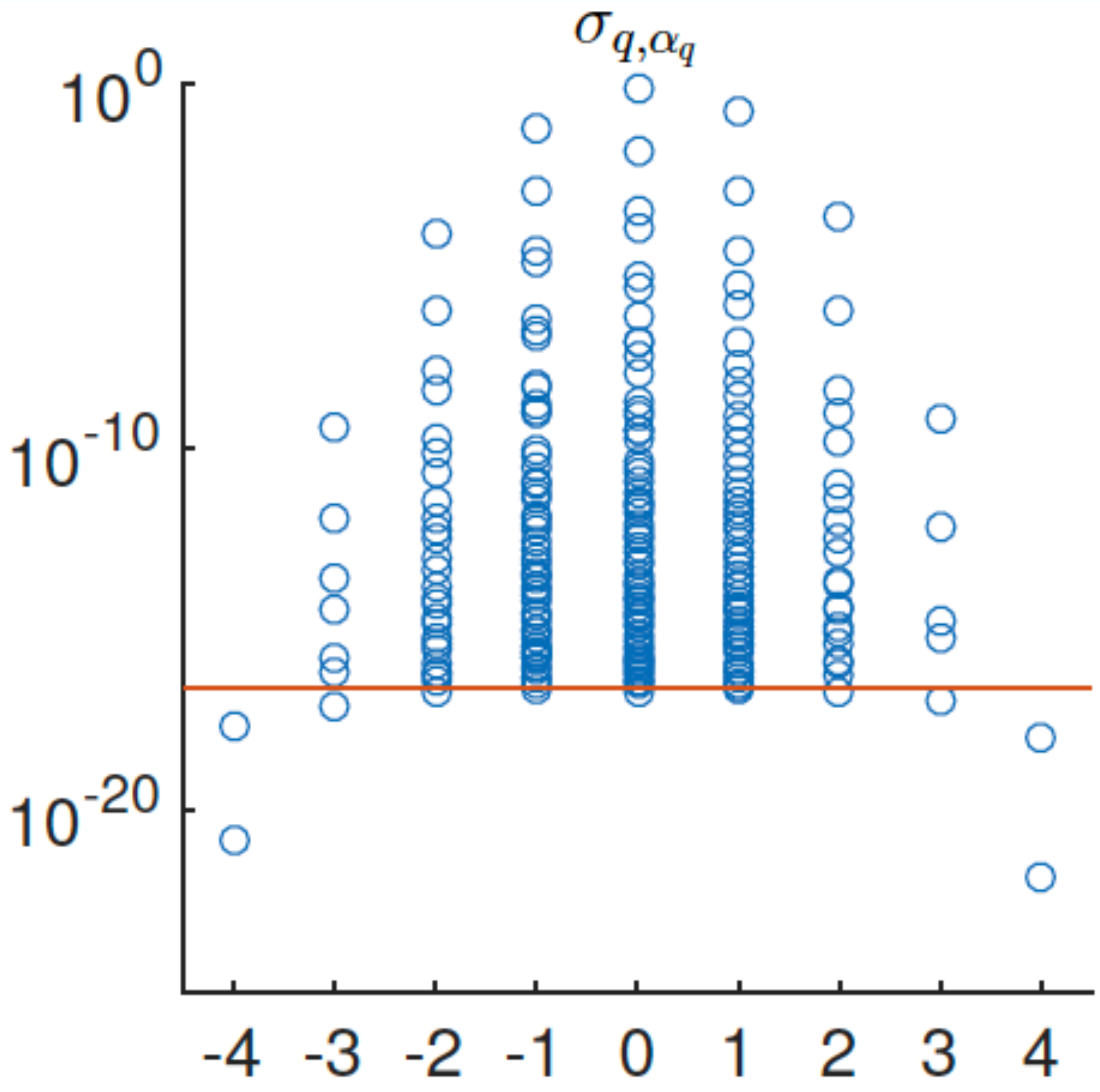
$$\ell = \ell_\beta \qquad\qquad \ell_\beta = \ell_\alpha + \frac{(-1)^i + (-1)^n}{2}$$

$$A_1^{1,p} = \begin{array}{cc} q\backslash r & \begin{array}{ccccc} \cdots & p-1 & p & p+1 & \cdots \end{array} \\ \begin{array}{c} \vdots \\ p \\ p+1 \\ p+2 \\ \vdots \end{array} & \left(\begin{array}{c|c|c|c|c} \ddots & \vdots & \vdots & \vdots & \iddots \\ \hline \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & a_1^{q,1} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline \cdots & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \begin{matrix}0&\cdots&0\\ \vdots&\ddots&\vdots\\ 0&\cdots&0\end{matrix} & \cdots \\ \hline \iddots & \vdots & \vdots & \vdots & \ddots \end{array}\right) \end{array}$$

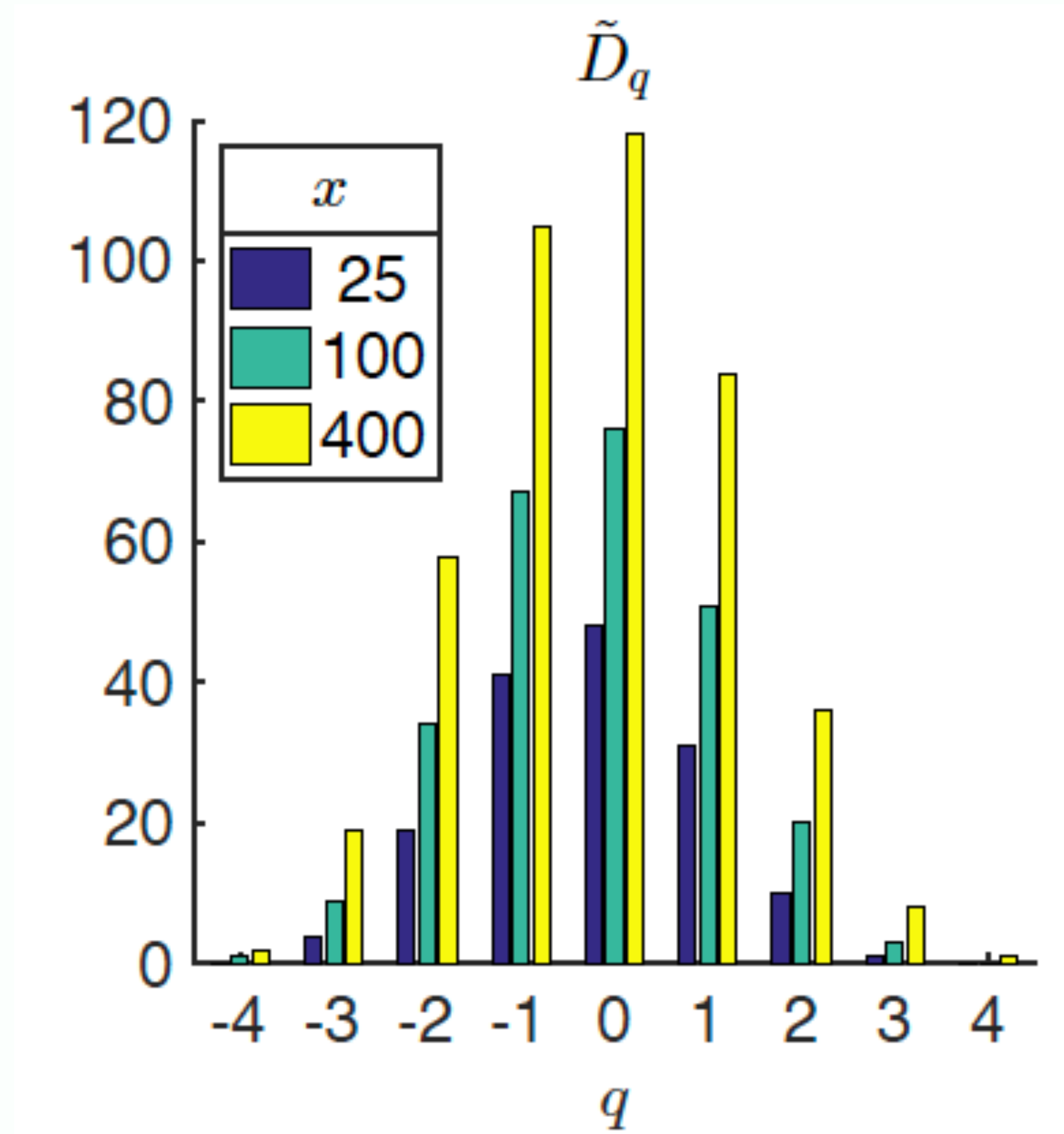
a maximum bond dimension per block

from B. Buyens

decay of Schmidt values



required D towards cont



Buyens et al PRD 95 (2017) 094509

D increases as $x^{1/2}$

finite density

spectrum

entropy

1+1D RESULTS

thermal equilibrium

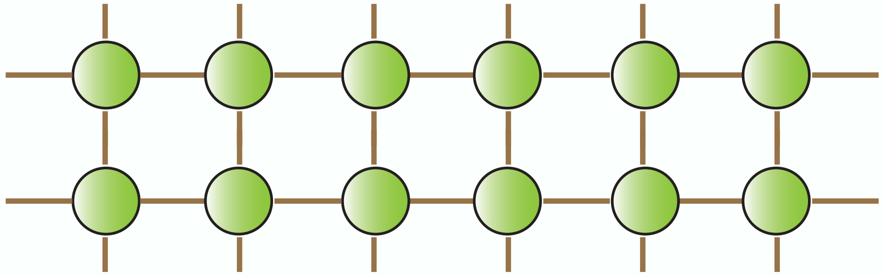
time evolution

thermal properties with MPO

$$\rho_{th}(\beta) = e^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H}$$

$$\langle O\rangle_{th} = \mathrm{tr}(Oe^{-\frac{\beta}{2}H}\mathbf{1}e^{-\frac{\beta}{2}H})$$

combining real time evolution can compute thermal response functions



$\rho_{th}(\beta)$

Barthel, NJP 2013
Karrasch et al NJP 2013

alternative method: METTS

White, PRL 2009
Binder, Barthel, PRB 2015

thermal properties with MPO

Scan parameters; perform extrapolations for each β

m/g chiral condensate as a function of temperature, in the continuum $x \to \infty$

$x \to \infty$

x $x \in [9,\ 1024]$

finite-size

N $N \propto \sqrt{x}$ (up to ~800)

extrapolation

δ sufficiently small for resolution

convergence

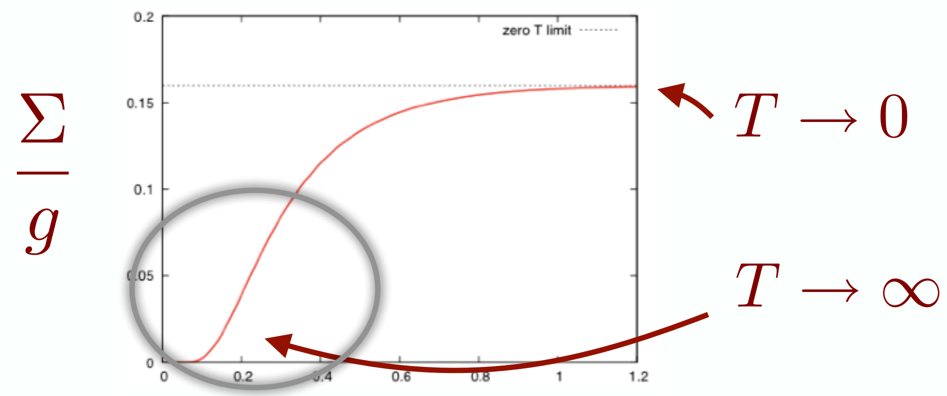
D $D \in [80,\ 160]$

PRD 92, 034519 (2015); PRD93, 094512 (2016)
Buyens PRD 94, 085018 (2016)

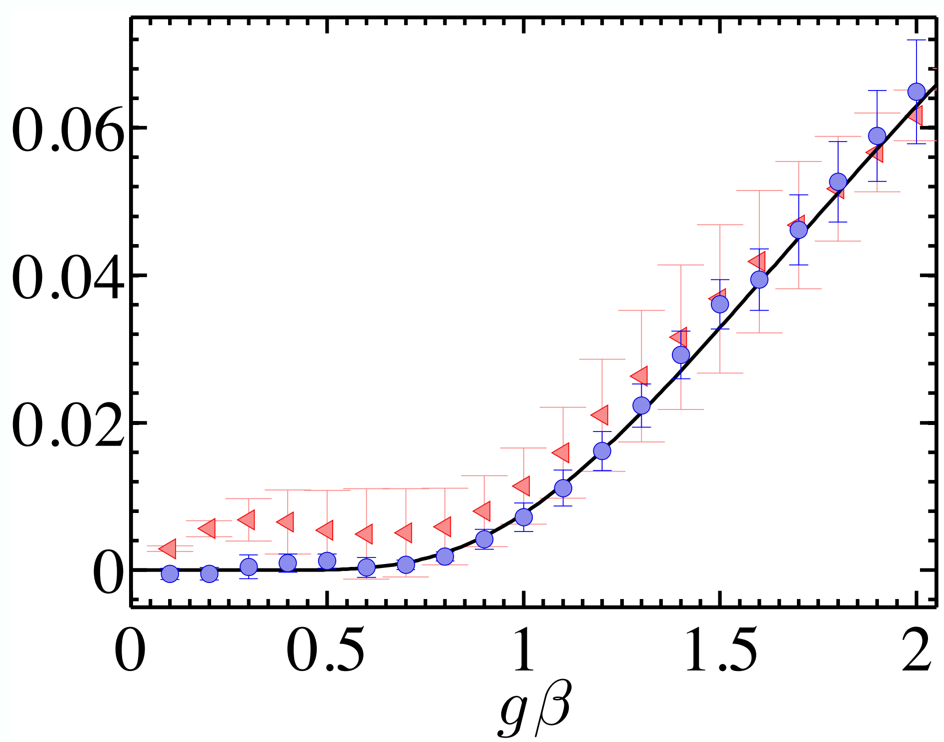
thermal properties schwinger

chiral condensate at finite T: analytical for $m/g=0$

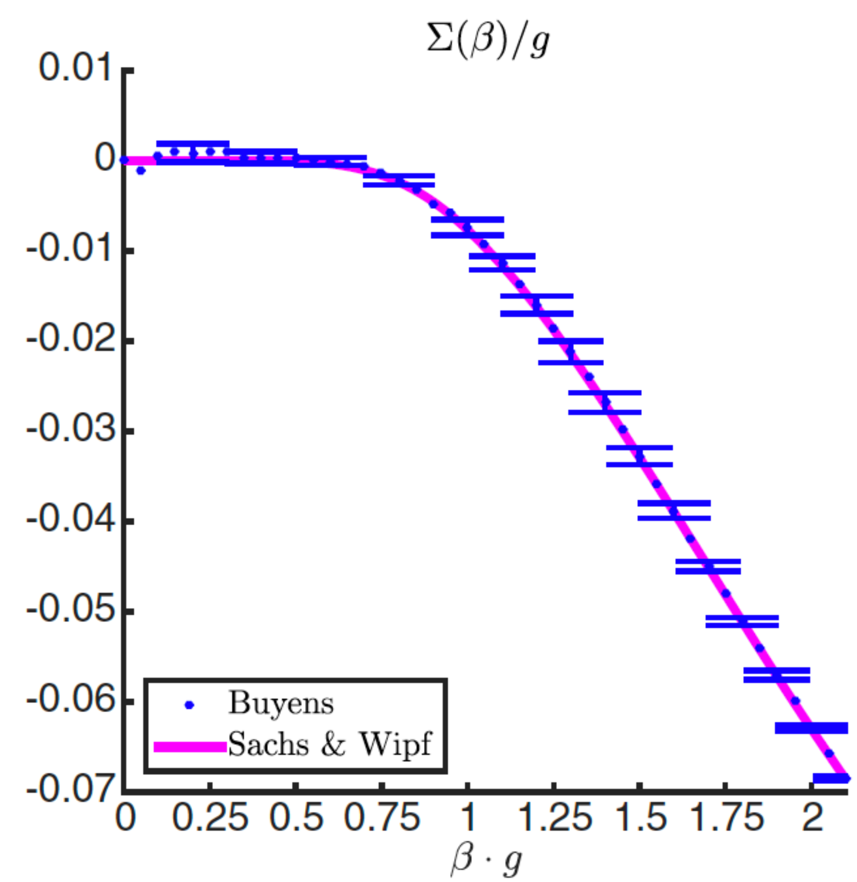
Sachs, Wipf 92



smooth restoration of chiral symmetry



PRD 92, 034519 (2015); PRD 93, 094512 (2016)



Buyens PRD 94, 085018 (2016)

finite density

spectrum

entropy

1+1D RESULTS

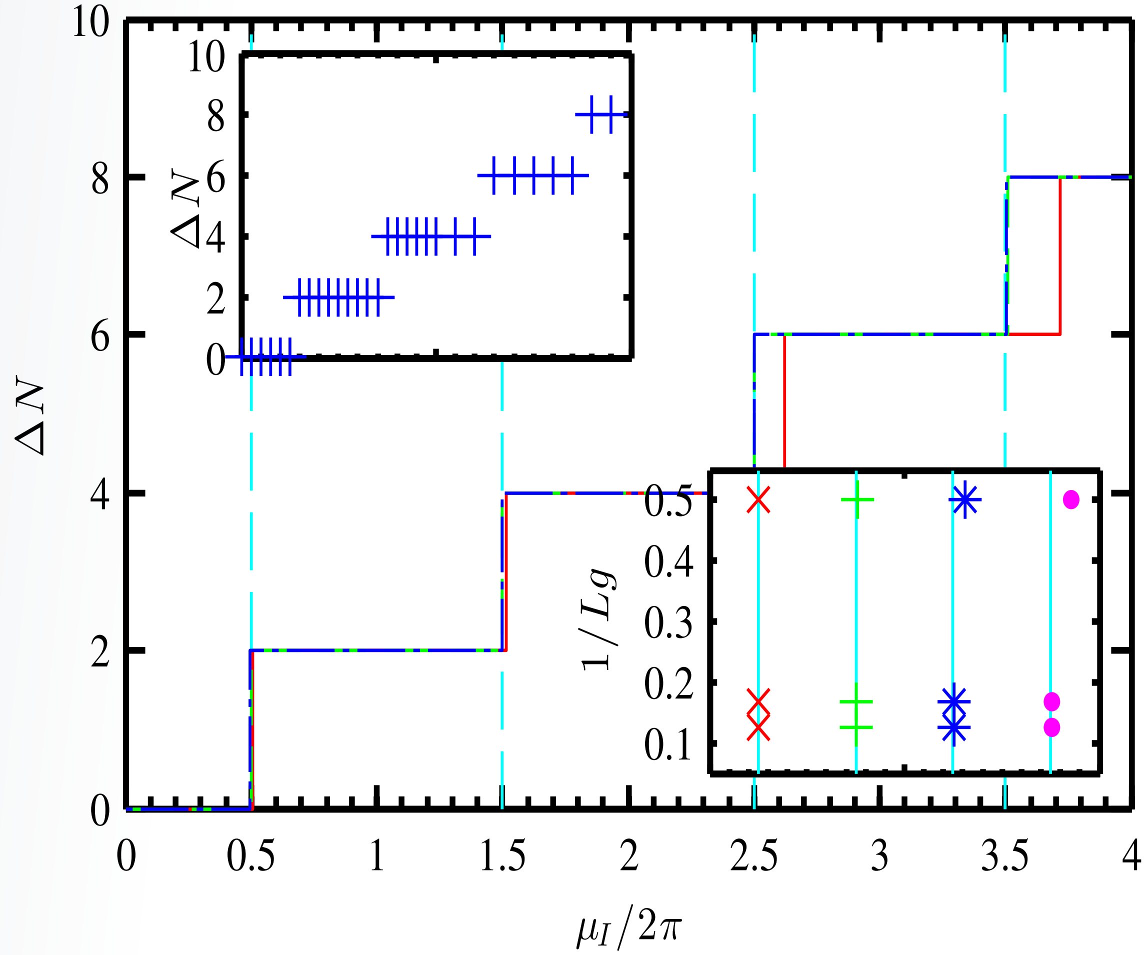
thermal equilibrium

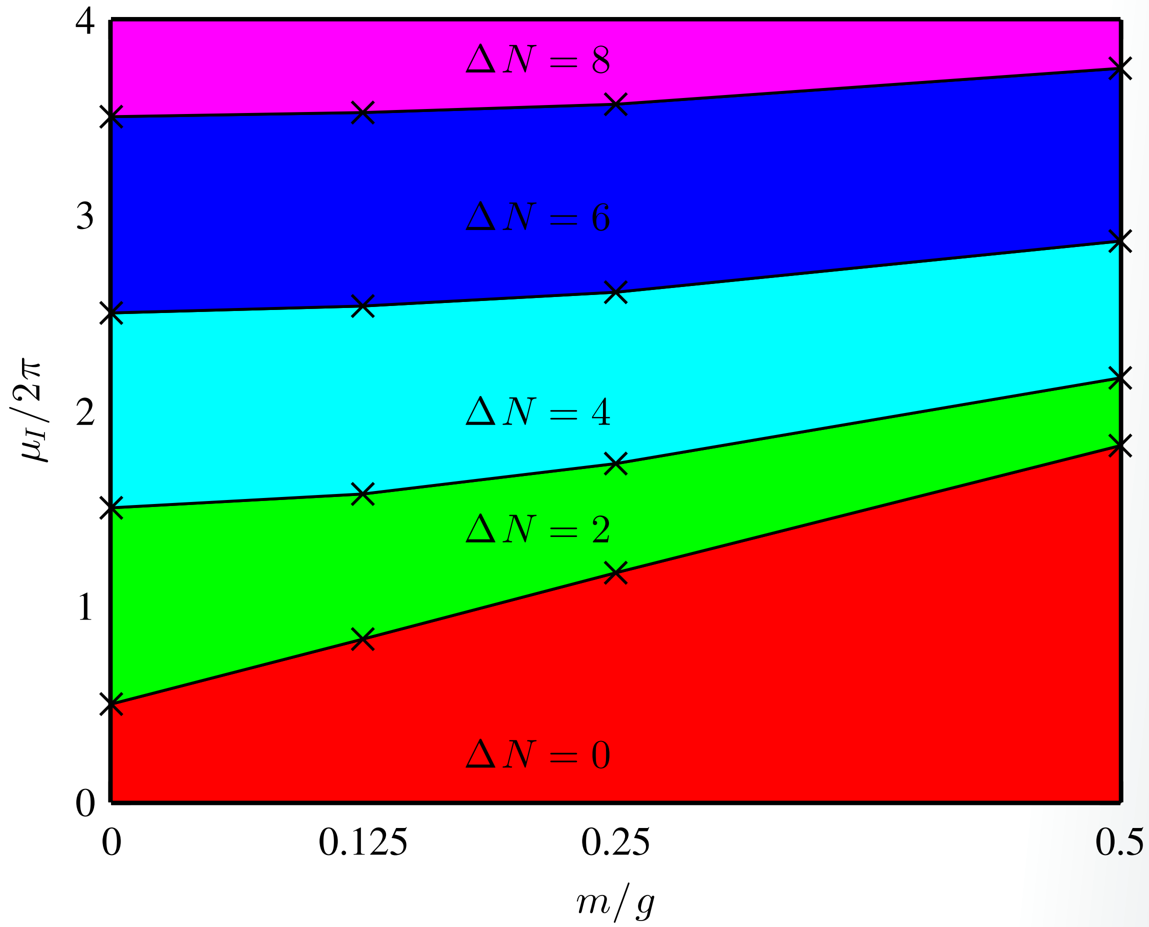
time evolution

finite density with MPS

Several fermion flavors, different chemical potentials

- ground state density changes (first order PT)
- Montecarlo has sign problem





S. Kuehn et al, PRL118 (2017) 071601

1+1D RESULTS

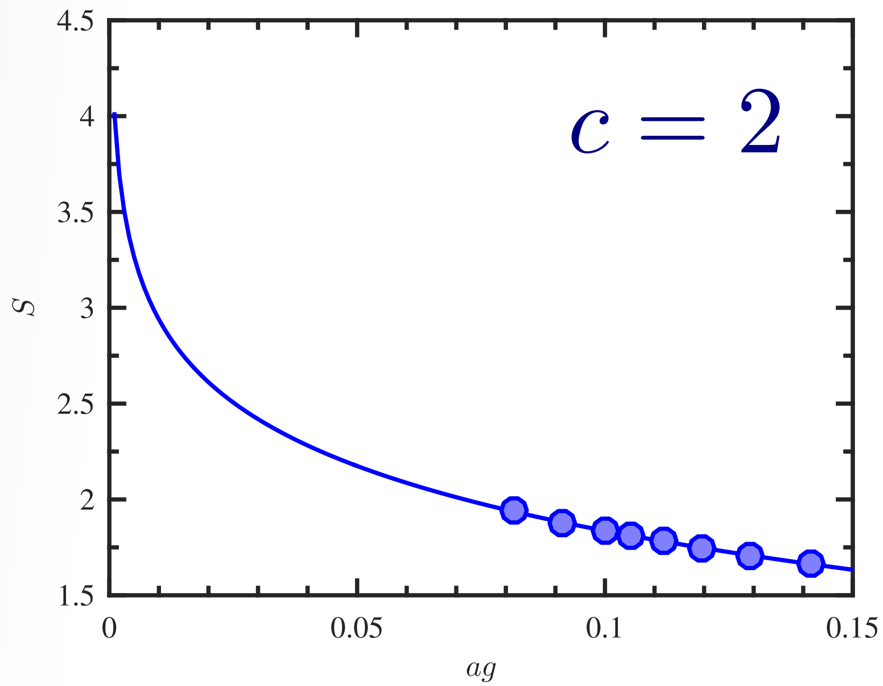
- finite density
- spectrum
- entropy
- thermal equilibrium
- time evolution

Entropy can be efficiently computed from MPS state

gauge constraints not purely local ⇒ not all entropy physical

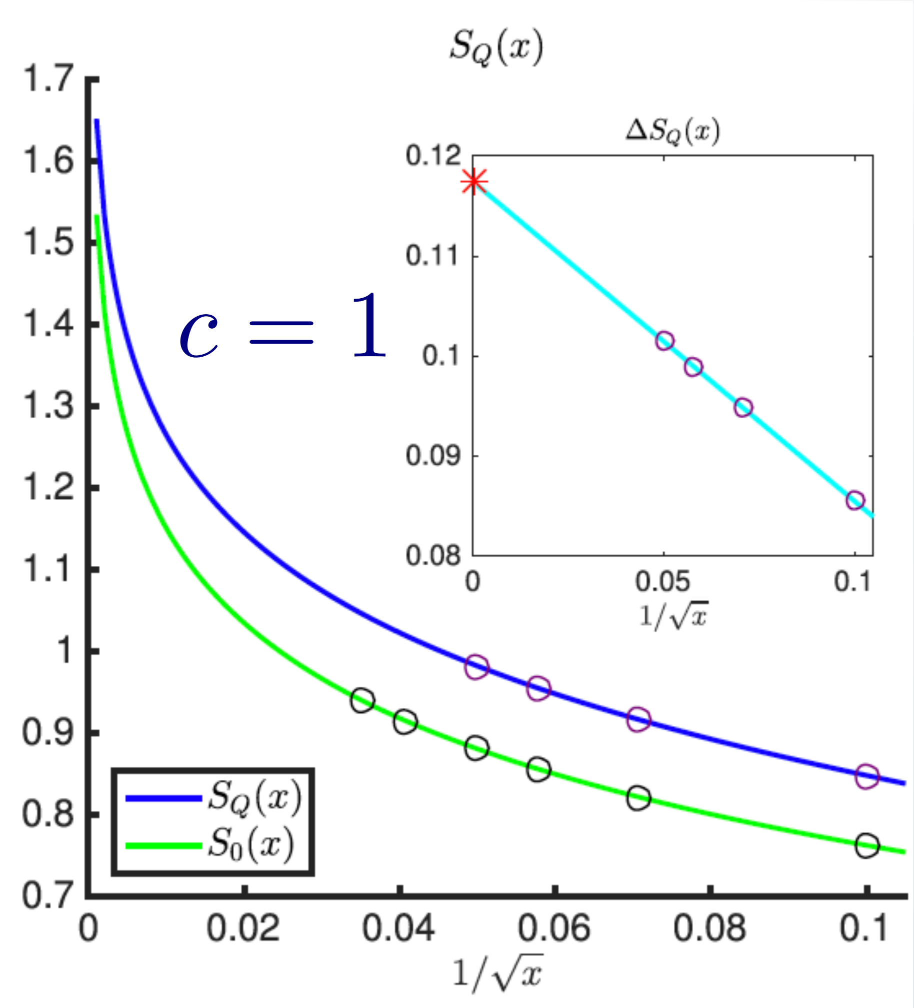
Casini et al 2014; Gosh et al JHEP 2015
Soni, Trivedi JHEP 2016; van Acoleyen et al PRL 2016

SU(2)



PRX7, 041046 (2017)

earlier for Schwinger



Buyens PRX6, 041040 (2016)

divergence in the continuum limit

$$S \propto \frac{c}{6} \log_2 \frac{\xi}{a}$$

Calabrese, Cardy JStatMech 2004

1+1D RESULTS

finite density

spectrum

entropy

thermal equilibrium

time evolution

basic evolution algorithms

Reliable for moderate times, or in some setups

Useful for quantum simulation

S. Kühn et al., Phys. Rev. A 90, 042305 (2014)
S. Kühn et al., JHEP 07 (2015) 130
Buyens et al., PRL 2014; PRX 2016
Rico et al., PRL 2014; NJP 2014; PRX 2016

Many results for finite lattices/discrete models (scattering, vacuum decay…)

quench scenario

Interacting vacuum (TI)

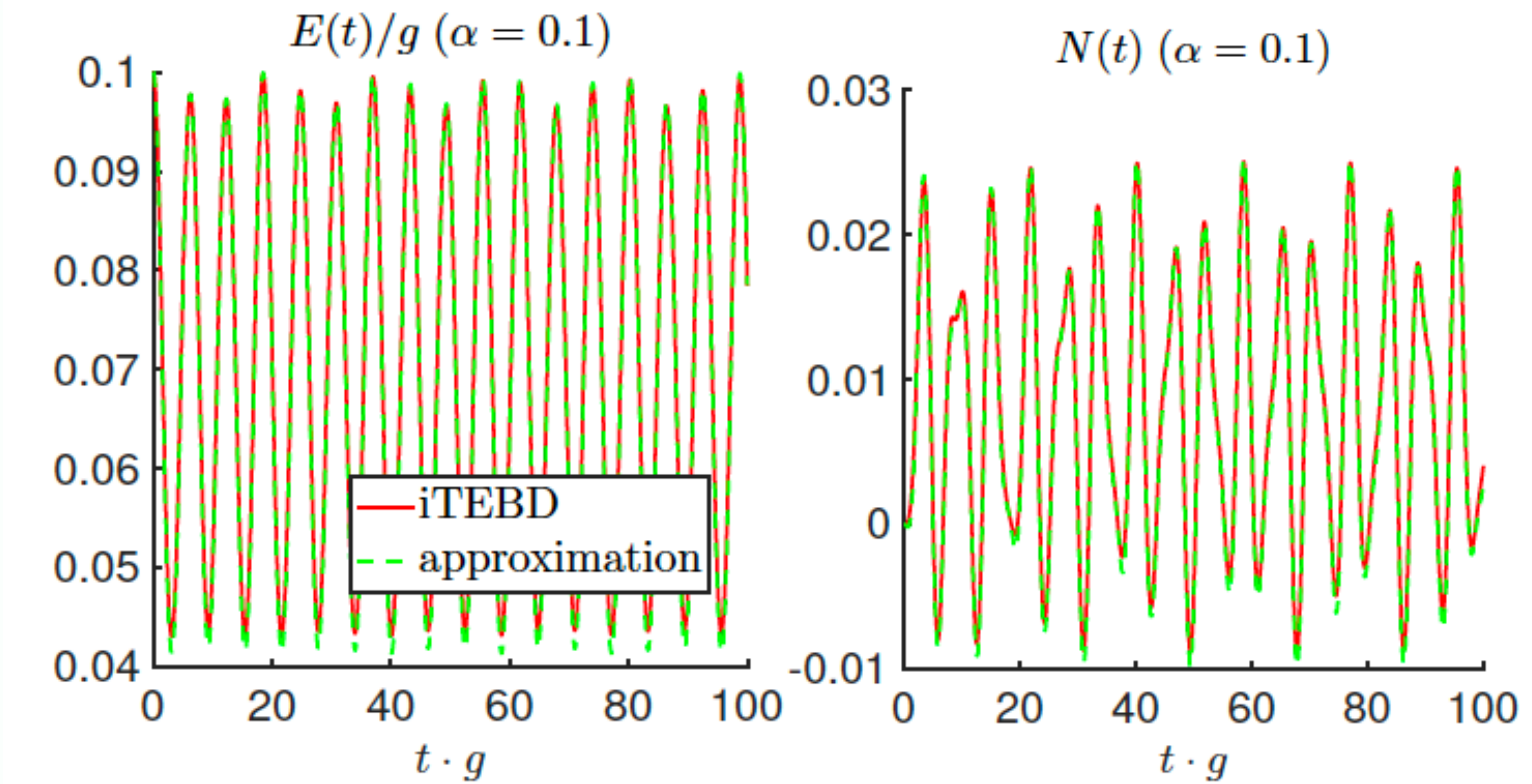
switch on background electric field α

$\Rightarrow$ pair production

iTEBD evolution

B. Buyens et al, PRD96, 114501 (2017)

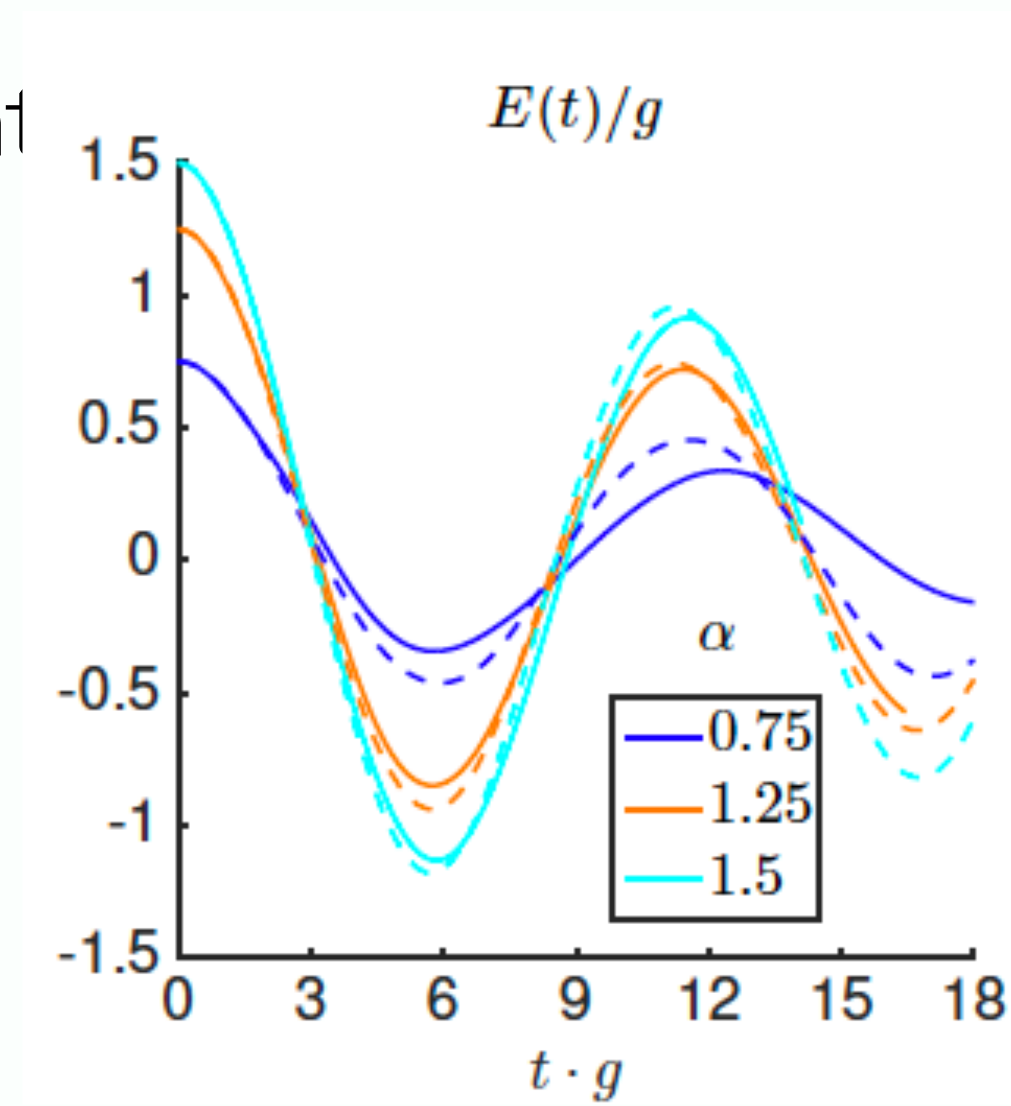
quench scenario

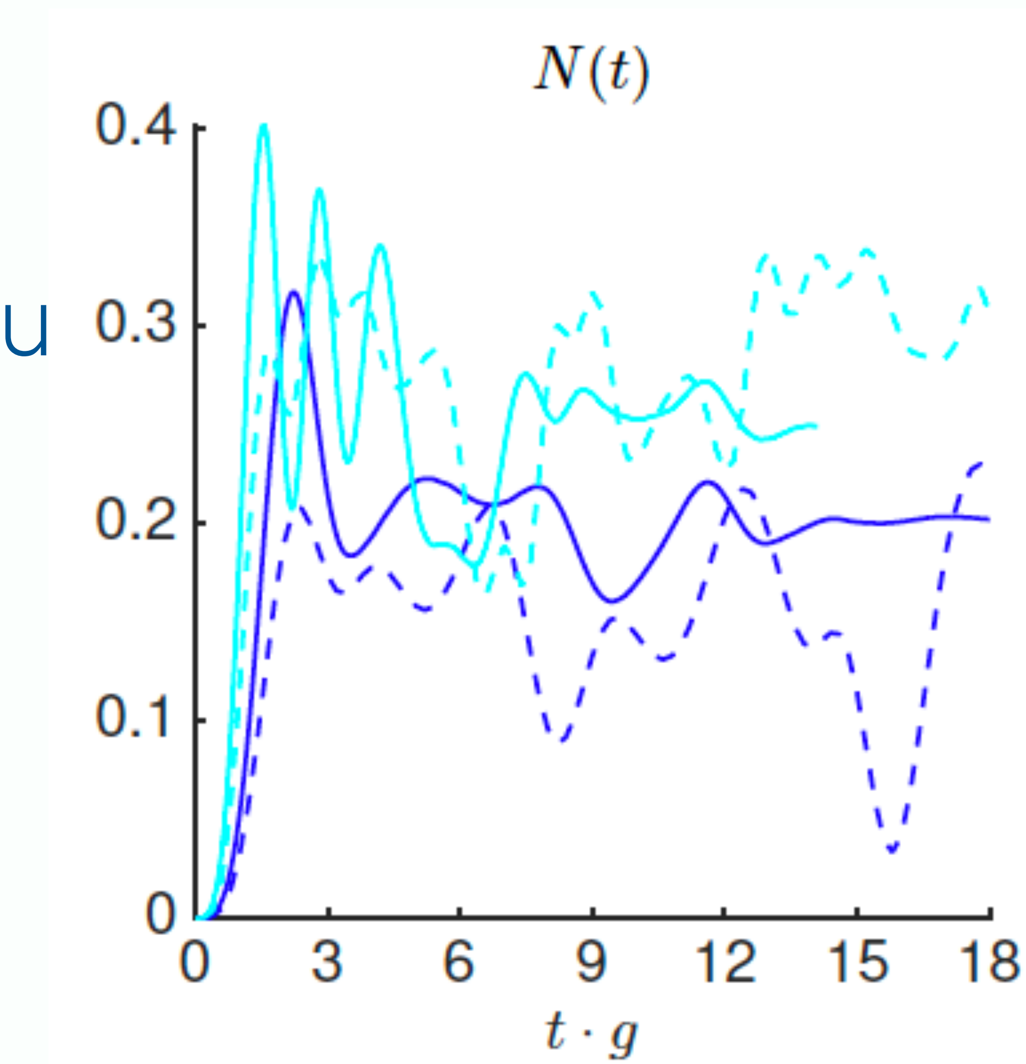


weak field

B. Buyens et al, PRD96, 114501 (2017)

quench scenario





strong field

Not a continuum extrapolation, but
results near the continuum limit

B. Buyens et al, PRD96, 114501 (2017)

PDF in Schwinger model

LATTICE'24 arXiv:2409.16996
arXiv:2504.07508

Krzysztof Cichy
(Poznan)

C.-J. David Lin
(NYCU)

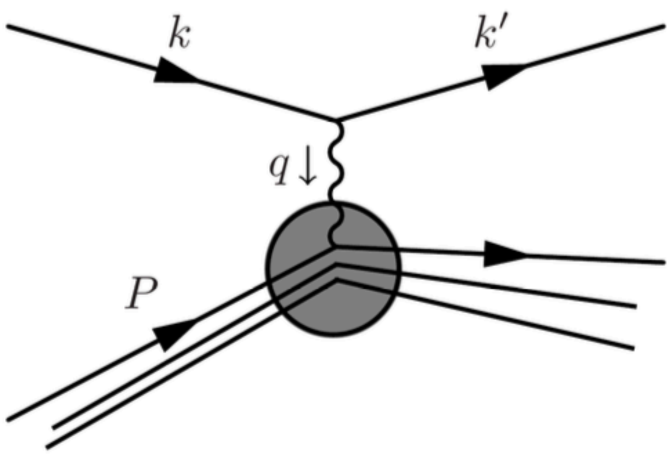
Manuel Schneider
(NYCU)

parton distribution functions

internal structure of hadrons (e.g. probed in DIS)

probability to find a constituent carrying a certain momentum fraction

scattering amplitude factorizes: perturbative part and PDF



[Schwartz 2014]

$$f_\Psi(\xi) = \int \frac{dz^-}{4\pi} e^{-i\xi P^+ z^-} \langle P|\overline{\Psi}(z^-) W_{(z^-,0)} \gamma^+ \Psi(0)|P\rangle$$

integral along light front direction

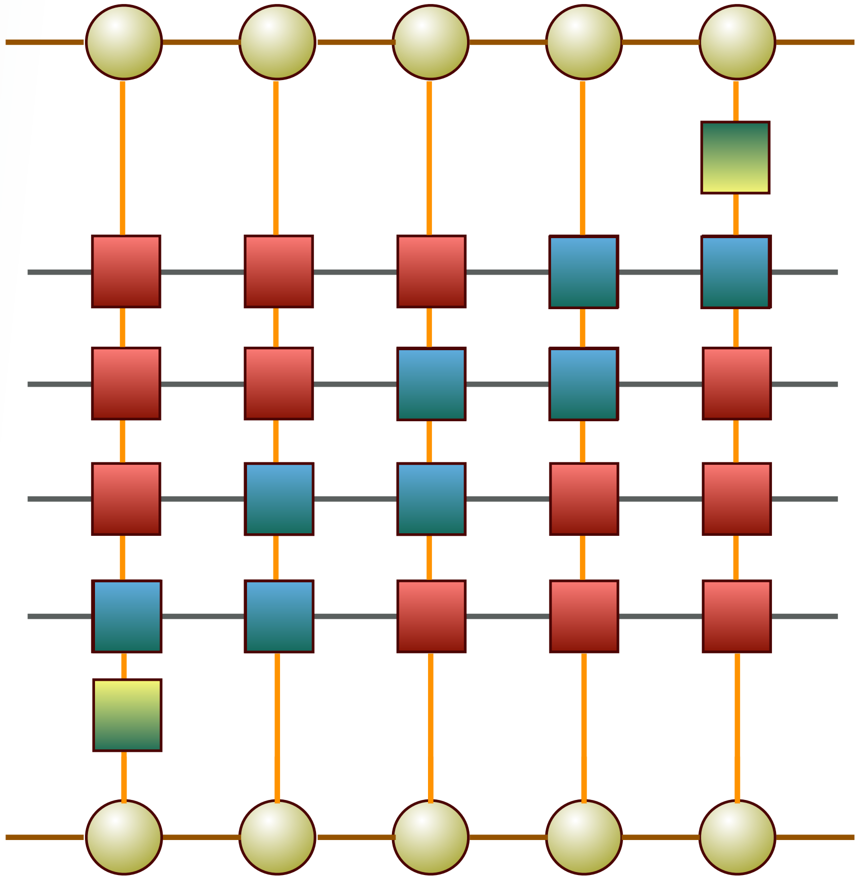
in LQCD only indirect access (Euclidean time)

fermionic PDF

$$\langle P|\overline{\Psi}(z^-)W_{(z^-,0)}\gamma^+\Psi(0)|P\rangle$$

lattice version (staggered fermions)

$$\langle h|e^{iH\frac{\Delta t}{2}t}\prod_{k<\Delta z}(i\sigma_k^z)\,\sigma_{\Delta z}^+ e^{-iH_{\delta z-1}\Delta t}\dots e^{-iH_3\Delta t}e^{-iH_1\Delta t}\prod_{k'<0}(-i\sigma_{k'}^z)\,\sigma_0^-|h\rangle$$



Jordan-Wigner spin mapping

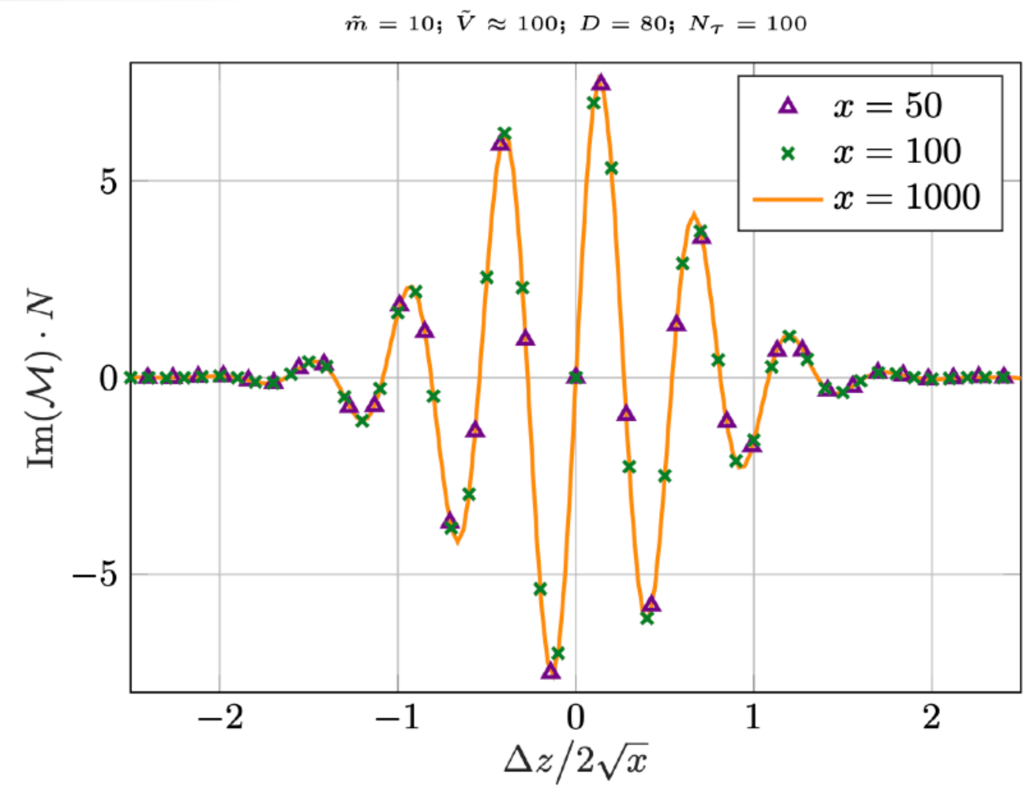
eigenstate

Trotterized evolution

zigzag light cone

MCB, Cichy, Lin, Schneider, arXiv:2504.07508
preliminary results presented in LATTICE'24 arXiv:2409.16996

fermionic PDF

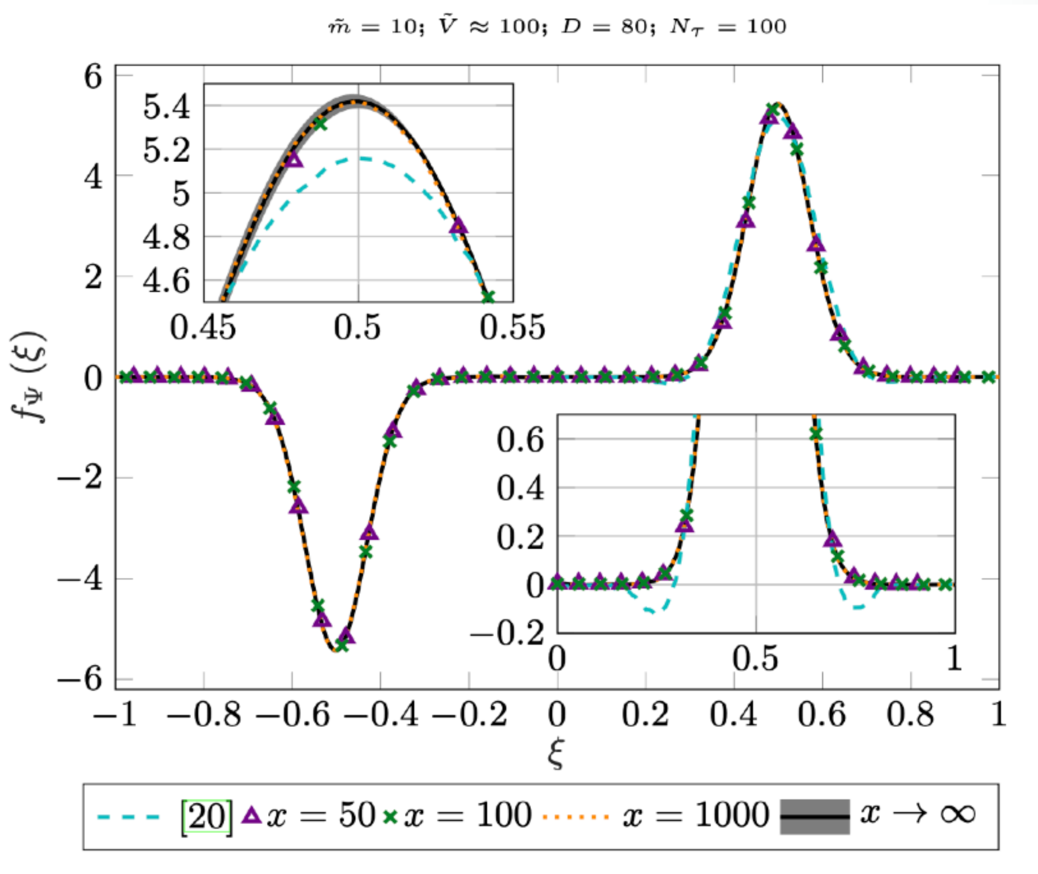


basic matrix element

$$x = \frac{1}{(ga)^2}$$

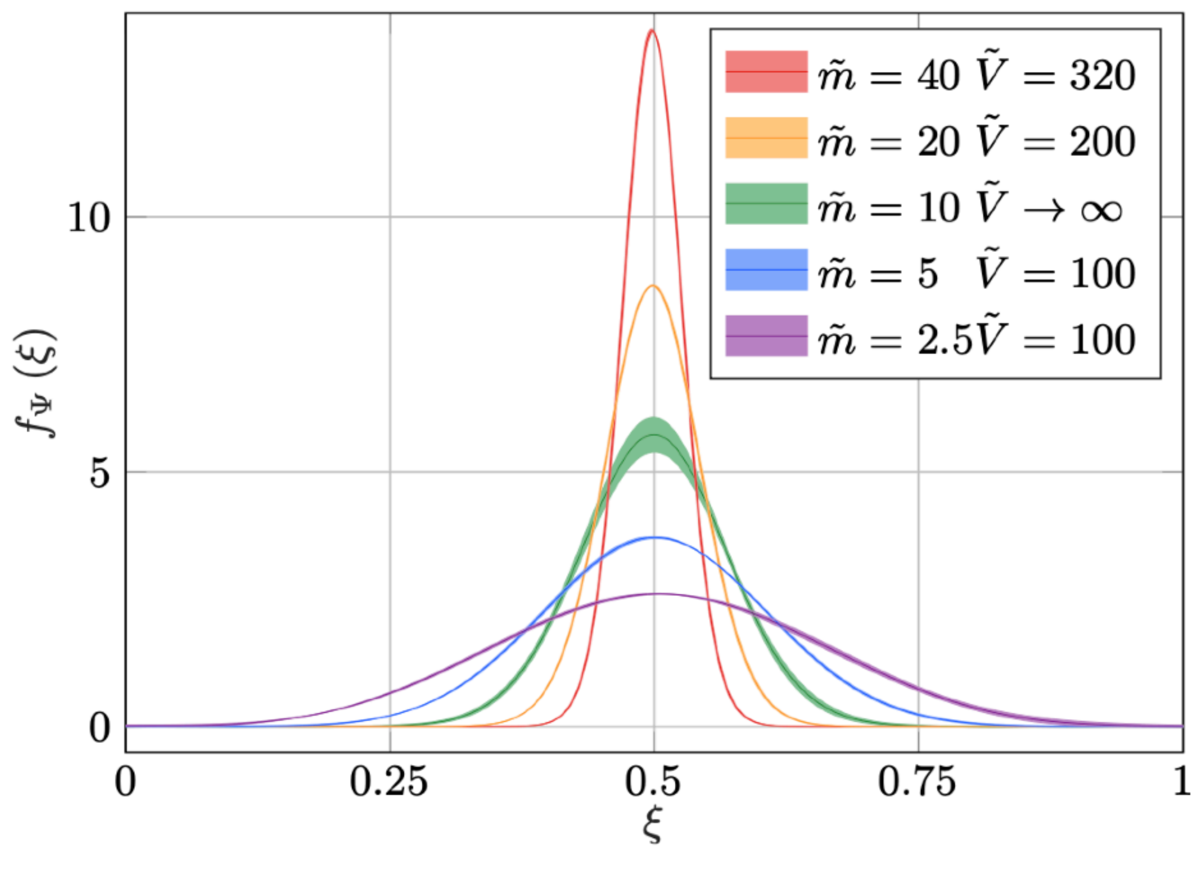
convergence with very moderate size of tensors

PDF (Fourier transform)



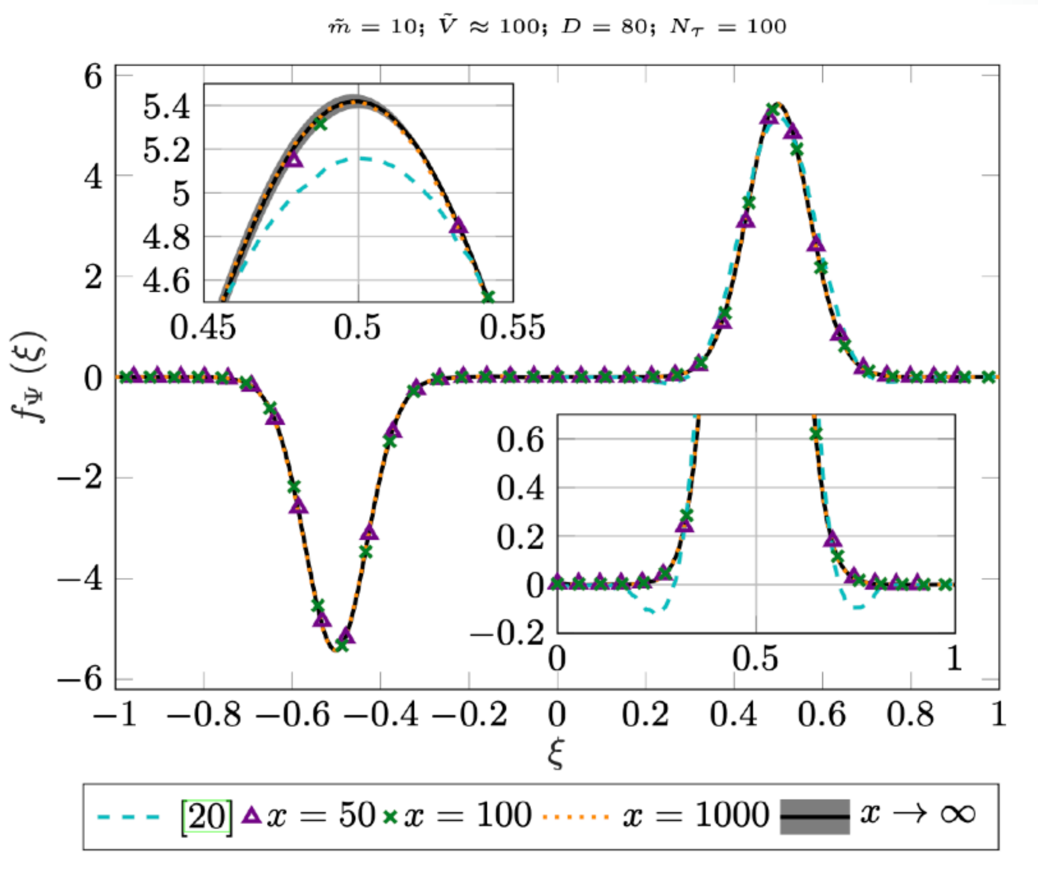
MCB, Cichy, Lin, Schneider, arXiv:2504.07508
preliminary results presented in LATTICE'24 arXiv:2409.16996

fermionic PDF



different fermion mass, continuum

PDF (Fourier transform)



convergence with very moderate size of tensors

MCB, Cichy, Lin, Schneider, arXiv:2504.07508
preliminary results presented in LATTICE'24 arXiv:2409.16996

beyond Schwinger

non-Abelian 1D models

SU(2) model

SU(2) matrices

$$H = \frac{1}{2a}\sum_{n,b,c}\left(\phi_n^{b\,\dagger} U_n^{bc}\phi_{n+1}^c + \text{h.c.}\right) + m\sum_{n,b}(-1)^n \phi_n^{b\,\dagger}\phi_n^b + \frac{ag^2}{2}\sum_n \mathbf{J}_n^2$$

Kogut-Susskind '75

plus non-Abelian Gauss' Law $G_n^\nu|\Psi_{\text{phys}}\rangle = 0$

$$G_n^\nu = L_n^\nu - R_{n-1}^\nu - Q_n^\nu,$$

$$Q_n^\nu = \sum_{bc}\frac{1}{2}\phi_n^{b\,\dagger}\sigma_{bc}^\nu\phi_n^c$$

a proper tensor product basis for MPS…

$$|\ldots n^1 n^2\ j\ell\ell'\ n^1 n^2 \ldots\rangle$$

SU(2) model

SU(2) matrices

$$H = \frac{1}{2a}\sum_{n,b,c}\left(\phi_n^{b\,\dagger} U_n^{bc}\phi_{n+1}^c + \mathrm{h.c.}\right) + m\sum_{n,b}(-1)^n \phi_n^{b\,\dagger}\phi_n^b + \frac{ag^2}{2}\sum_n \mathbf{J}_n^2$$

Kogut-Susskind '75

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$$Q_n^\nu = \sum_{bc}\frac{1}{2}\phi_n^{b\,\dagger}\sigma_{bc}^\nu\phi_n^c$$

a proper tensor product basis for MPS…

$$|\ldots n^1 n^2 \; j\ell\ell' \; n^1 n^2 \ldots\rangle$$

truncate gauge dof

integrate out gauge

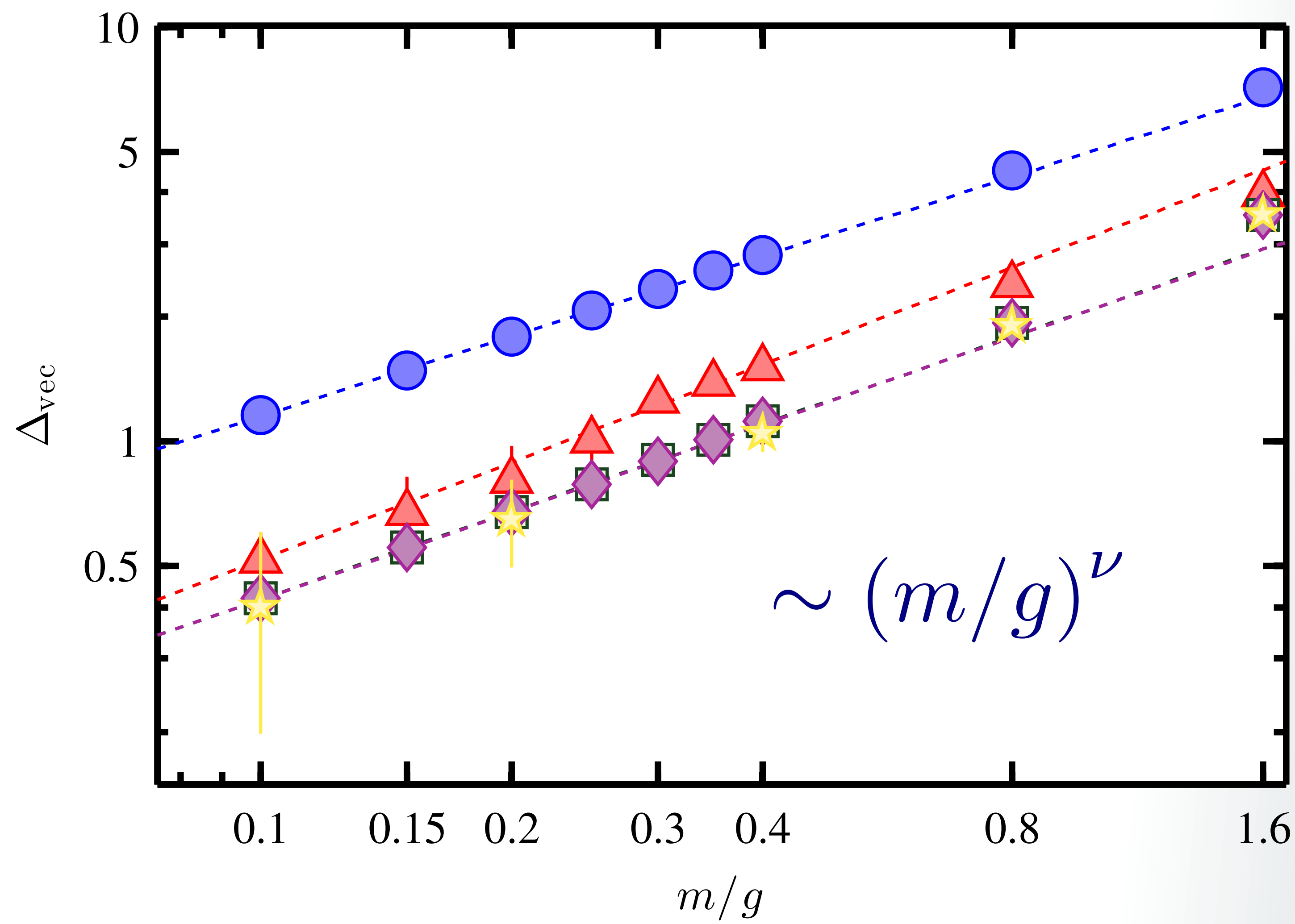
also P. Sala et al, PRB 98, 034505 (2018)

SU(2) model

e.g. vector meson mass gap

critical exponent

$j_{\max} = 1/2$	*~0.639*
$j_{\max} = 1$	0.781(93)(75)
$j_{\max} = 3/2$	0.700(29)(11)
$j_{\max} = 2$	0.700(29)(11)



MCB, Cichy, Cirac, Jansen, Kühn arXiv:1707.06434

beyond ID

PEPS for 2+1 D LGT

some results with other TNS

Tagliacozzo, Vidal PRB 2011
Felser et al. PRX 2020
Magnifico et al. Nat. Comm. 2022

explicitly gauge invariant PEPS

Tagliacozzo et al PRX 2014
Haegeman et al PRX 2014
Zohar et al Ann Phys 2015

restricted ansatz calculations

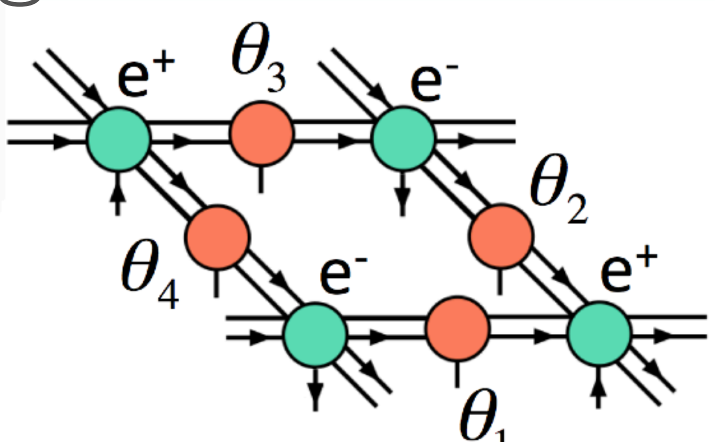
also Gaussian PEPS

Zohar, Cirac PRD 2018
Emonts et. al., PRD 102, 074501 (2020)

standard PEPS toolbox contains all ingredients for full variational computation

computational cost, required D

plaquette terms



Zapp, Orús PRD 2017

PEPS for 2+1 D LGT

no continuum extrapolations yet

results with iPEPS imaginary time evolution

use it to study the phase diagram of Z_3 pure LGT

D. Robaina, MCB, J. I. Cirac, PRL126, 050401 (2021)

recent results with finite PEPS

Wu, Lio, arXiv:2503.20566

another approach

restricted ansatzes

MERA: study of Z_2 2+1D model

Tagliacozzo, Vidal PRB83, 115127 (2011)

Tree TN: U(1) 2+1D model at finite density

Felser et al. PRX10, 041040 (2020)

gauged Gaussian PEPS: Z3 on 2+1D finite PBC lattice

Emonts, MCB, Cirac, Zohar, PRD102, 074501 (2020)

Trees: first 3+1D simulations for U(1) LGT

Magnifico et al. Nat. Comm. 12, 3600 (2021)

related topics…

Related, but I didn't talk about…

TNS for other field theories

Thirring model, PRD100, 094504 (2019)
Thermal QFT dynamics, PRR2, 033301 (2020)

continuous TNS for QFT

Verstraete, Cirac PRL104, 190405 (2010)
Tilloy, Cirac PRX9, 021040 (2019)

proposals for quantum simulation of LGT with ultracold atoms

Zohar et al. PRL 2010, 2012 ,
Tagliacozzo et al., Nat. Comm. 2013
Banerjee et al., PRL 2012

Rico et al. PRL 2014
Pichler et al, PRX 2016
Zohar, Burrello, PRD 2015

DFG TRR 360

to conclude…

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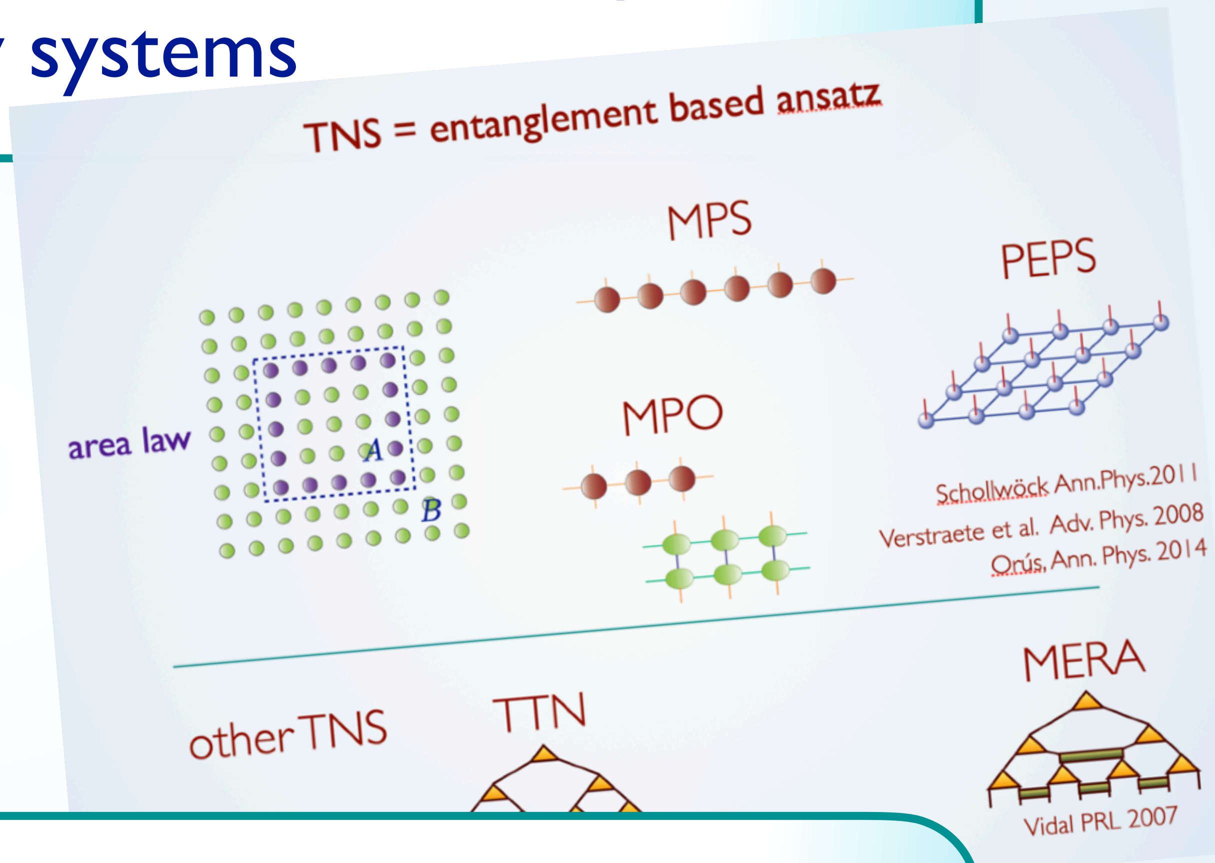
T-NiSQ

TNS: quantum inspired methods to describe quantum many-body systems

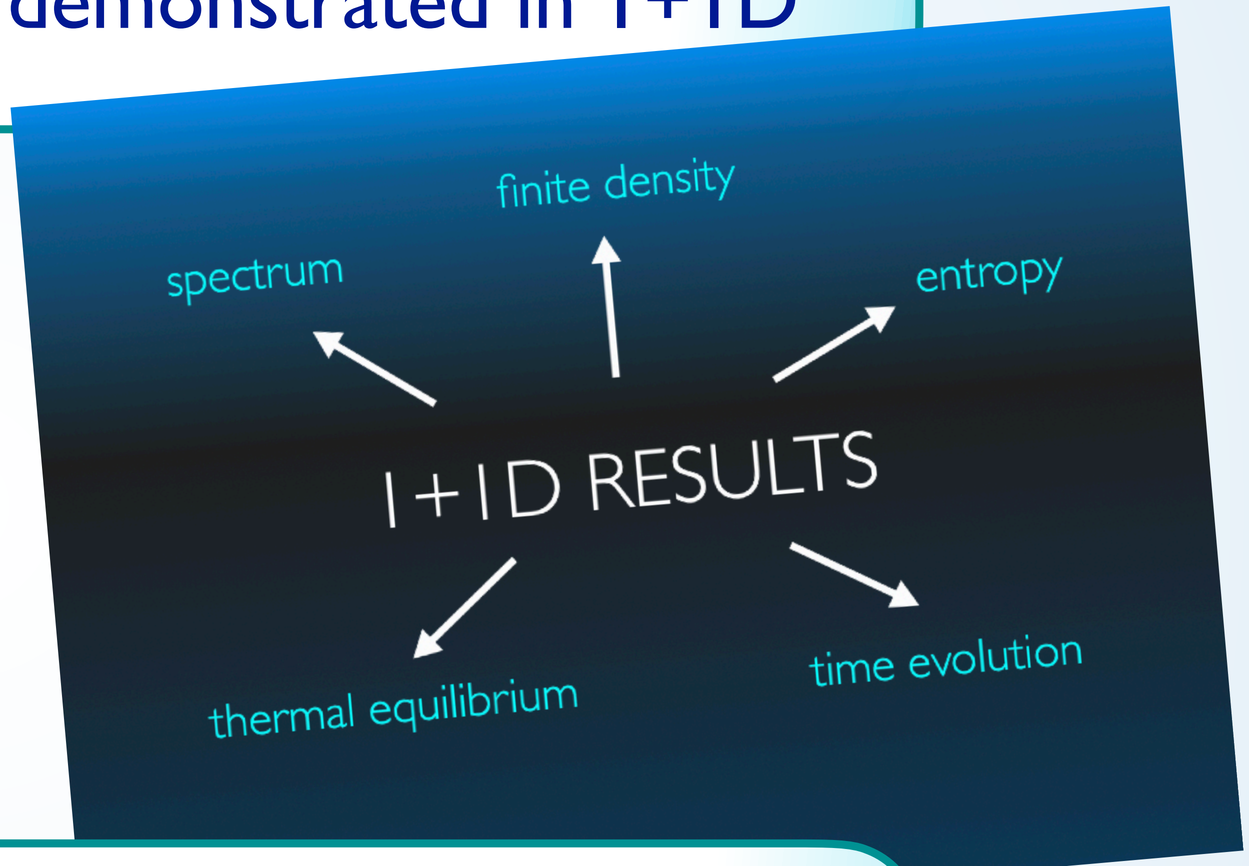


efficient algorithms can be also used to find thermal states, evolve master equations and more

simulating time evolution/arbitrary quantum circuits requires truncation ⇒ limited!

to conclude…

feasibility of TNS for LQFT demonstrated in 1+1D



strong connections to quantum simulation proposals

already results with PEPS for 2+1D LGT

with restricted ansatzes 3+1D

DFG TRR 360

DFG FOR 5522

T-NiSQ

THANKS!

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Review on novel methods for lattice gauge theories, MCB, K. Cichy, Reports on Progress in Physics 83, 024401 (2020),arXiv:1910.00257

Simulating lattice gauge theories within quantum technologies, MCB, R. Blatt, Catani et al., Eur. Phys. J. D 74, 165 (2020), arXiv:1911.00003

Quantum Simulation for High-Energy Physics, C. Bauer, Z. Davoudi et al., PRX Quantum 4, 027001 (2023).

Cold-atom quantum simulators of gauge theories, J. C. Halimeh, M. Aidelsburger, F. Grusdt, P. Hauke, and B. Yang, arXiv:2310.12201

…